# Log - Frequency Spectrogram

**STFT**

$$\underset{\text{Fourier coefficient}}{X(m,k)} = \sum_{n=0}^{N-1} x(n + \overset{\text{frame index}}{m}\underset{\text{hop size}}{H})\, \overset{\text{window}}{w(n)} \exp(-2\pi i kn/\overset{\text{window size}}{N})$$

Window $w : [0:N-1] \to \mathbb{R}$

$$F_{coef}(k) = k \cdot \frac{F_s}{N}, \quad T_{coef}(m) = m \cdot \frac{H}{F_s}$$

**Pitch binning:**

$$F_{pitch}(p) = 2^{(p-69)/12}\ 440\ Hz, \quad p \in [0:127]$$

$$P(p) = \{ k : F_{pitch}(p-0.5) \leq F_{coef}(k) \leq F_{pitch}(p+0.5)\}$$

Bandwidth $BW(p) = \underbrace{F_{pitch}(p+0.5) - F_{pitch}(p-0.5)}_{\text{cut-off frequencies}}$

$$Y_{LF}(m,p) = \sum_{k \in P(p)} |X(m,k)|^2$$

**Problem:** Bandwidth becomes smaller for decreasing $p$

Frequency resolution is linear when using STFT

**Example:**

$$\left.\begin{array}{l} F_s = 44100\ Hz \\ N = 4096,\ H = 2048 \end{array}\right\} \text{Freq. res.} = \frac{F_s}{N} \approx 10.8\ Hz$$

$p = 69$:

$F_{pitch}(p-0.5) = 427.5\ Hz$

$F_{pitch}(p+0.5) = 452,9\ Hz$

$BW(p) = 25.4$

$F_{coef}(39) = 419,9$

$\quad\text{"}\ (40) = 430,7$

$\quad\text{"}\ (41) = 441,4$

$\quad\text{"}\ (42) = 452,2$

$\quad\text{"}\ (43) = 463,0$

$P(p) = \{40, 41, 42\}$