INTERNATIONAL AUDIO LABORATORIES ERLANGEN

AUDIO LABS

Hochschule für Musik Karlsruhe

Blockvorlesung

Advanced Audio-Based Music Processing

2. Music Theory Basics

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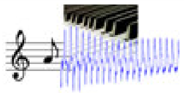
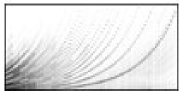
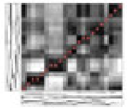
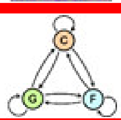
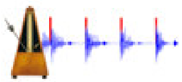
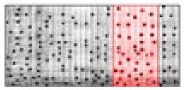
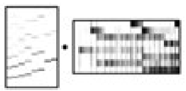
Fraunhofer IIS

Book: Fundamentals of Music Processing

Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
483 p., 249 illus., hardcover
ISBN: 978-3-319-21944-8
Springer, 2015

Accompanying website:
www.music-processing.de

Book: Fundamentals of Music Processing

Chapter		Music Processing Scenario
1		Music Representations
2		Fourier Analysis of Signals
3		Music Synchronization
4		Music Structure Analysis
5		Chord Recognition
6		Tempo and Beat Tracking
7		Content-Based Audio Retrieval
8		Musically Informed Audio Decomposition

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5.1 Basic Theory of Harmony

Dissertation: Tonality-Based Style Analysis

Christof Weiß
Computational Methods for Tonality-Based Style Analysis of Classical Music Audio Recordings
PhD thesis, Ilmenau University of Technology, 2017
https://www.db-thueringen.de/receive/dbt_mods_00032890

Chapter 2: Musicological Foundations

Music Theory Basics

Overview

Part I:

- Pitches and Intervals
- Tuning and Enharmonic Equivalence
- Scales

Part II:

- Chords
- Keys and the Circle of Fifths

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Tone and Pitch

Harmonic series | overtone series

Mix

Harmonics: Frequency = integer multiples of fundamental frequency

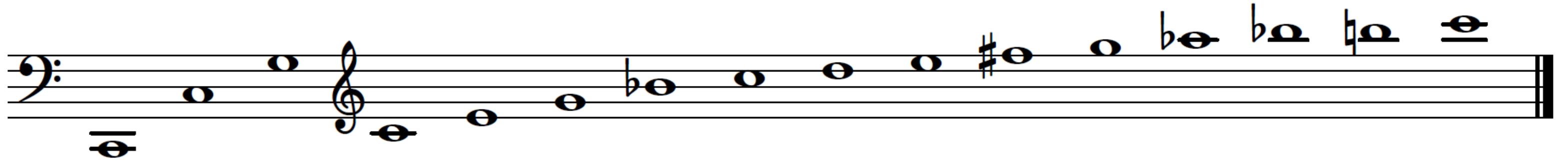
Deviation in cents: +2 -14 +2 -31 +4 -14 -49 +2 +41 -31 -12

MIDI: Frequency = fundamental frequency of MIDI pitch

Stereo file: Harmonics vs. MIDI

Tone and Pitch

Harmonic series | overtone series



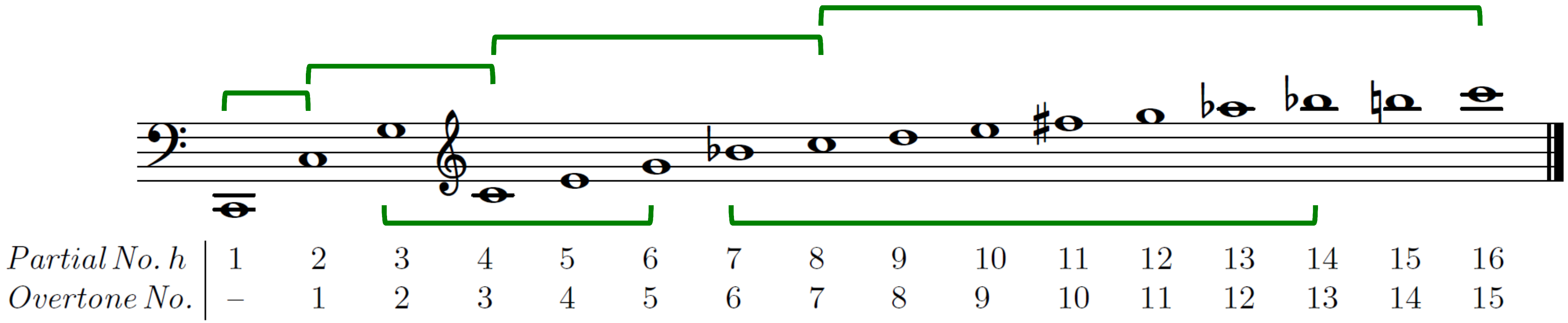
Partial No. h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Overtone No.	–	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$$f_{\text{Part}}(h) := h \cdot f_0$$

- Notation: only approximation
- *Mathematical*: harmonics (integer multiples)
- *Physical*: partials/overtones – not the same (inharmonicity)
- Counting of fundamental: harmonics/partials vs. overtones

Intervals

Harmonic series | overtone series



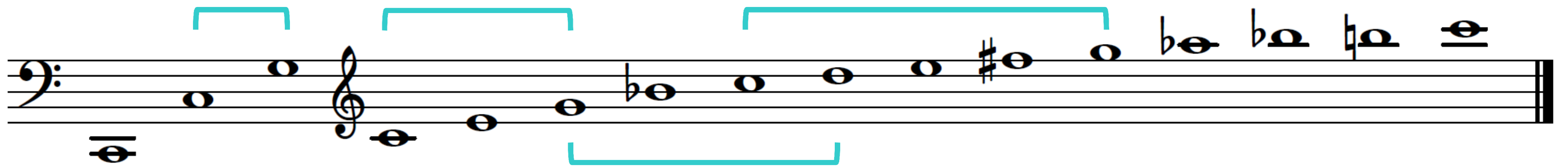
$$f_{\text{Part}}(h) := h \cdot f_0$$

Intervals: harmonic frequency ratios $\dfrac{f^b}{f^a}$

- 2:1 – Octave

Intervals

Harmonic series | overtone series



Partial No. h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Overtone No.	–	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

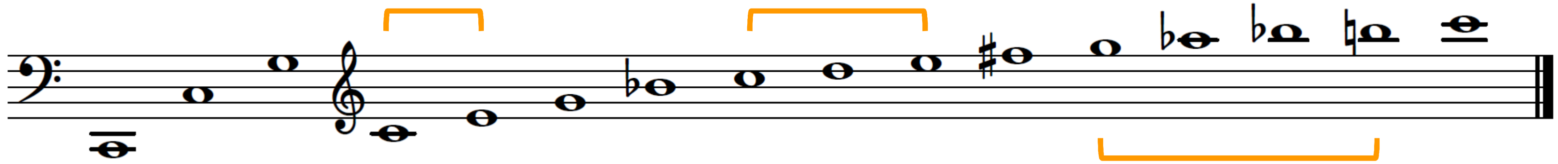
$$f_{\text{Part}}(h) := h \cdot f_0$$

Intervals: harmonic frequency ratios $\dfrac{f^b}{f^a}$

- 2:1 – Octave
- 3:2 – Fifth

Intervals

Harmonic series | overtone series



Partial No. h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Overtone No.	–	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$$f_{\mathrm{Part}}(h) := h \cdot f_0$$

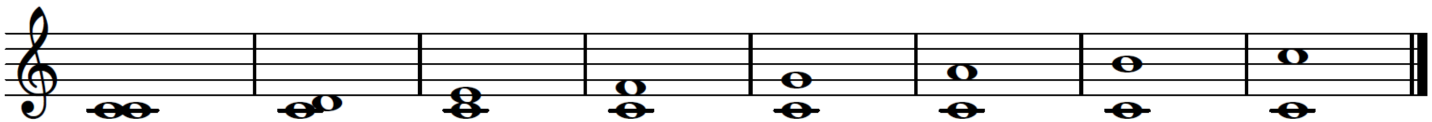
Intervals: harmonic frequency ratios $\dfrac{f^b}{f^a}$

- 2:1 – Octave
- 3:2 – Fifth
- 5:4 – Major Third

With perfect mathematical ratios: **pure intervals**

Intervals

Generic Intervals



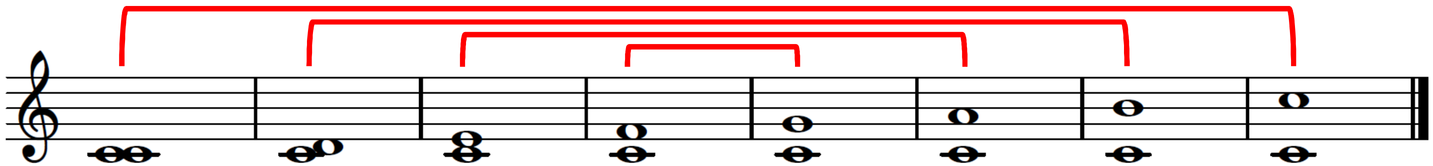
No. of Steps	0	1	2	3	4	5	6	7
Diatonic size	1	2	3	4	5	6	7	8
Generic name	Unison	Second	Third	Fourth	Fifth	Sixth	Seventh	Octave

- **Generic intervals**: only diatonic size (ignoring accidentals)
- Obtained by counting distance in staff lines & spaces

- **Simple intervals**: Up to the octave
- **Compound intervals**: Larger than octave
- *Compound = Simple + Octave(s)*

Intervals

Generic Intervals



No. of Steps	0	1	2	3	4	5	6	7
Diatonic size	1	2	3	4	5	6	7	8
Generic name	Unison	Second	Third	Fourth	Fifth	Sixth	Seventh	Octave

- High similarity of octave-related pitches (same **pitch class**!)
- → high similarity of intervals with octave mutation (**inversion**)
- → **Complementary intervals**
- *Interval + Complementary = Octave*

Intervals

Specific Intervals

Specific name	Perf. unison	Aug. unison	Min. second	Maj. second
Complementary	Perf. octave	Dim. octave	Maj. seventh	Min. seventh
Abbreviation	P1 / P8	+1 / °8	m2 / M7	M2 / m7
Semitone distance Δ	0 / 12	1 / 11	1 / 11	2 / 10

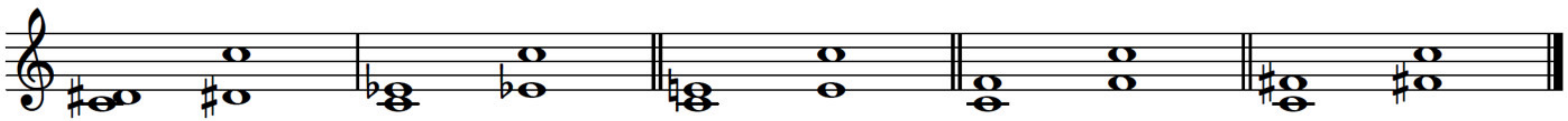
Aug. second	Min. third	Maj. third	Perf. fourth	Aug. fourth
Dim. seventh	Maj. sixth	Min. sixth	Perf. fifth	Dim. fifth
+2 / °7	m3 / M6	M3 / m6	P4 / P5	+4 / °5
3 / 9	3 / 9	4 / 8	5 / 7	6 / 6

- With accidentals: several „versions“ of intervals
- Different „exact size“ (semitone distance)
- Notation: *Specific interval* = *Modifier* + *Generic interval* **(need both!)**
- Complementary: perfect ↔ perfect | major ↔ minor | dimin. ↔ augm.

Intervals

Specific Intervals

Specific name	Perf. unison	Aug. unison	Min. second	Maj. second
Complementary	Perf. octave	Dim. octave	Maj. seventh	Min. seventh
Abbreviation	P1 / P8	+1 / °8	m2 / M7	M2 / m7
Semitone distance Δ	0 / 12	1 / 11	1 / 11	2 / 10

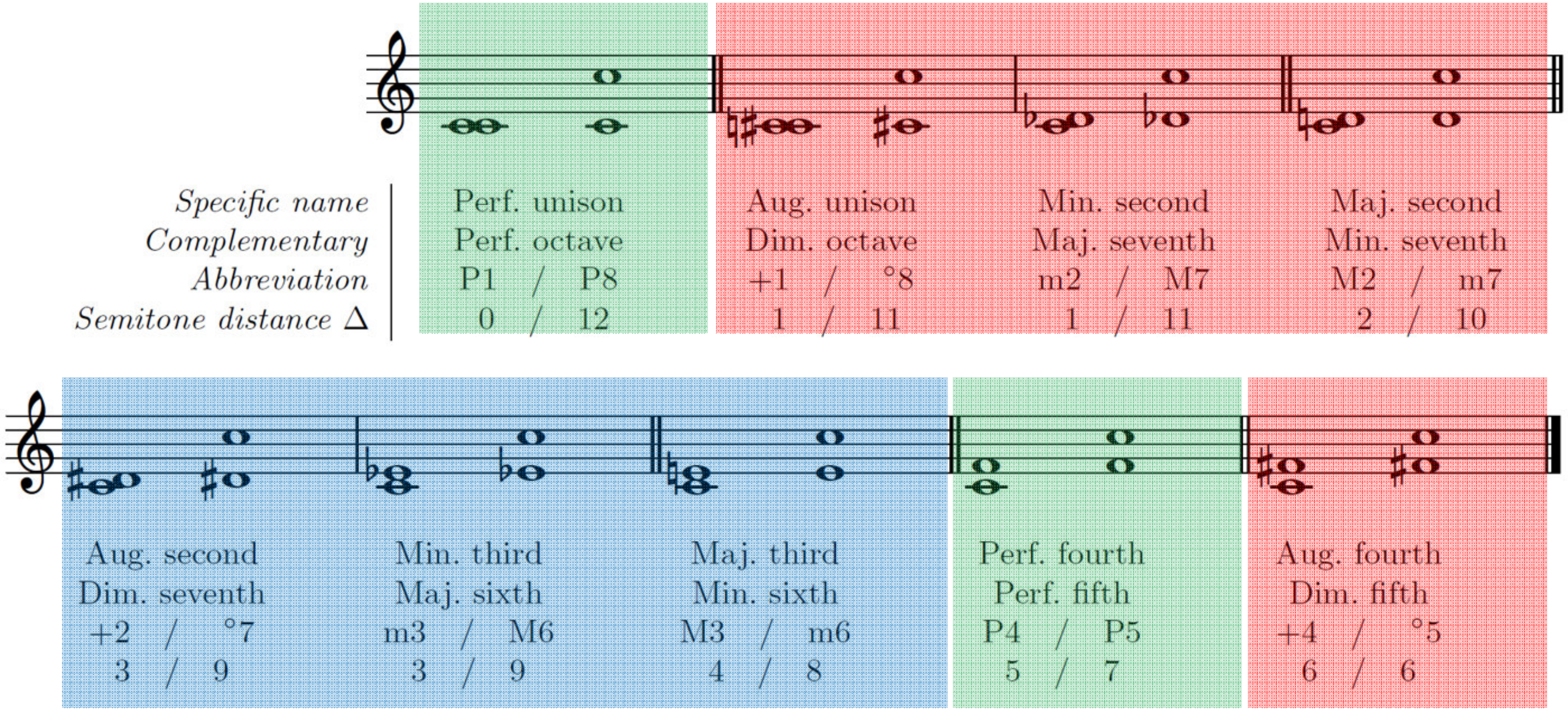


Aug. second	Min. third	Maj. third	Perf. fourth	Aug. fourth
Dim. seventh	Maj. sixth	Min. sixth	Perf. fifth	Dim. fifth
+2 / °7	m3 / M6	M3 / m6	P4 / P5	+4 / °5
3 / 9	3 / 9	4 / 8	5 / 7	6 / 6

- Perfect intervals: 1 – 4 – 5 – 8
- Others: Major and minor
- All: Diminished and augmented
- In major scale: upward intervals always **perfect** or **major**

Intervals

Consonance & Dissonance



- Perfect consonances
- Imperfect consonances
- Dissonances

Intervals

Specific Intervals

Δ	Interval name	Interval	JI ratio	Pyt. ratio
0	(Perfect) unison	C4 – C4	1:1	1:1
1	Minor second	C4 – D♭4	15:16	$3^5:2^8$
2	Major second	C4 – D4	8:9	$2^3:3^2$
3	Minor third	C4 – E♭4	5:6	$3^3:2^5$
4	Major third	C4 – E4	4:5	$2^6:3^4$
5	(Perfect) fourth	C4 – F4	3:4	$3:2^2$
6	Tritone	C4 – F♯4	32:45	$2^9:3^6$ or $3^6:2^{10}$
7	(Perfect) fifth	C4 – G4	2:3	2:3
8	Minor sixth	C4 – A♭4	5:8	$3^4:2^7$
9	Major sixth	C4 – A4	3:5	$2^4:3^3$
10	Minor seventh	C4 – B♭4	5:9	$3^2:2^4$
11	Major seventh	C4 – B4	8:15	$2^7:3^5$
12	(Perfect) octave	C4 – C5	1:2	1:2

Intervals

Intervals in context

- **Harmonic intervals:** describing the relationships of concurrently sounding pitches (no „direction“)
- **Melodic intervals:** describing the relationships of successively sounding pitches (with direction)
- On the pitch class level: An interval progressions corresponds to the **complementary** interval progression in **opposite direction**

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Enharmonic Equivalence

Intervals

Specific name	Perf. unison	Aug. unison	Min. second	Maj. second
Complementary	Perf. octave	Dim. octave	Maj. seventh	Min. seventh
Abbreviation	P1 / P8	+1 / °8	m2 / M7	M2 / m7
Semitone distance Δ	0 / 12	1 / 11	1 / 11	2 / 10

- Different specific intervals with same semitone distance
- → **Enharmonically equivalent intervals**
- Involve enharmonically **equivalent pitches**

Enharmonic Equivalence

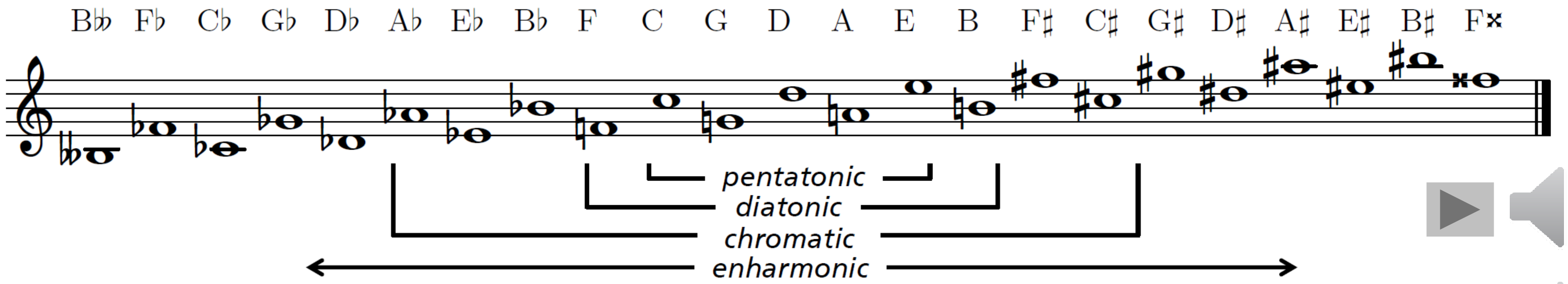
Pitch classes

- Overtone series: Fifths as most important (non-octave) interval
- Construct scales from fifth series
  - Pentatonic 
  - Diatonic 
  - Chromatic 

Enharmonic Equivalence

Pitch classes

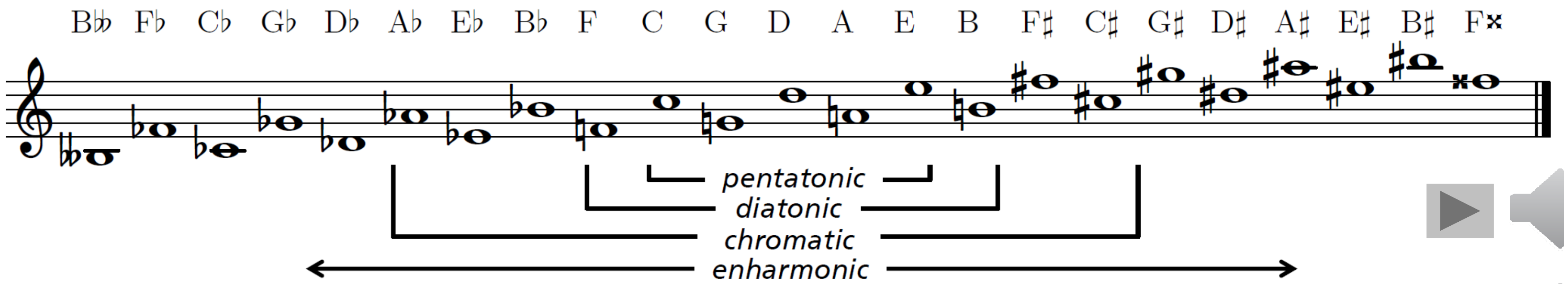
- Overtone series: Fifths as most important (non-octave) interval
- Scales as excerpts from fifth series



- Enharmonic: More than twelve fifths-related pitch classes
- → **enharmonically equivalent pitch classes**
- A **spiral**, not a **circle**!

Enharmonic Equivalence

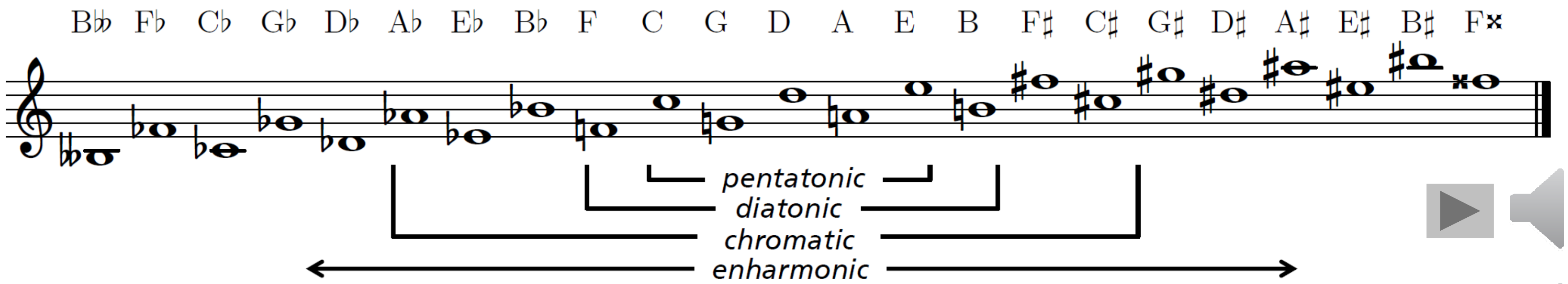
Pitch classes



- Construction of pitch frequencies from pure **perfect fifths intervals** with ratio 3:2 → **Pythagorean tuning**
- Problem: 12 fifths are not exactly 7 octaves!
- → „Pythagorean comma“:
  - Ratio: $$\frac{(3/2)^{12}}{2^7} \approx 1.0136$$
  - Distance in cents: $\log_2(1.0136) \cdot 1200 \approx 23.5\ \textbf{Cent}$

Enharmonic Equivalence

Pitch classes



- Consequence: Pure intervals (beating-free) and enharmonic equivalence **not possible** at the same time
- → Pythagorean comma needs to be „tempered“
- Different kinds of „temperament“
- **Twelve-tone equal temperament:**
  - Pythagorean comma **equally distributed**
  - Perfect fifth of size 23.5 / 12 ≈ **2 Cents** smaller than pure fifth

Global tuning

Concert pitch

- Global tuning: shift of all frequencies
- Given by concert pitch (frequency of MIDI pitch 69 $\triangleq$ **A4**)
  - Standard: $f_{\mathrm{concert}} := 440\ \mathrm{Hz}$
  - Historical tuning: $f_{\mathrm{concert}}^{\mathrm{hist}} := 415\ \mathrm{Hz}$

- Compute frequency from MIDI pitch number

$$f_0(p) = 2^{(p-69)/12} \cdot f_{\mathrm{concert}}$$

Further Computations

Equal temperament

- Pitch class numbers: $q \in [0:11]$

$$(0, 1, \dots, 11) \;\widehat{=}\; (\mathrm{C}, \mathrm{C}\sharp, \dots, \mathrm{B})$$

- Pitch class from MIDI pitch: $q(p) = p \mod 12$
- Interval in semitones: $\Delta(p^a, p^b) = p^b - p^a$
- Simple from compound interval: $\Delta_{\text{simple}} = \Delta_{\text{compound}} \mod 12$
- Complementary from original: $\Delta_{\text{complementary}} = 12 - \Delta_{\text{original}}$

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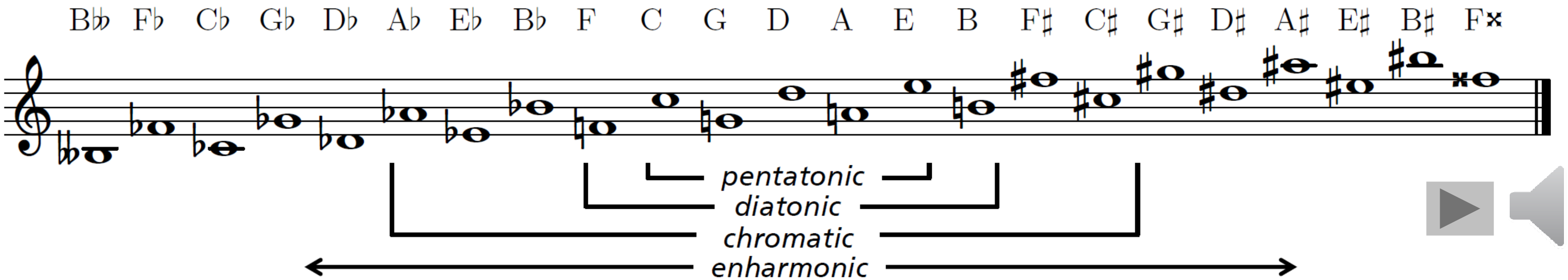
Scale Families

Pitch class content



- Scale family only defines a specific **pitch class content**
  - Can be **transposed** (shifted) in different ways
  - Different **referential pitch classes** (tonic notes)

Scale Transpositions

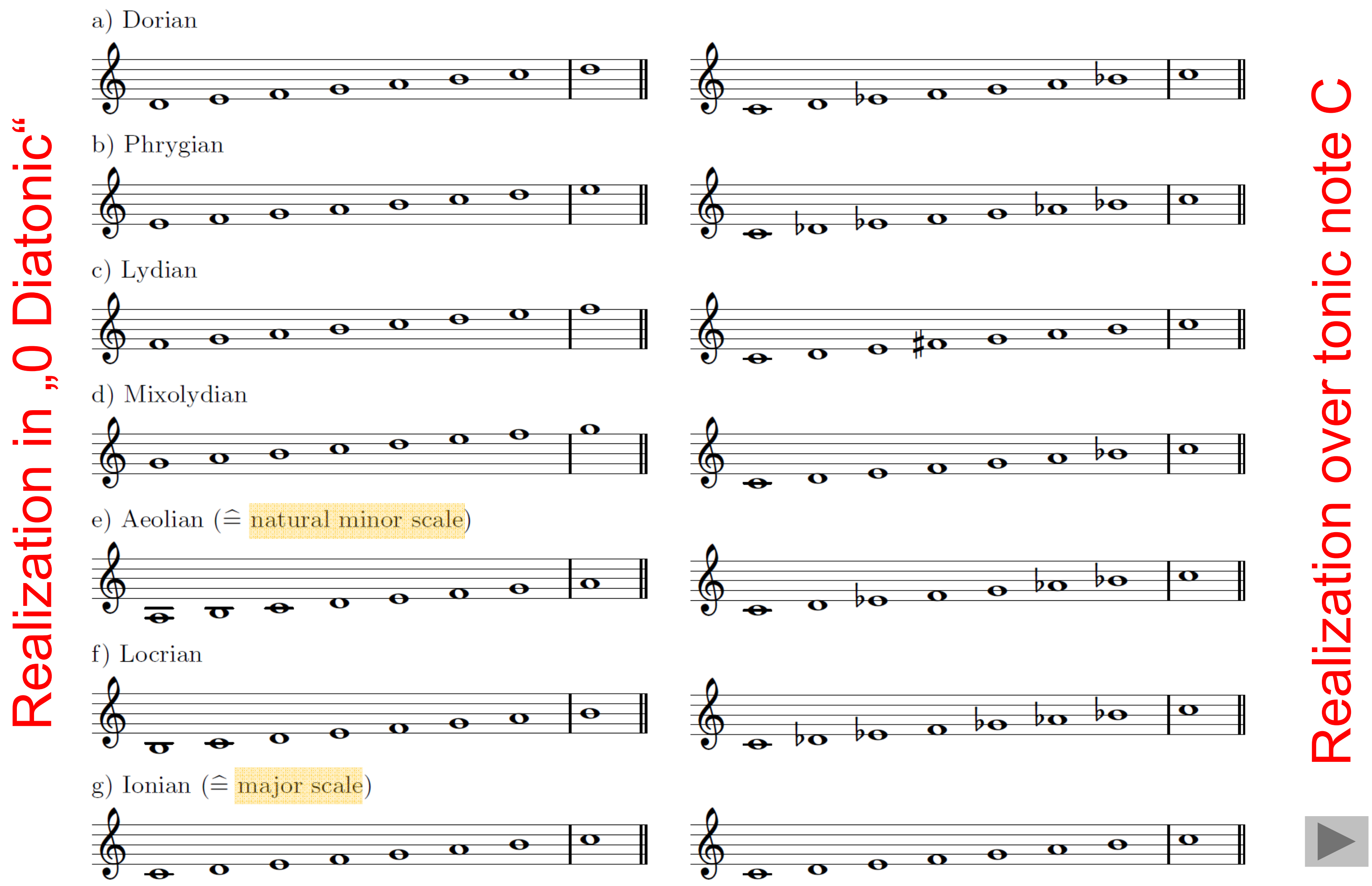
Diatonic Scales

- Transposition corresponds to a **shift** in the fifth series
- Naming convention: according to the **accidentals** (key signature)
- Allows for measuring **distances** between diatonic scales

Specific Scales

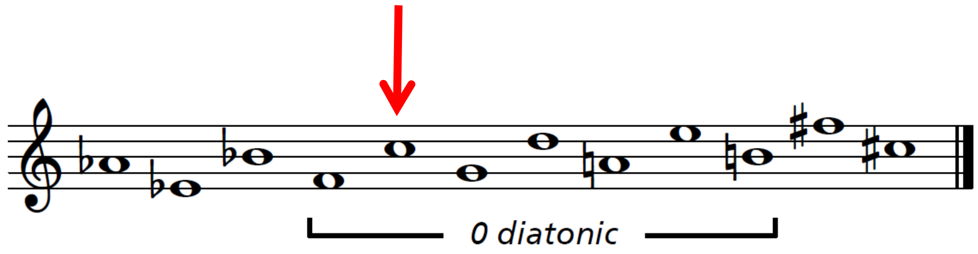
Diatonic scales

- Different referential pitch classes (tonic notes): **(church) modes**

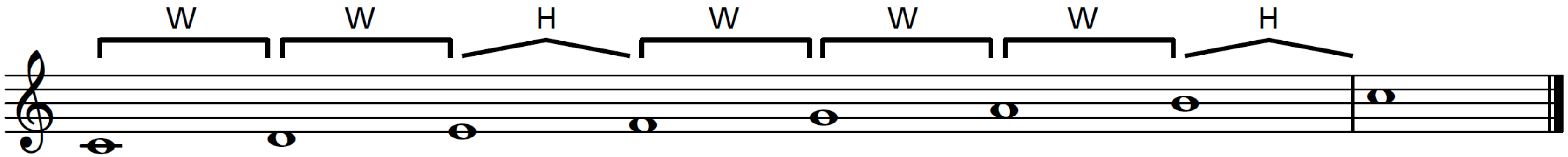


Specific Scales

Major scale



- Diatonic scale based on second pitch class in fifth series
- Results in semitones between scale degrees **3–4** and **7–8** (7–1)



Degree	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{8} = \hat{1}$
Name	Tonic	Supertonic	Mediant	Subdominant	Dominant	Submediant	Leading Tone	Tonic
Solfège	*do*	*re*	*mi*	*fa*	*sol*	*la*	*ti*	*do*

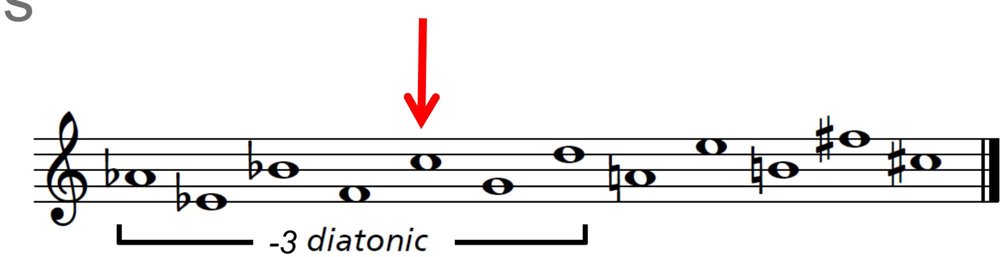
Specific Scales

Minor scales

- **Natural** minor scale: Diatonic scale based on fifth pitch class in fifth series

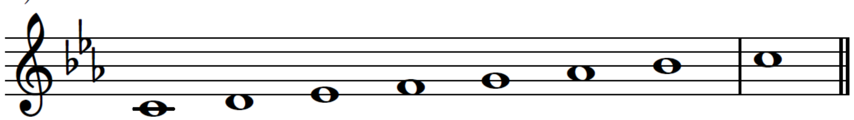
Specific Scales

Minor scales

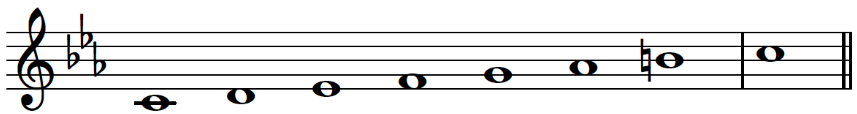


- **Natural** minor scale: Diatonic scale based on fifth pitch class in fifth series
- Results in semitones between scale degrees **2–3** and **5–6** (7–1)

a) Natural



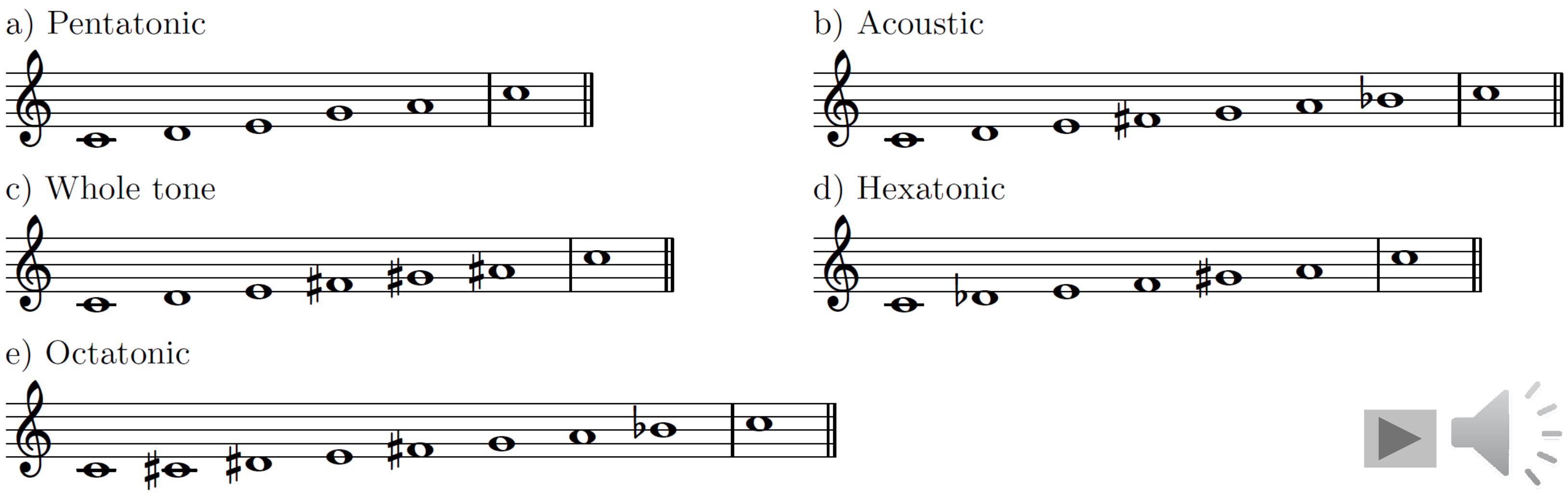
b) Harmonic



c) Melodic

Specific Scales

Non-diatonic scales



- Symmetry in the equal-tempered scale: pitch class activation vectors (templates):

$$\mathbf{T}^{\mathrm{Wholetone}} = (1,0,1,0,1,0,1,0,1,0,1,0)^{\mathrm{T}}$$

$$\mathbf{T}^{\mathrm{Hexatonic}} = (1,1,0,0,1,1,0,0,1,1,0,0)^{\mathrm{T}}$$

$$\mathbf{T}^{\mathrm{Octatonic}} = (1,1,0,1,1,0,1,1,0,1,1,0)^{\mathrm{T}}$$

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Chord
Definition

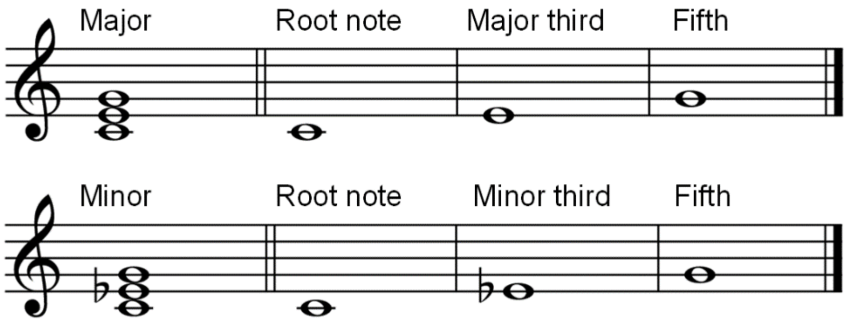
- „Sets of pitches that are perceived as an entity“
- Usually three (**triads**) or more pitches (**seventh chords**, …)
- Can be realized in different ways, referring to the same „abstract“ chord

Triads

Basic types

Thirds	(M3, m3)	(m3, M3)	(m3, m3)	(M3, M3)
Frame interval	P5	P5	°5	+5
Triad type name	Major	Minor	Diminished	Augmented
Abbreviation	M	m	°	+

- Three notes in **tertian structure** („snowman“)
- Structure:



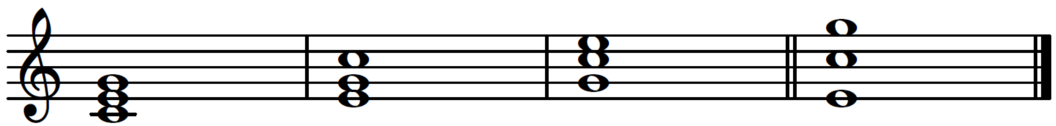
C major (C)

C minor (Cm)

- Stability according to frame interval (**fifth**)

Triads

Inversions



Inversion	Root pos.	1st inv.	2nd inv.	1st inv. (open pos.)
Figured bass notation	$\left(\begin{smallmatrix}5\\3\end{smallmatrix}\right)$	^{6}	$\begin{smallmatrix}6\\4\end{smallmatrix}$	^{6}
Bass note	Root	Third	Fifth	Third

- Only bass pitch class is important
- Root position is most stable
- Caution: **Root pitch class ≠ bass pitch class!**

Triads

Pitch class sets

- Pitch class activation vectors (independent of inversion)
  - Major: $\mathbf{T}^{\mathrm{CM}} = (1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0)^{\mathrm{T}}$
  - Minor: $\mathbf{T}^{\mathrm{Cm}} = (1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0)^{\mathrm{T}}$
  - Diminished: $\mathbf{T}^{\mathrm{C}^{\circ}} = (1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0)^{\mathrm{T}}$
  - Augmented: $\mathbf{T}^{\mathrm{C+}} = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0)^{\mathrm{T}}$

Seventh Chords

Basic types

Thirds	(M3, m3, M3)	(M3, m3, m3)	(m3, M3, m3)	(m3, m3, M3)	(m3, m3, m3)
Triad+7	(M, M7)	(M, m7)	(m, m7)	(°, m7)	(°, °7)
Name	Major 7	Dominant 7	Minor 7	Half-diminished 7	Diminished 7
Abbr.	$\mathrm{M}^{\mathrm{maj7}}$	M^7	m^7	^{ø7}	$^{\circ 7}$

- Three concatenated thirds
- Basic triad types + seventh (above the root)
- Other extensions as well (6, 9, 11, 13, …) → jazz harmony

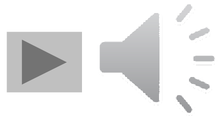
Figuration

Types

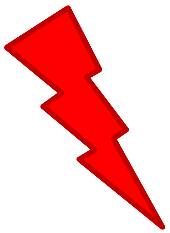
- Homophonic texture (no figuration): harmonic rhythm = rhythm
- **Figuration**: rhythm faster than harmonic rhythm
  - **Rhythmic** figuration: repeated chords / notes
  - **Harmonic** figuration: different chord notes (arpeggio)
  - **Melodic figuration:** involving non-chord tones (usually dissonant!)

Melodic Figuration

Non-chord tones



- Make harmony interesting
- „Chord modifications“ – no actual „chords“!
- Types
  - Pedal points
  - Passing tone
  - Neighbor note
  - Anticipation
  - Suspension

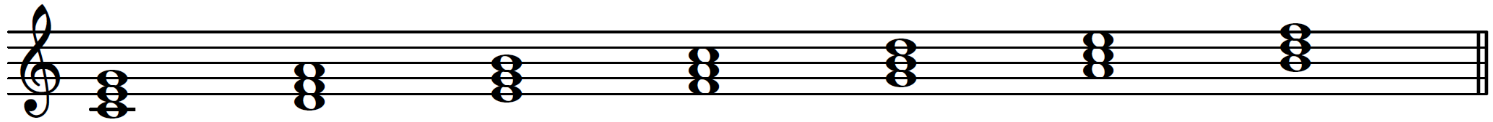


~~„suspended chord“~~

J. S. Bach, Jesu meine Freude BWV 227

Functional Harmony

Chord functions & Roman numerals



Chord type	M	m	m	M	M	m	°
Function name	Tonic	Subdominant parallel	Dominant parallel	Subdominant	Dominant	Tonic parallel	~~Incomplete dom.^{7}~~
Function short	T	Sp	Dp	S	D	Tp	~~D^7~~
Roman numeral	I	ii	iii	IV	V	vi	vii°

m	°	M	m	M	M	M	°
Tonic	~~Incomplete dom. par.^{7}~~	Tonic parallel	Subdominant	Dominant	Subdominant parallel	Dominant parallel	~~Incomplete dom.^{7}~~
t	~~dP^7~~	tP	s	D	sP	dP	~~D^7~~
i	ii°	III	iv	V	VI	VII	vii°

- Capitals: Major & augmented chords
- Lowercase: Minor & diminished chords
- No „incomplete chords“

Functional Harmony

Chord relationships

- Parallel chords (e.g. C major – A minor): $\mathrm{M} \underset{\text{up m3}}{\overset{\text{down m3}}{\rightleftarrows}} \mathrm{m}$
- Contrast chords (e.g. C major – E minor): $\mathrm{M} \underset{\text{down m3}}{\overset{\text{up m3}}{\rightleftarrows}} \mathrm{m}$
- Major (minor) variant (e.g. C major – C minor): $\mathrm{M} \rightleftarrows \mathrm{m}$
- $\rightarrow$ share each two pitch classes!

Functional Harmony

Chord progressions

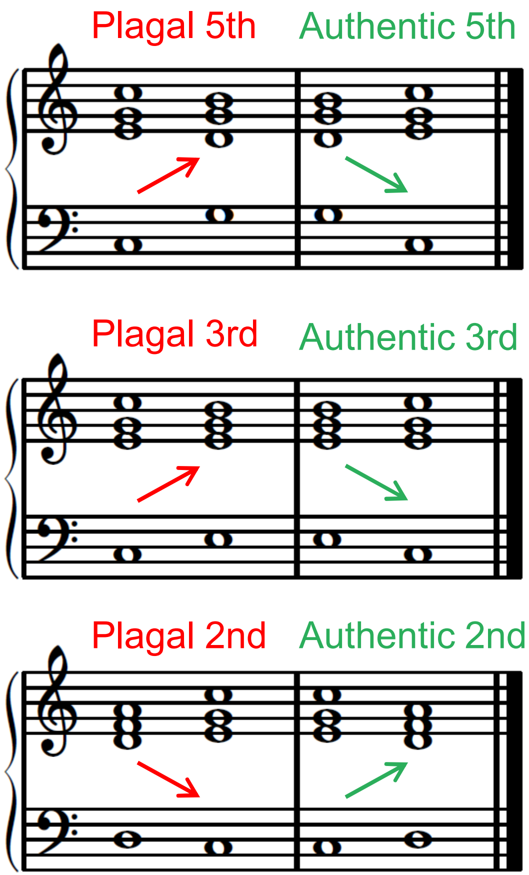
Types:

- **Pendulum**: chord change and reverse (e.g. I – V – I)
- **Sequence**: repetition of same diatonic step (e.g. III – VI – II – V – I)
- **Cadence**: ending formula, often with closing character (e.g. II – V – I)

Functional Harmony

Chord progressions

- Authentic progressions: „falling“, „moving forward“, „directional“
- Plagal progressions: „opening“, „archaic“ („A-men“), colorful



Interval	Δ	Complem.	Δ	Quality
P1	0	P8 $\searrow$	-12	None
m2 $\nearrow$	+1	M7 $\searrow$	-11	Authentic
M2 $\nearrow$	+2	m7 $\searrow$	-10	Authentic
m3 $\nearrow$	+3	M6 $\searrow$	-9	Plagal
M3 $\nearrow$	+4	m6 $\searrow$	-8	Plagal
P4 $\nearrow$	+5	P5 $\searrow$	-7	Authentic
+4 $\nearrow$	+6	$^\circ$5 $\searrow$	-6	None
P5 $\nearrow$	+7	P4 $\searrow$	-5	Plagal
m6 $\nearrow$	+8	M3 $\searrow$	-4	Authentic
M6 $\nearrow$	+9	m3 $\searrow$	-3	Authentic
m7 $\nearrow$	+10	M2 $\searrow$	-2	Plagal
M7 $\nearrow$	+11	m2 $\searrow$	-1	Plagal
P8 $\nearrow$	+12	P1	0	None

Music Theory Basics
Overview

Part I:

- Pitches and Intervals
- Tuning and Enharmonic Equivalence
- Scales

Part II:

- Chords
- Keys and the Circle of Fifths

Key
Definition

- „A set of pitch relationships that establishes a specific major or minor triad as a **tonal center**"
- Example: „F major" = **tonic note** *F*, **mode** *major,* **tonic chord** *F major*
- With enharmonic equivalence: 24 keys
- Change of key: **Modulation**
- Types of modulation:
  - **Diatonic** modulations: **pivot chord** obtains new function
  - **Chromatic** modulation: one note or chord chromatically **altered**
  - **Enharmonic** modulation: **re-spelling** of pitch to obtain new function

Key

Key relationships

Special relationships & common modulations:

- **Relative** keys (same key signature, different tonic):

$$\text{F major} \xrightarrow{\text{down m3}} \text{D minor}$$

- **Parallel** keys (same tonic note, different mode):

$$\text{F major} \xrightarrow{\text{P1}} \text{F minor}$$

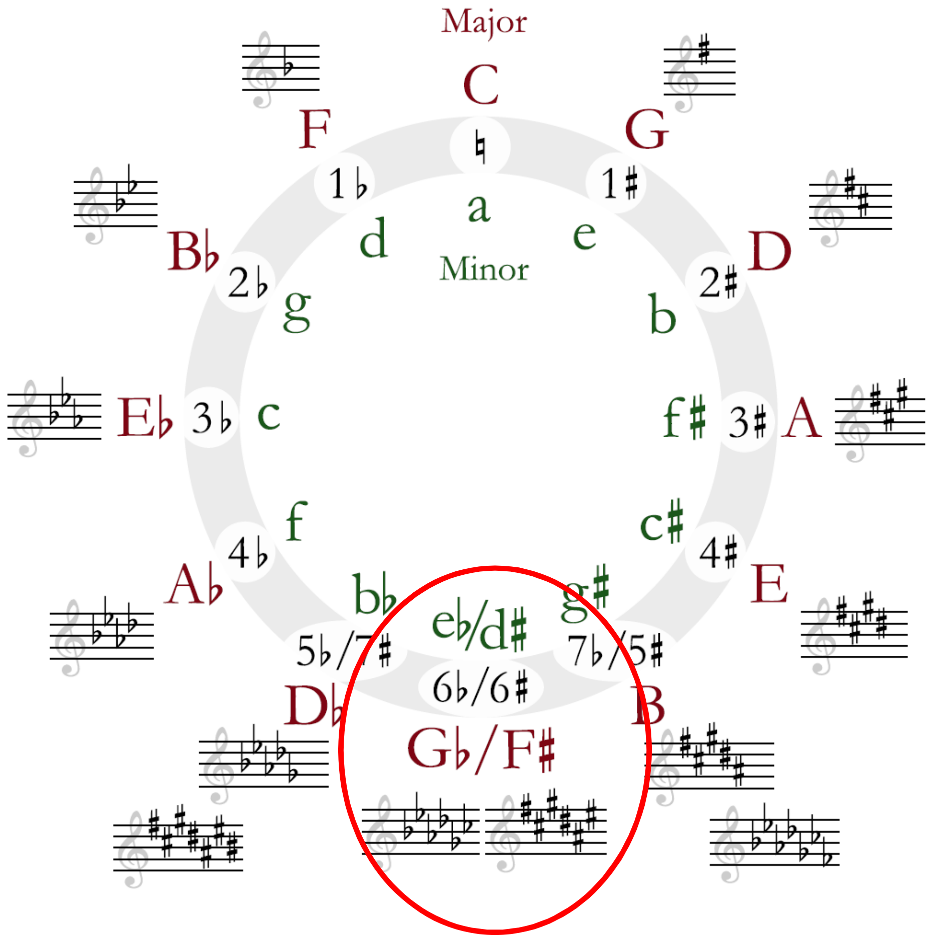
- **Fifth-related** keys (differ in one scale pitch class):

$$\text{F major} \xrightarrow{\text{up P5}} \text{C major}$$

- **Mediant** keys (third-related), e.g.:

$$\text{F major} \xrightarrow{\text{down m3}} \text{D major}$$

Key
Circle of fifths



- Actually (without enharmonic equivalence): **spiral** not **circle!**
- Use **series of fifths** instead…