Hochschule für Musik Karlsruhe

Blockvorlesung

Advanced Audio-Based Music Processing

4. Harmony Analysis

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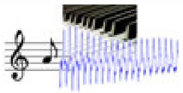
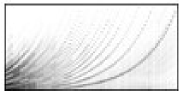
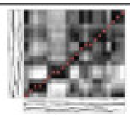
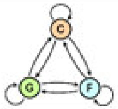
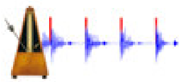
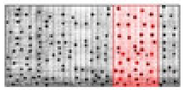
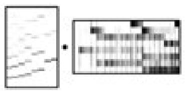
Book: Fundamentals of Music Processing

Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
483 p., 249 illus., hardcover
ISBN: 978-3-319-21944-8
Springer, 2015

Accompanying website:
www.music-processing.de

Book: Fundamentals of Music Processing

Chapter		Music Processing Scenario
1		Music Representations
2		Fourier Analysis of Signals
3		Music Synchronization
4		Music Structure Analysis
5		Chord Recognition
6		Tempo and Beat Tracking
7		Content-Based Audio Retrieval
8		Musically Informed Audio Decomposition

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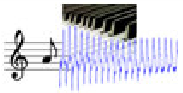
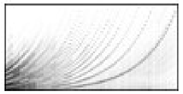
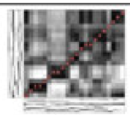
1		Music Representations
2		Fourier Analysis of Signals
3		Music Synchronization
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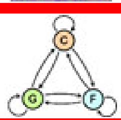
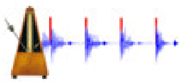
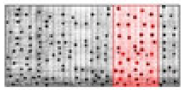
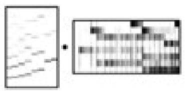
5		Chord Recognition
6		Tempo and Beat Tracking
7		Content-Based Audio Retrieval
8		Musically Informed Audio Decomposition

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Dissertation: Tonality-Based Style Analysis

Christof Weiß
Computational Methods for Tonality-Based Style Analysis of Classical Music Audio Recordings
PhD thesis, Ilmenau University of Technology, 2017
https://www.db-thueringen.de/receive/dbt_mods_00032890

Chapter 5: Analysis Methods for Key and Scale Structures

Harmony Analysis

Overview

- Template-based Chord Recognition
- HMM-based Chord Recognition
- Local Key Detection
- Application for Musicology

Harmony Analysis

Overview

- Template-based Chord Recognition
- HMM-based Chord Recognition
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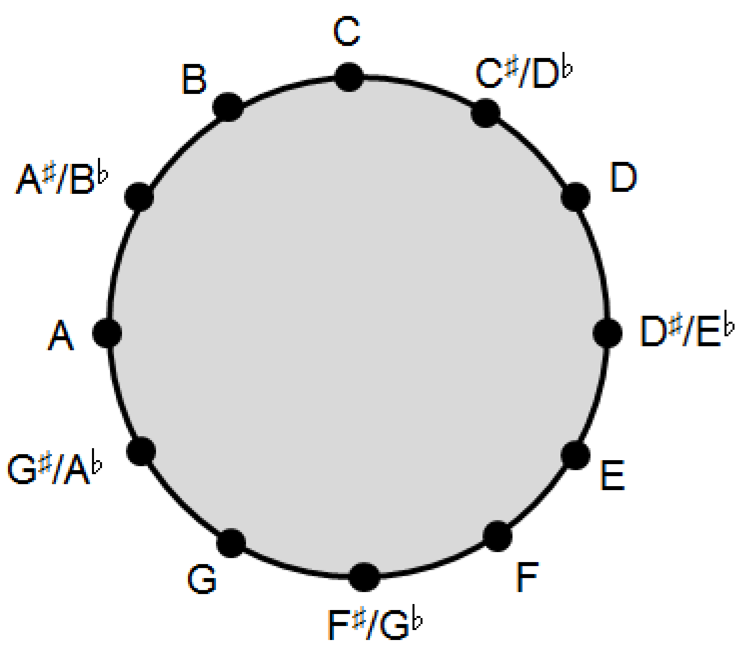
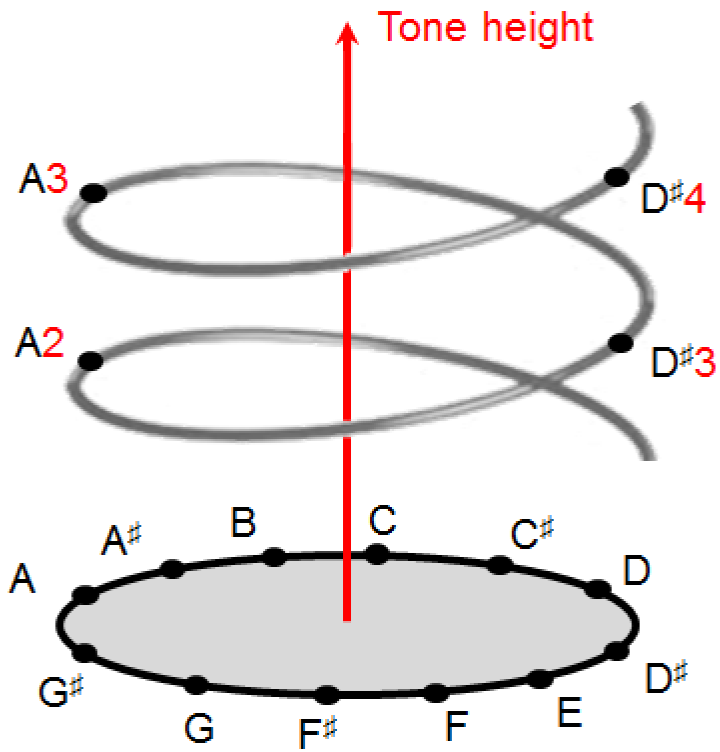
Recall: Chroma Features

- Human perception of pitch is periodic
- Two components: tone height (octave) and chroma (pitch class)

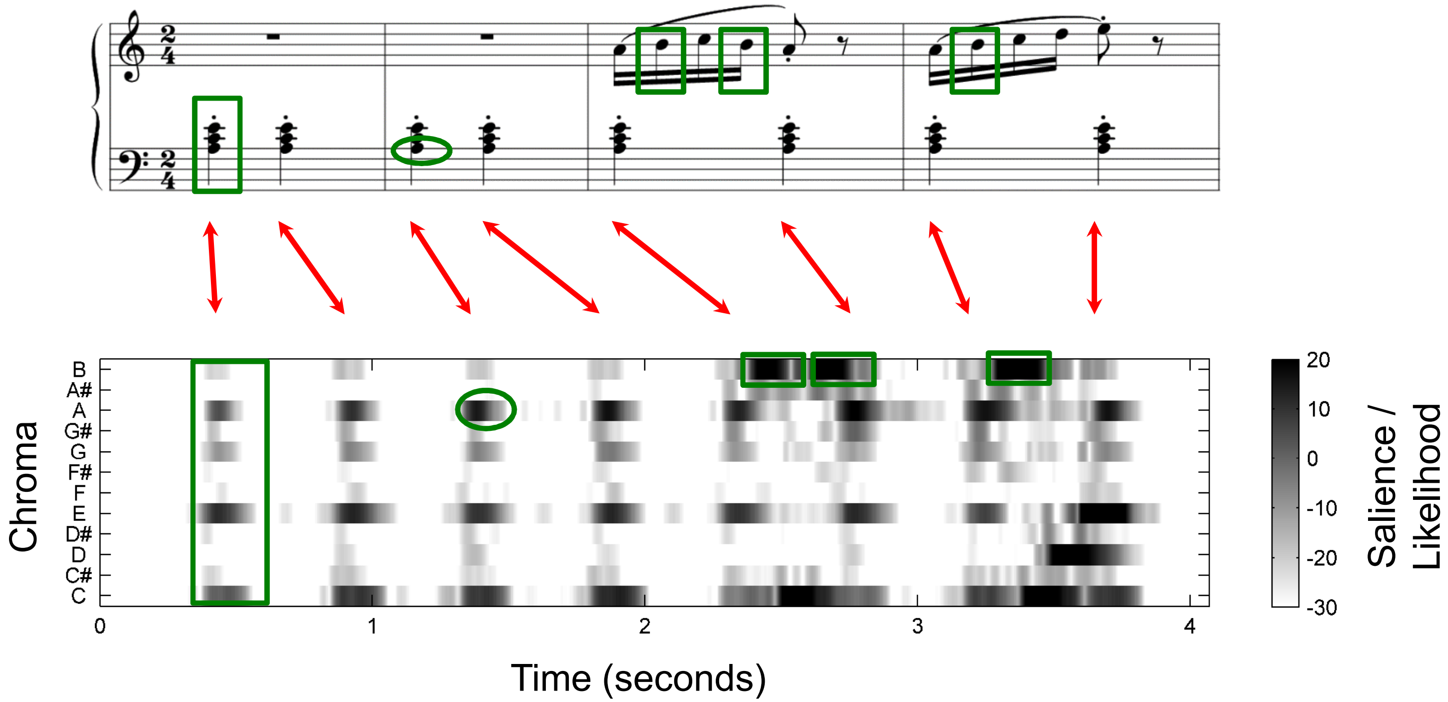
Chromatic circle



Shepard's helix of pitch



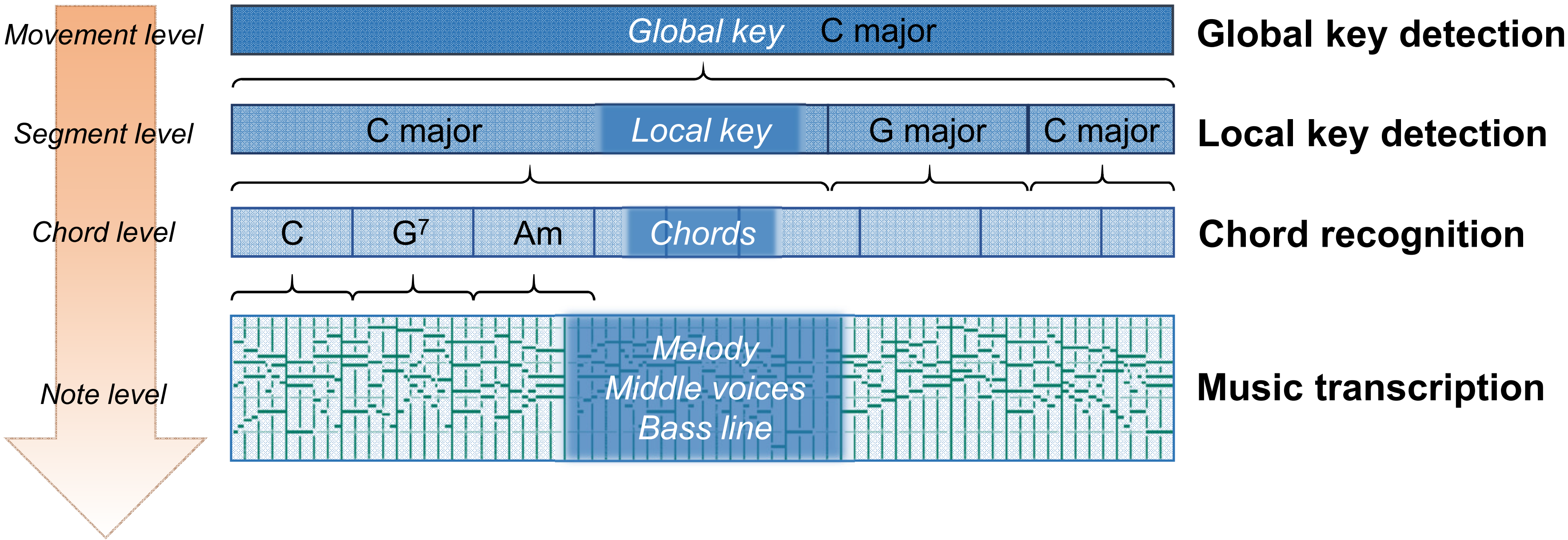
Recall: Chroma Features



→ capture harmonic progression

Harmony Analysis: Overview

- Western music (and most other music): Different aspects of harmony
- Referring to different time scales



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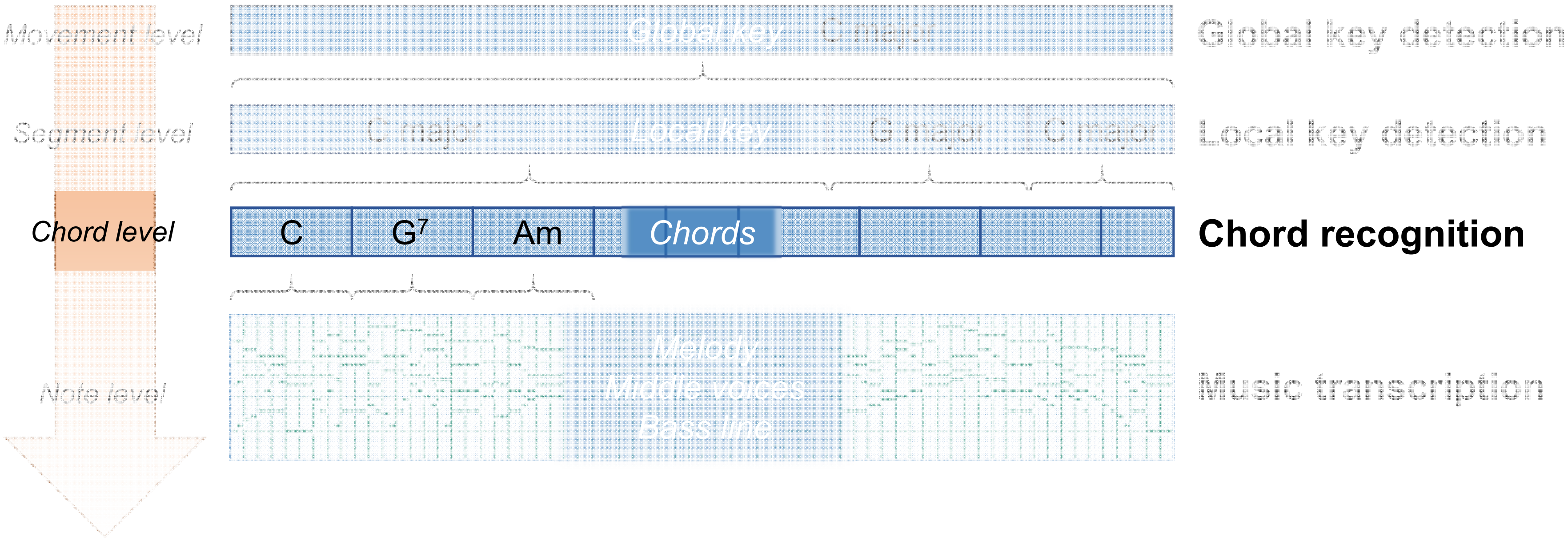
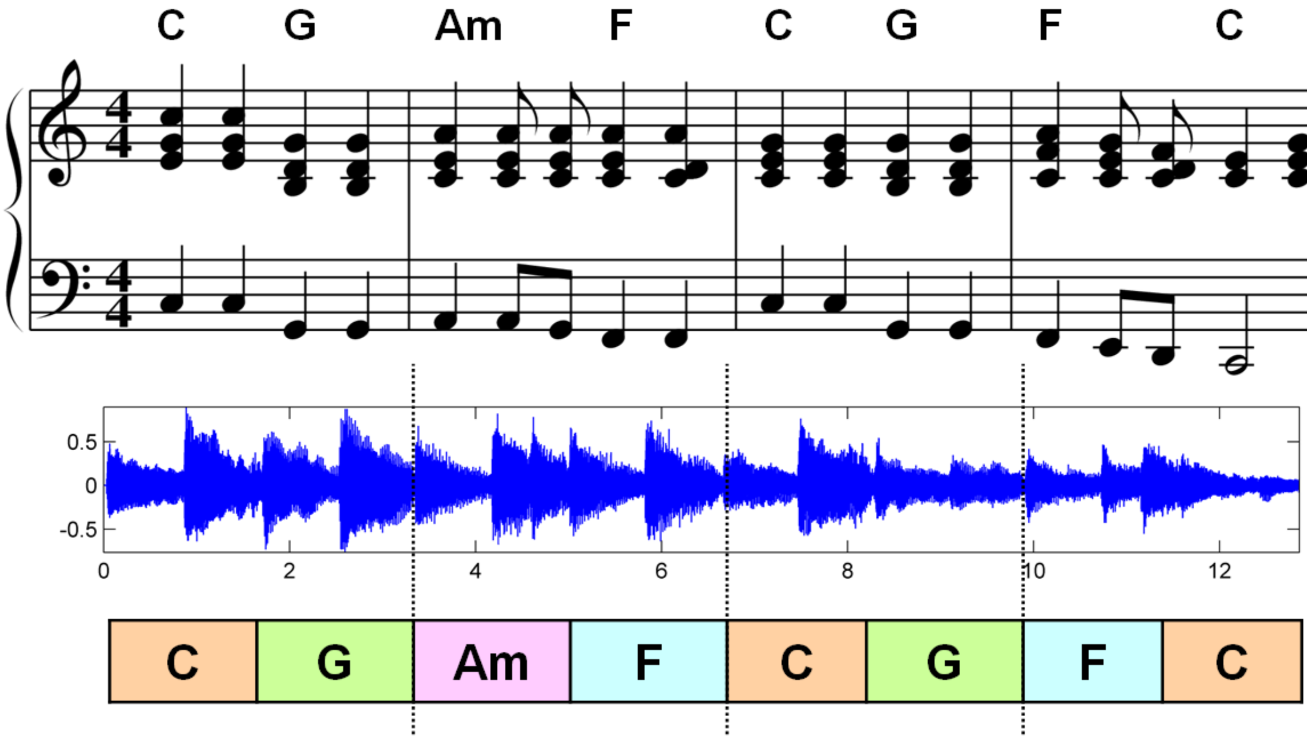


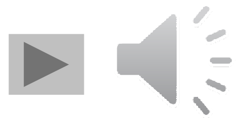
Chord Recognition

```
Let It Be chords
The Beatles 1970  (Let It Be)

[Intro]

C G Am F C G
F C Dm C

[Verse 1]

       C              G                 Am           F
When I find myself in times of trouble, Mother Mary comes to me
C                 G              F C Dm C
Speaking words of wisdom, let it be

    C             G                 Am                 F
And in my hour of darkness, she is standing right in front of me
C                 G              F C Dm C
Speaking words of wisdom, let it be

[Chorus]
```

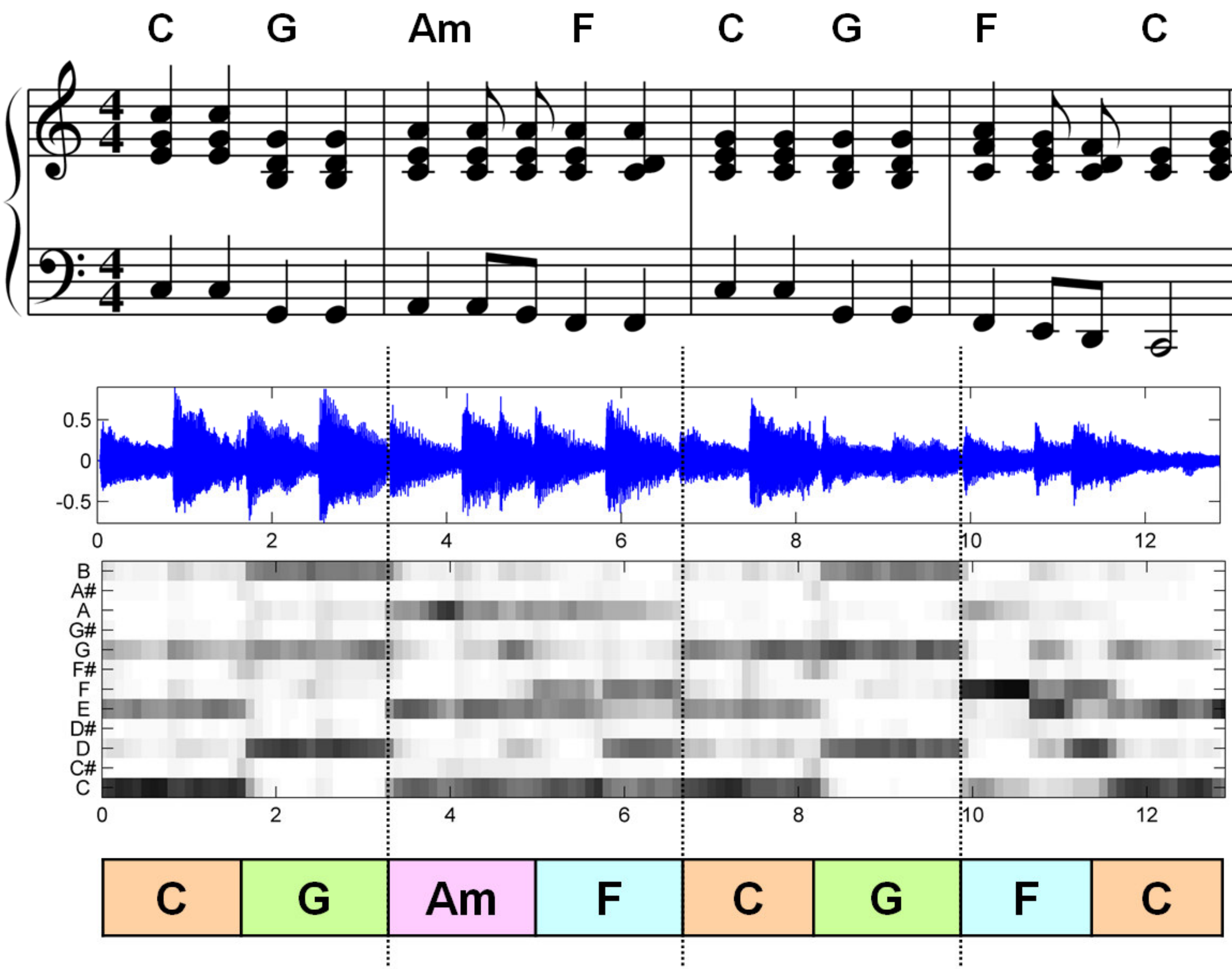
Source: www.ultimate-guitar.com

Chord Recognition

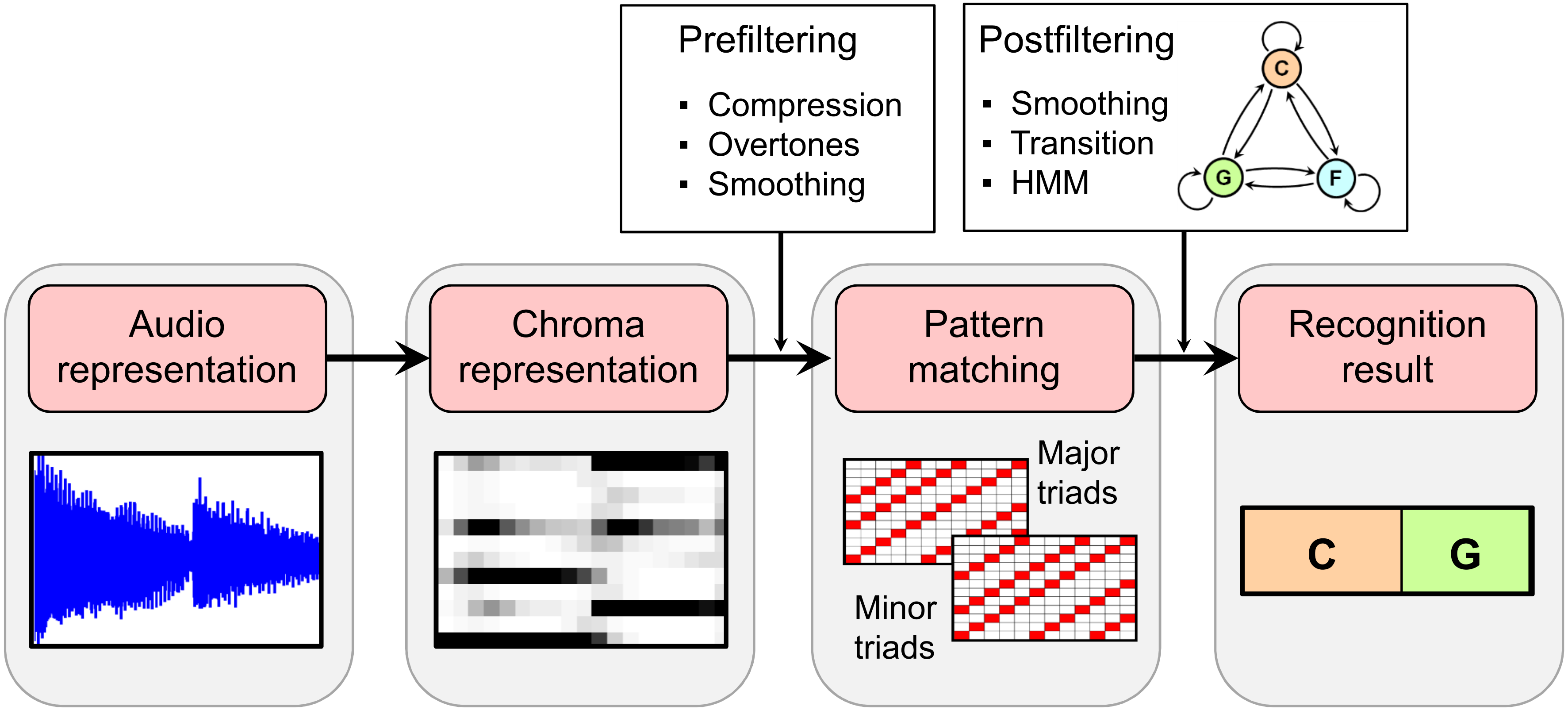




Chord Recognition



Chord Recognition

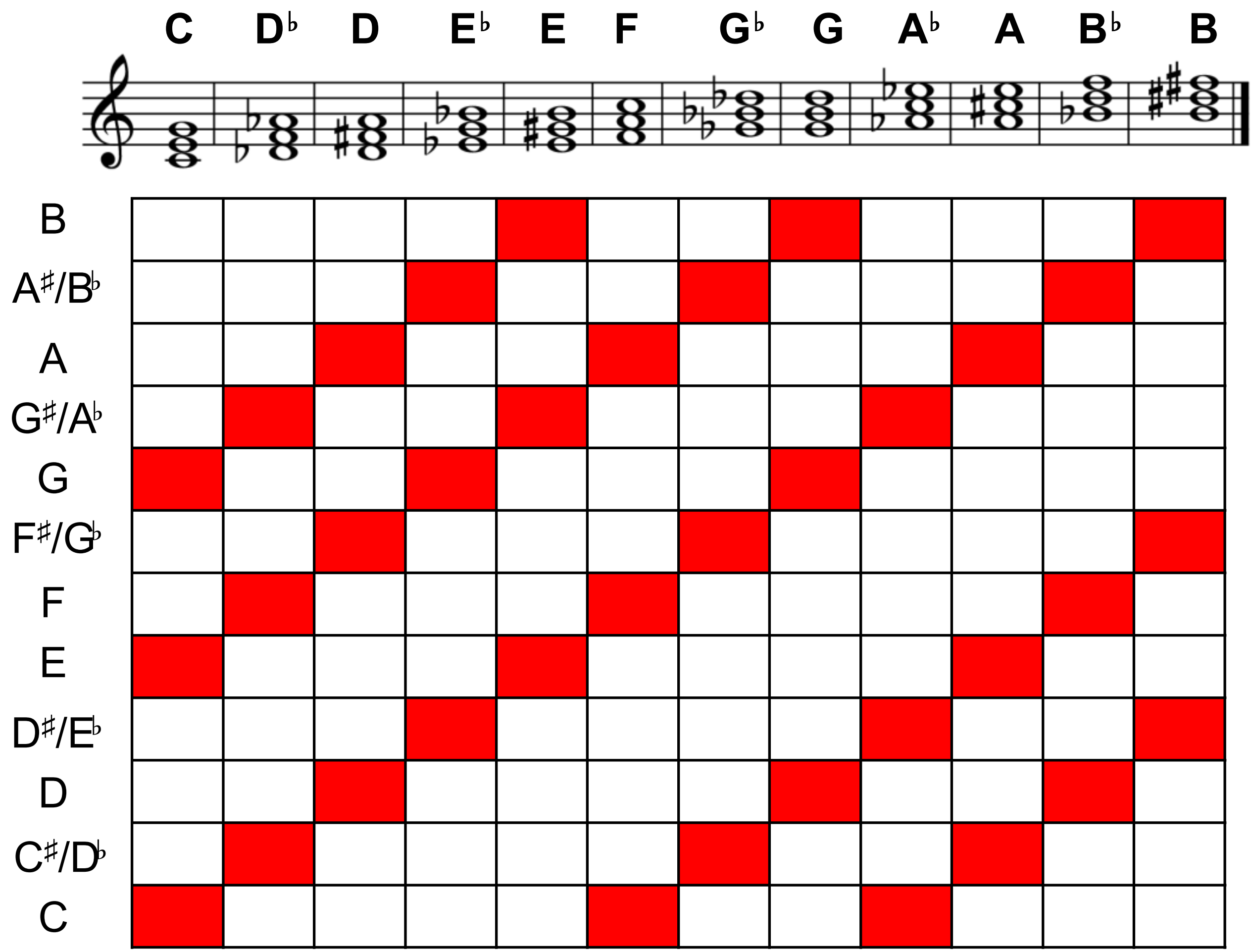


Chord Recognition: Basics

- Chords as *Pitch Class Sets*
- *Activation Vectors* or *Templates:*
  - Major: $\mathbf{T}^{\mathrm{CM}} = (1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0)^{\mathrm{T}}$
  - Minor: $\mathbf{T}^{\mathrm{Cm}} = (1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0)^{\mathrm{T}}$

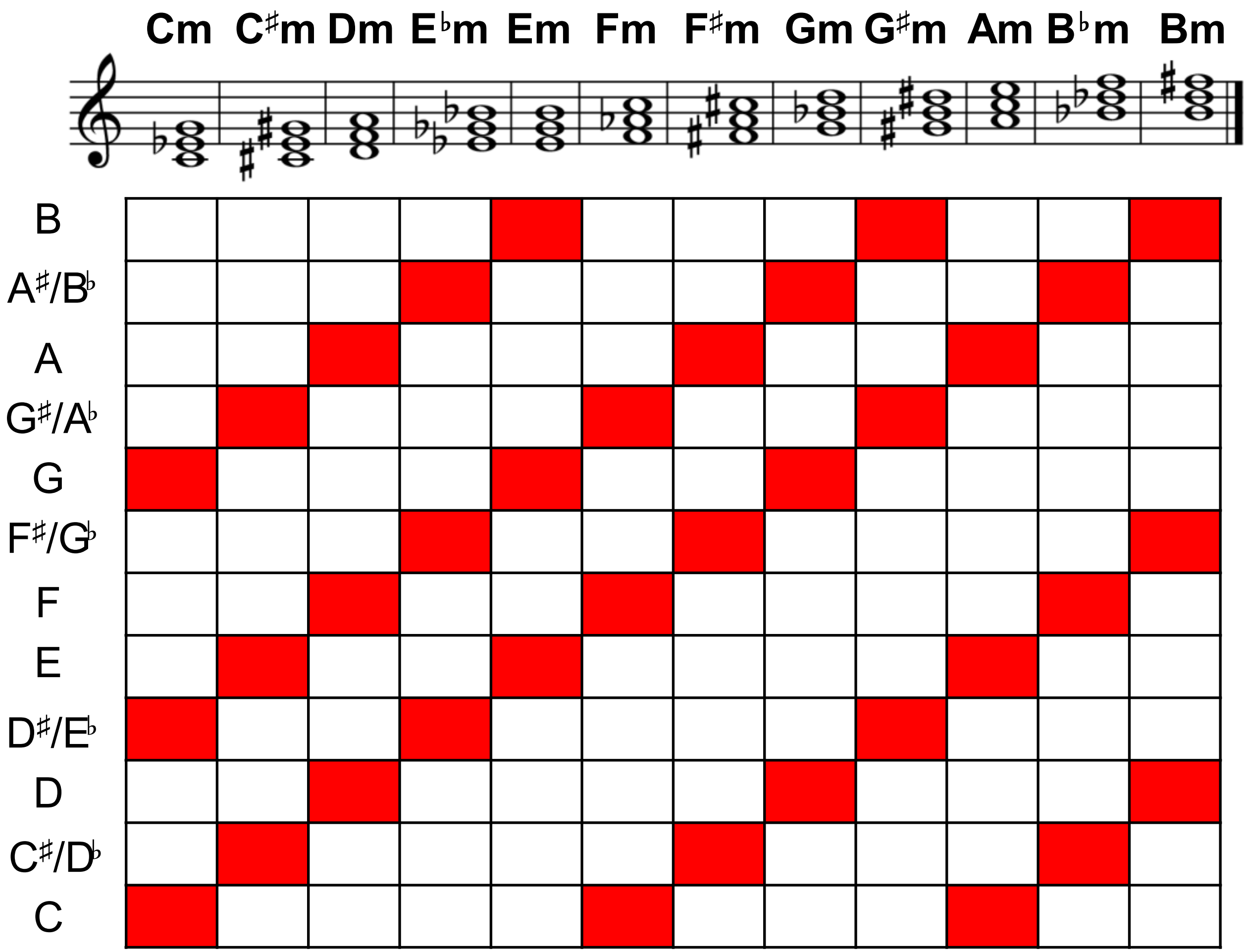
Chord Recognition: Basics

- Templates: **Major Triads**

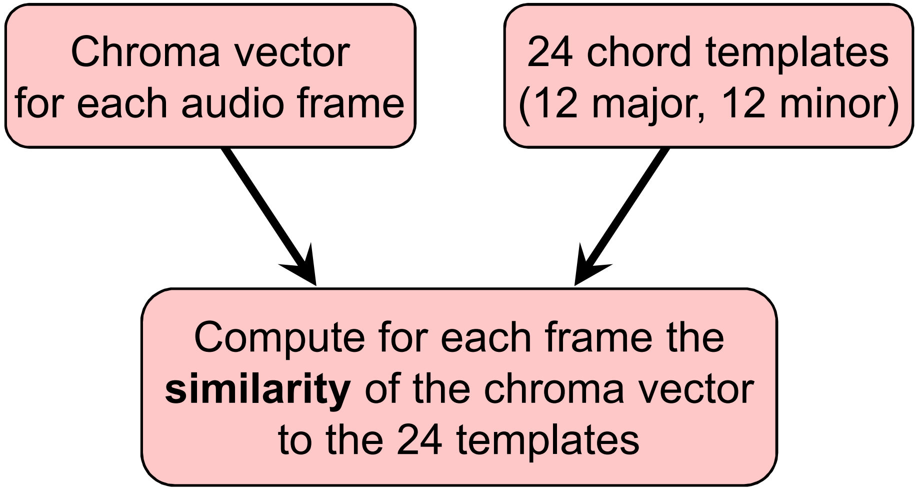


Chord Recognition: Basics

- Templates: **Minor Triads**



Chord Recognition: Template Matching



	C	C$^\sharp$	D	…	Cm	C$^\sharp$m	Dm	…
B	0	0	0	…	0	0	0	…
A$^\sharp$	0	0	0	…	0	0	0	…
A	0	0	1	…	0	0	1	…
G$^\sharp$	0	1	0	…	0	1	0	…
G	1	0	0	…	1	0	0	…
F$^\sharp$	0	0	1	…	0	0	0	…
F	0	1	0	…	0	0	1	…
E	1	0	0	…	0	1	0	…
D$^\sharp$	0	0	0	…	1	0	0	…
D	0	0	1	…	0	0	1	…
C$^\sharp$	0	1	0	…	0	1	0	…
C	1	0	0	…	1	0	0	…

Chord Recognition: Template Matching

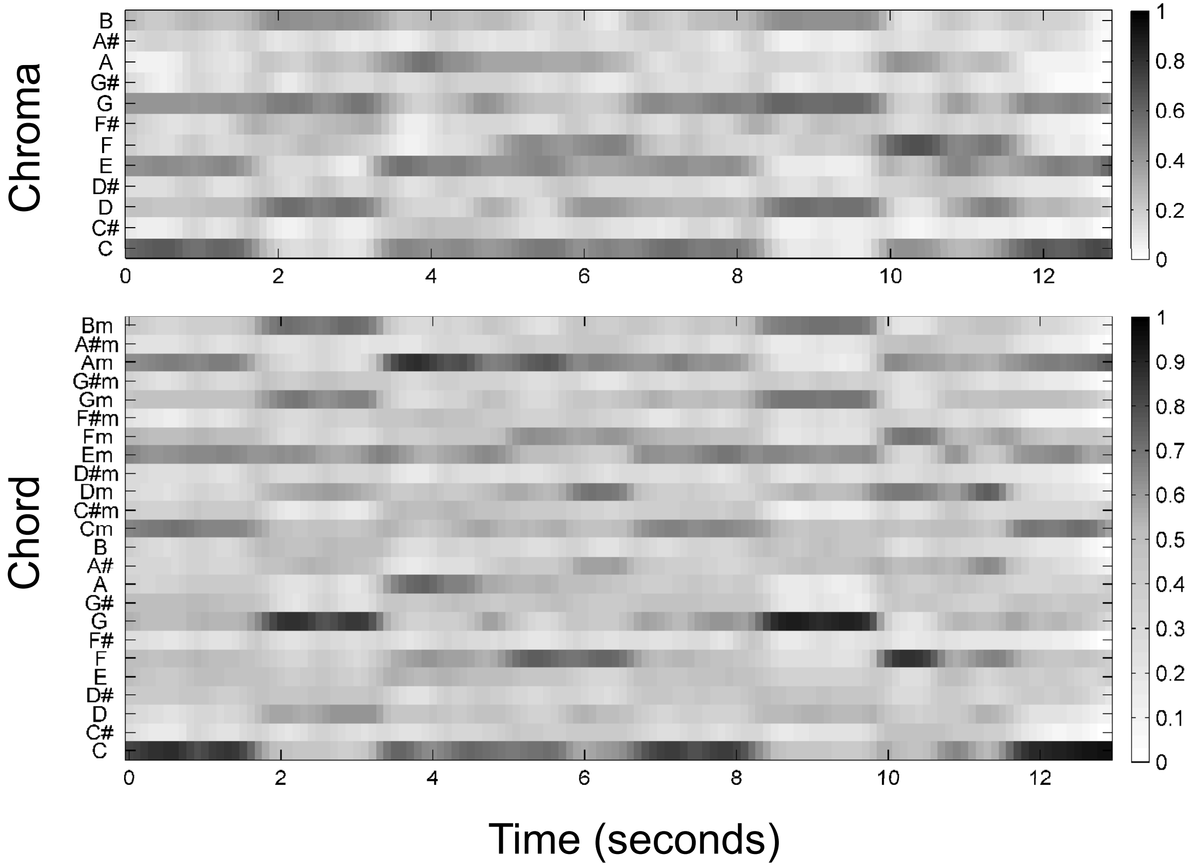
- Similarity measure: Cosine similarity (inner product of normalized vectors)

Chord template: $\boldsymbol{t} \in \mathbb{R}^{12}$

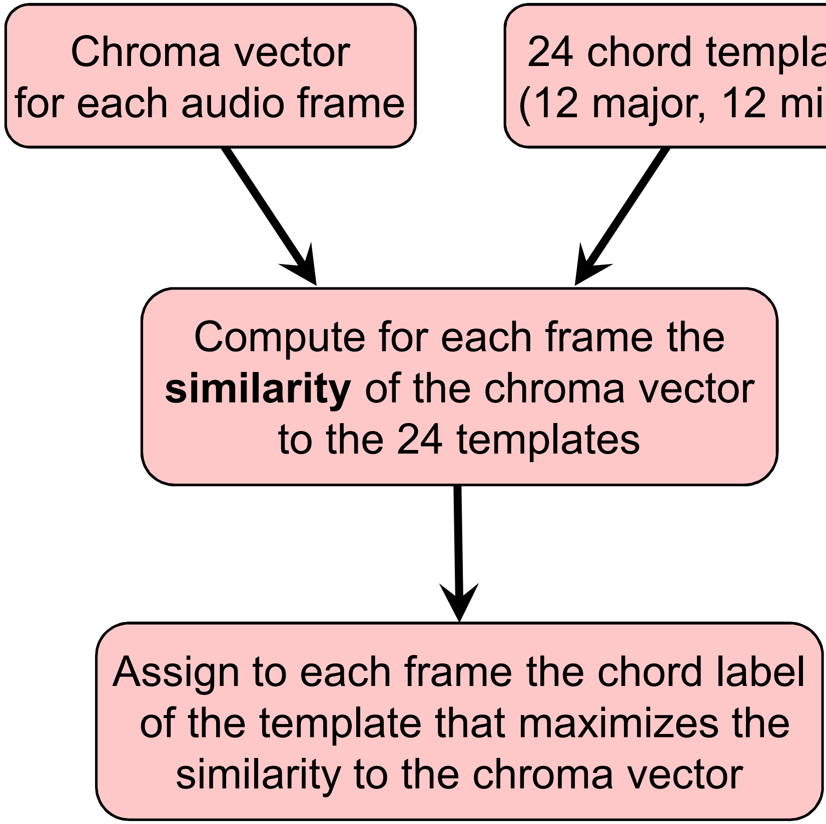
Chroma vector: $\boldsymbol{c} \in \mathbb{R}^{12}$

Similarity measure: $s(\boldsymbol{t}, \boldsymbol{c}) = \dfrac{\langle \boldsymbol{t} | \boldsymbol{c} \rangle}{||\boldsymbol{t}|| \cdot ||\boldsymbol{c}||}$

Chord Recognition: Template Matching

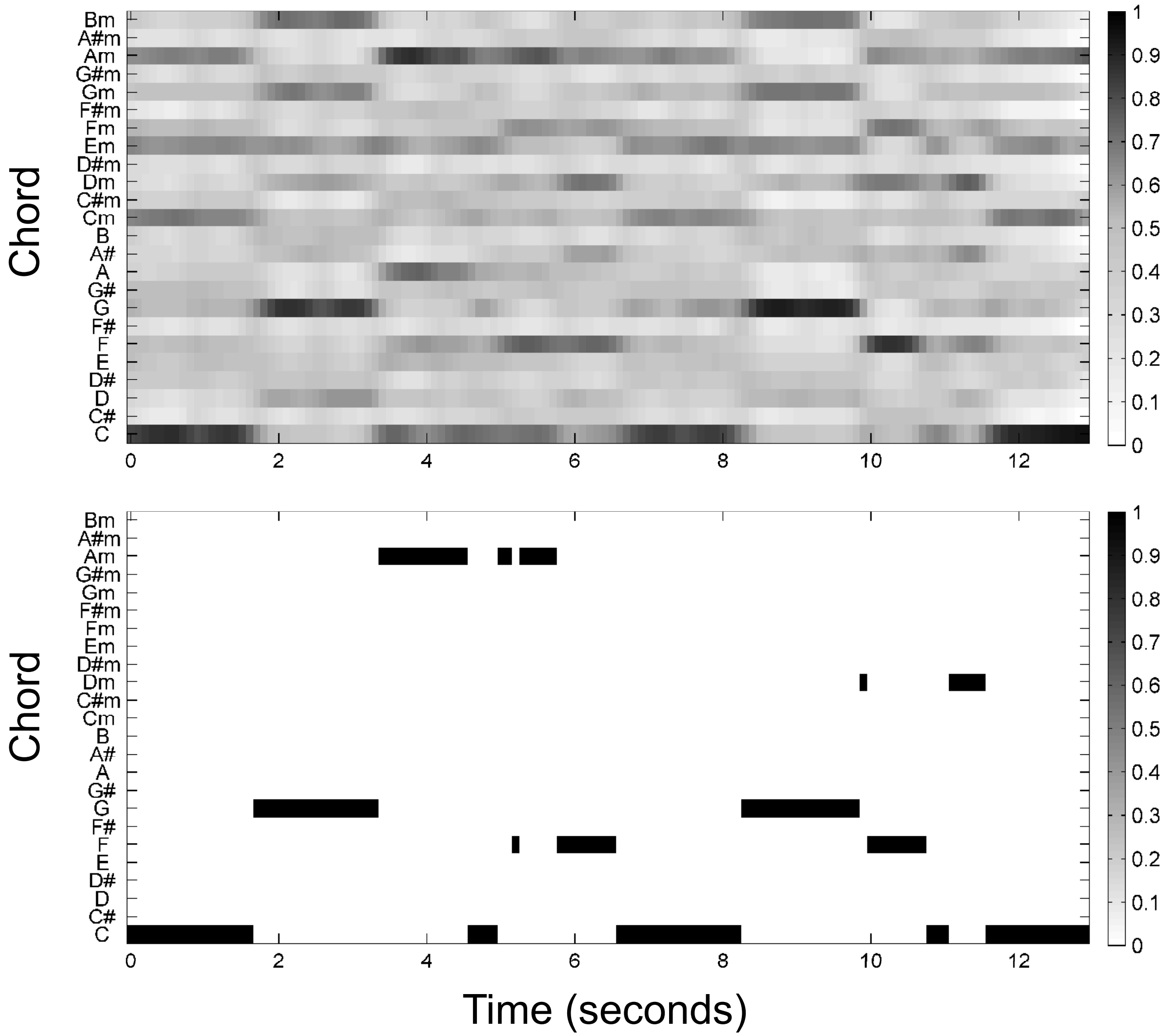


Chord Recognition: Label Assignment

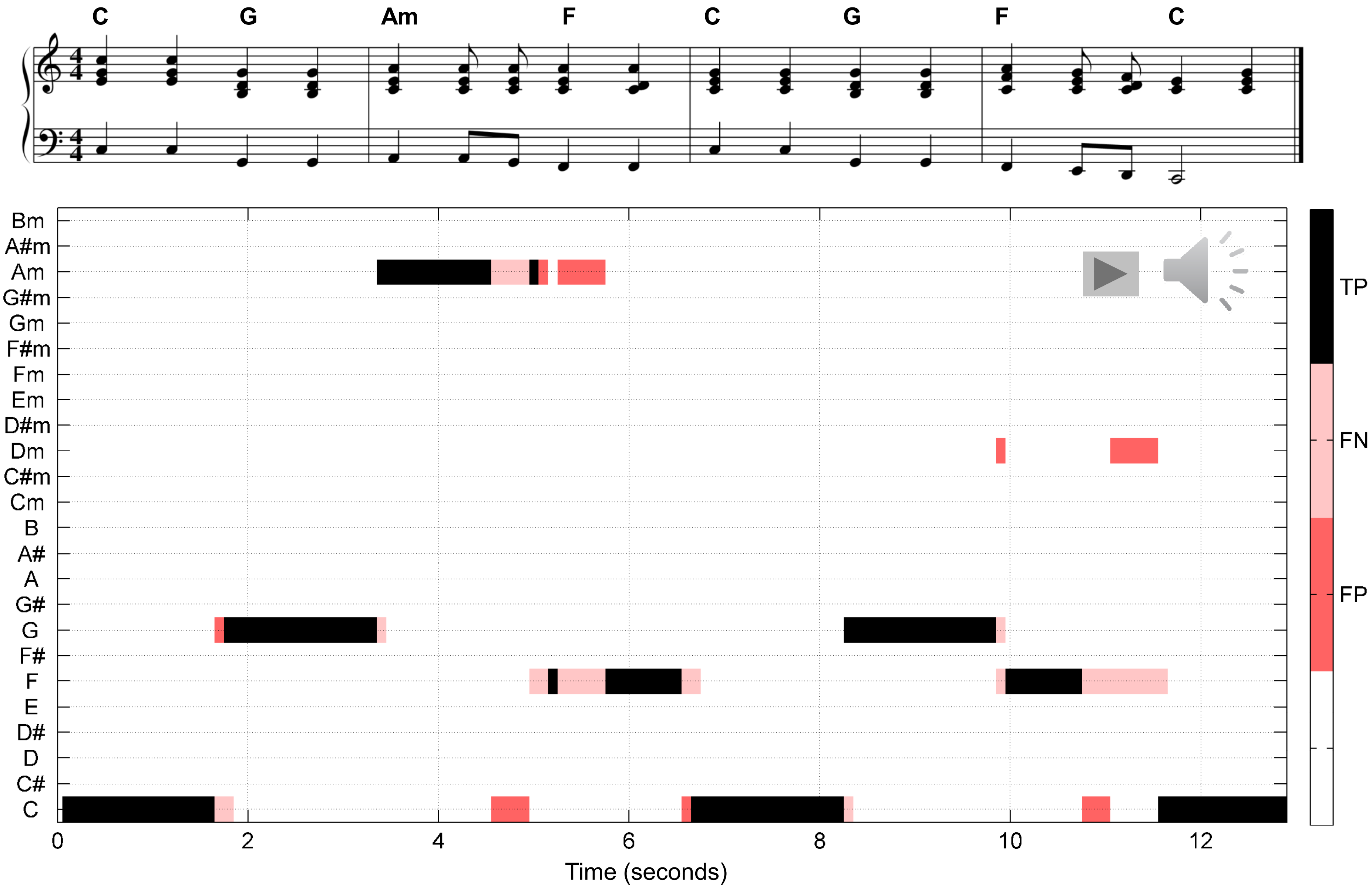


	C	C$^\sharp$	D	…	Cm	C$^\sharp$m	Dm	…
B	0	0	0	…	0	0	0	…
A$^\sharp$	0	0	0	…	0	0	0	…
A	0	0	1	…	0	0	1	…
G$^\sharp$	0	1	0	…	0	1	0	…
G	1	0	0	…	1	0	0	…
F$^\sharp$	0	0	1	…	0	0	0	…
F	0	1	0	…	0	0	1	…
E	1	0	0	…	0	1	0	…
D$^\sharp$	0	0	0	…	1	0	0	…
D	0	0	1	…	0	0	1	…
C$^\sharp$	0	1	0	…	0	1	0	…
C	1	0	0	…	1	0	0	…

Chord Recognition: Label Assignment



Chord Recognition: Evaluation



Chord Recognition: Evaluation

- "No-Chord" annotations: not every frame labeled

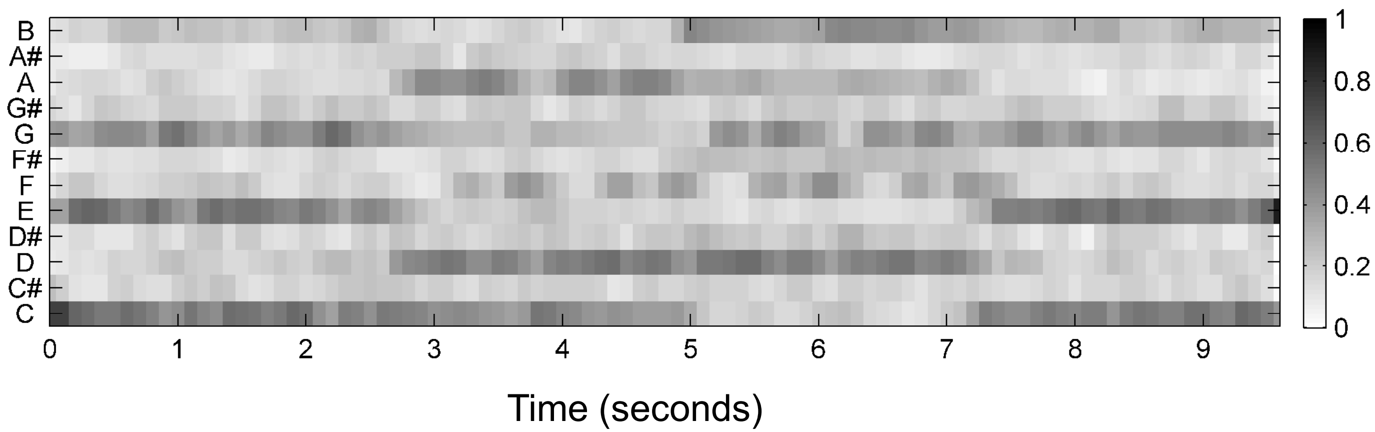
- Different evaluation measures:
  - Precision: $$P = \frac{\#\mathrm{TP}}{\#\mathrm{TP} + \#\mathrm{FP}}$$ „how many predicted chords are correct“
  - Recall: $$R = \frac{\#\mathrm{TP}}{\#\mathrm{TP} + \#\mathrm{FN}}$$ „how many annotated chords are recognized“
  - F-Measure (balances precision and recall):

$$F = \frac{2 \cdot P \cdot R}{P + R}$$ harmonic mean of P and R

- Without "No-Chord" label: $P = R = F$

Chord Recognition: Smoothing

- Apply average filter of length $L \in \mathbb{N}$:



Chord Recognition: Smoothing

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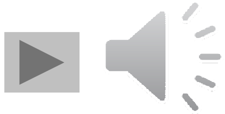
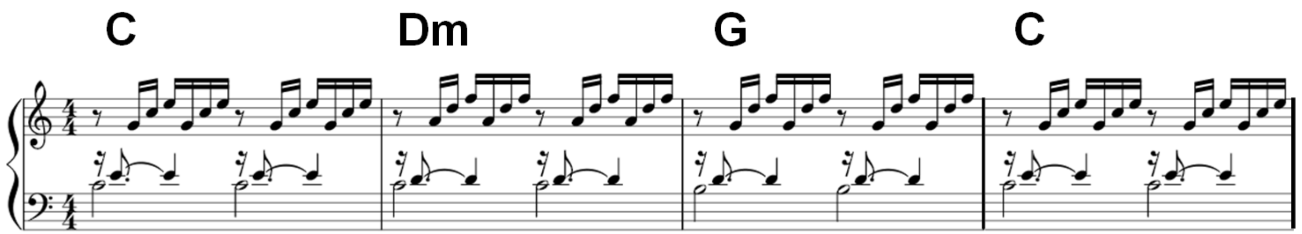
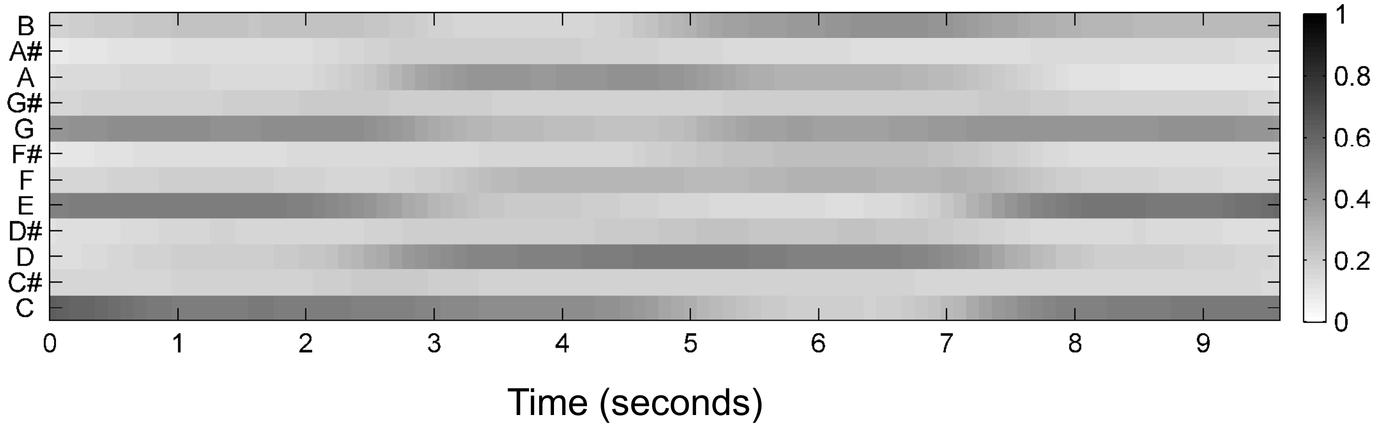
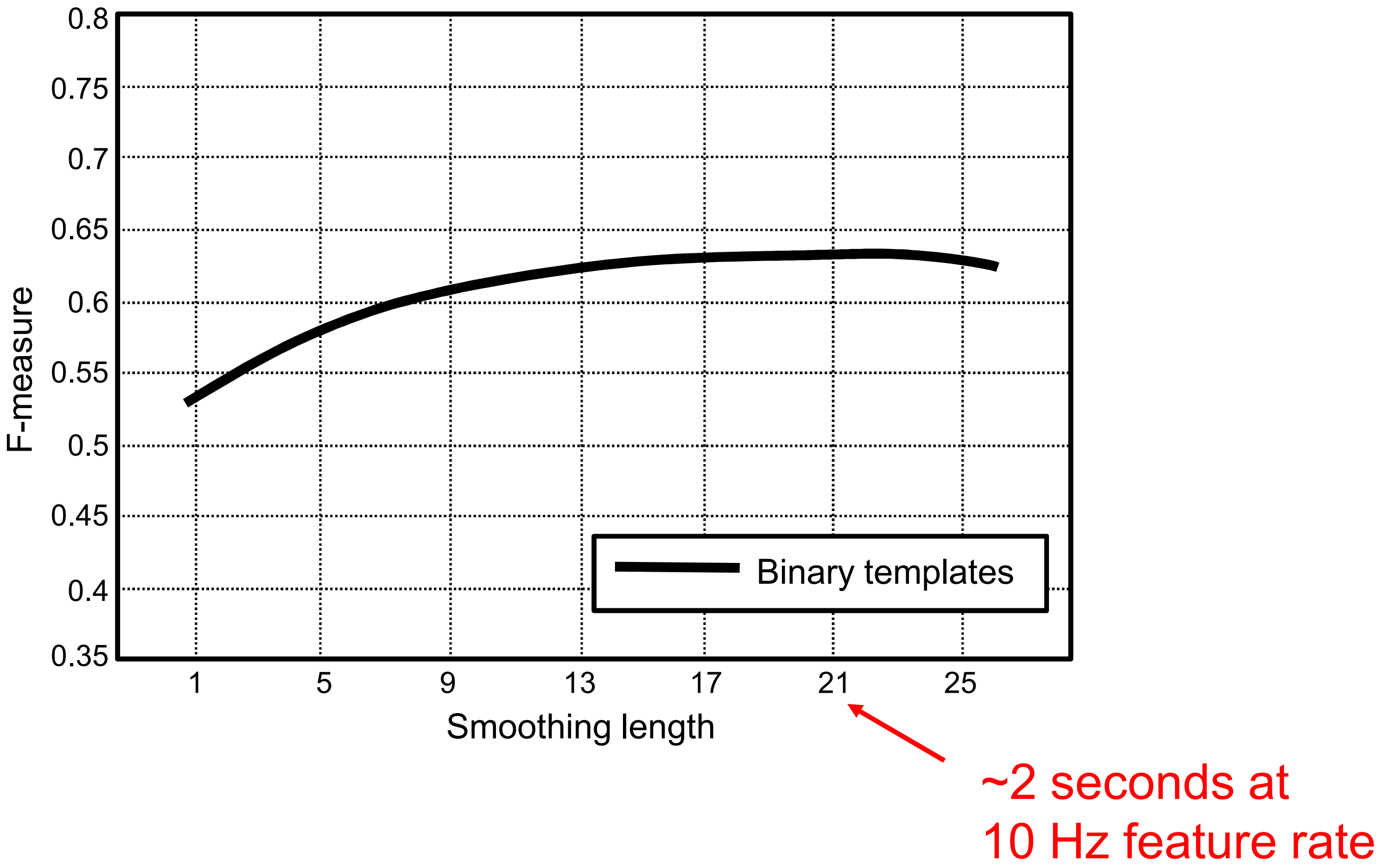






Chord Recognition: Smoothing

- Evaluation on all 180 Beatles songs (10 studio albums)

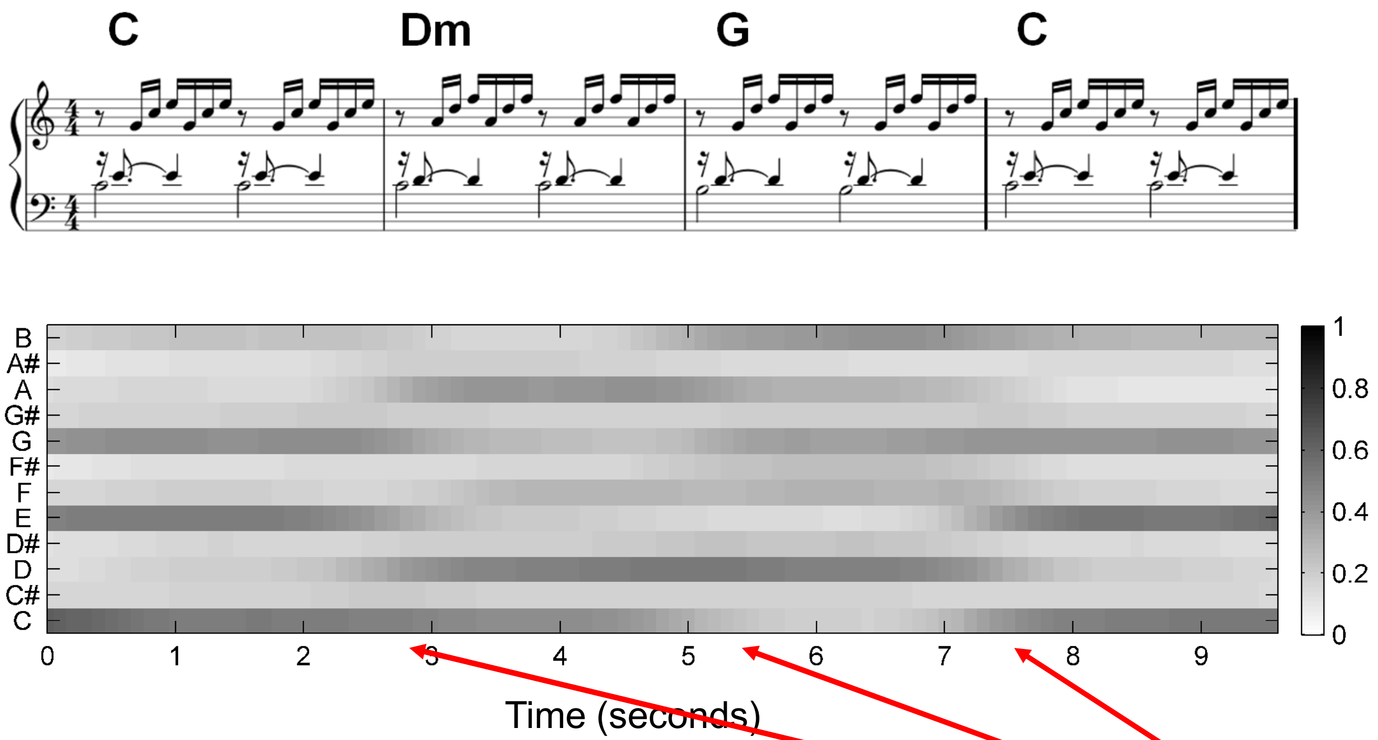


Harmony Analysis
Overview

- Template-based Chord Recognition
- HMM-based Chord Recognition
- Local Key Detection
- Application for Musicology

Template-Based Chord Recognition: Smoothing

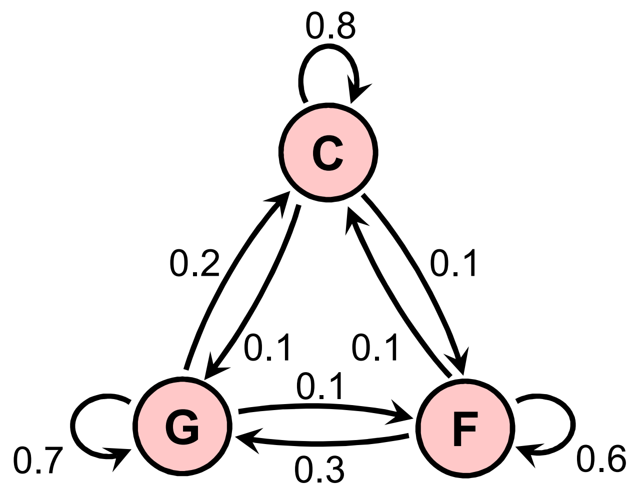
- Apply average filter of length $L \in \mathbb{N}$:



blurring of boundaries!

Markov Chains

- Probabilistic model for sequential data
- Markov property: Next state only depends on current state (transition model – time-invariant, no "memory")
- Consist of:
  - Set of states
  - State transition probabilities
  - *Initial state probabilities*



Markov Chains

Notation:

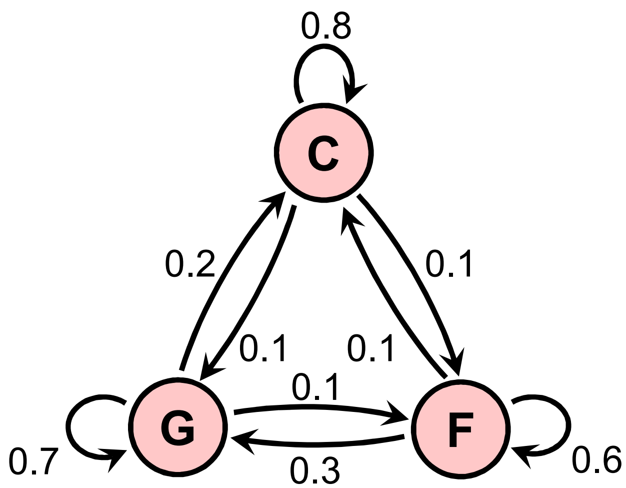
States $\qquad \alpha_i$ for $i \in [1:I]$

State transition probabilities $\qquad a_{ij}$

A	α_1	α_2	α_3
α_1	a_{11}	a_{12}	a_{13}
α_2	a_{21}	a_{22}	a_{23}
α_3	a_{31}	a_{32}	a_{33}

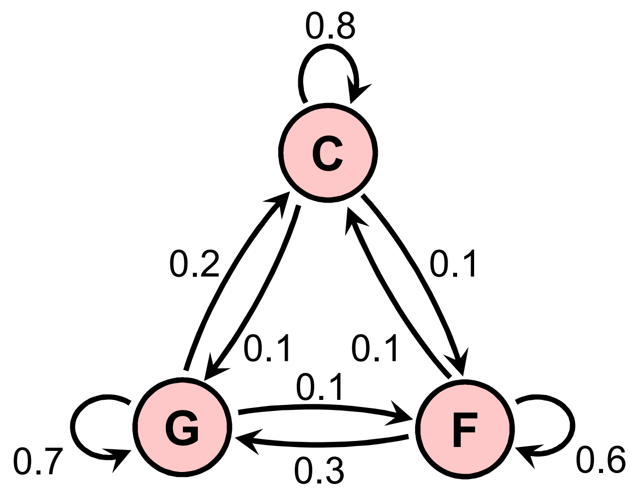
Initial state probabilities $\qquad c_i$

C	α_1	α_2	α_3
	c_1	c_2	c_3

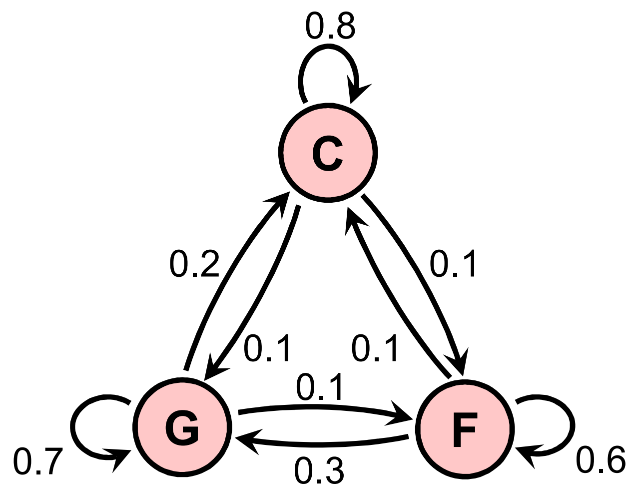


Markov Chains

- Application examples:
  - Compute probability of a sequence using given a model (evaluation)
  - Compare two sequences using a given model
  - Evaluate a sequence with two different models (classification)



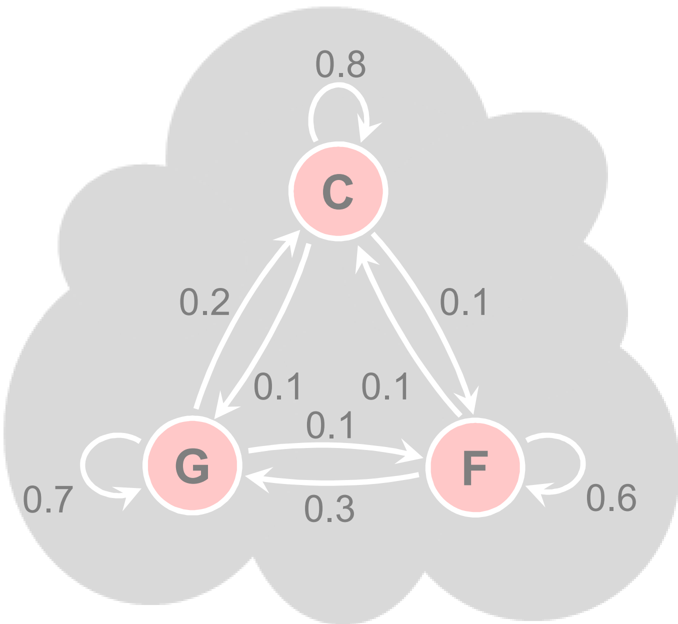
Hidden Markov Model



Hidden Markov Model

- States as **hidden** variables

- Consist of:
  - Set of states (hidden)
  - State transition probabilities
  - *Initial state probabilities*



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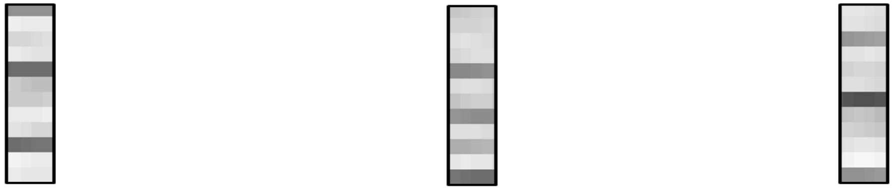
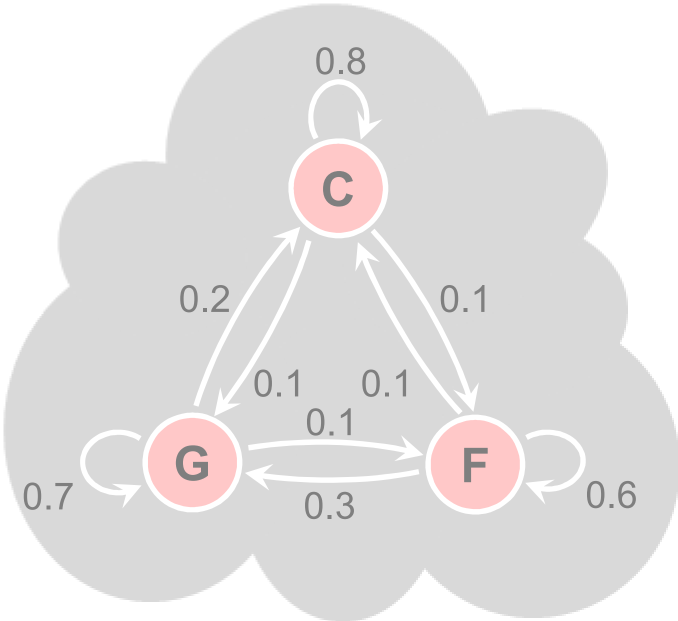
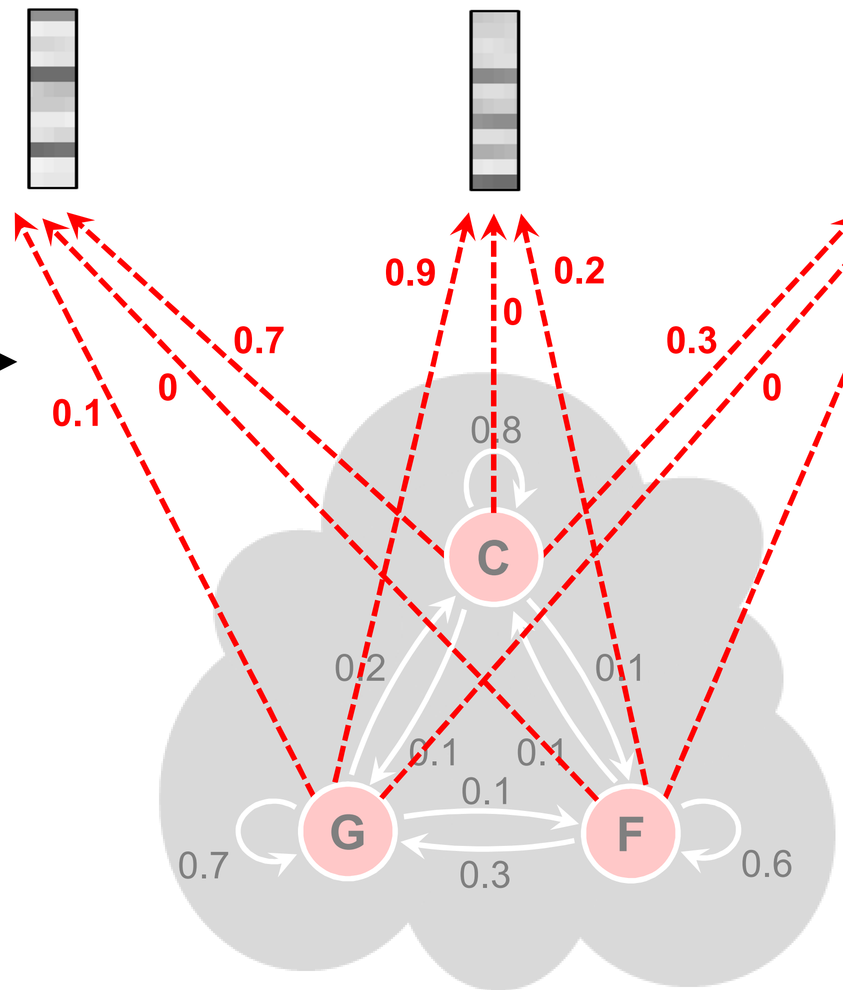


- Consist of:
  - Set of states (hidden)
  - State transition probabilities
  - *Initial state probabilities*
  - Observations (visible)



Hidden Markov Model

- States as **hidden** variables
- Consist of:
  - Set of states (hidden)
  - State transition probabilities
  - *Initial state probabilities*
  - Observations (visible)
  - Emission probabilities



Hidden Markov Model

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States — α_i for $i \in [1:I]$

State transition probabilities — a_{ij}

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α1	a_{11}	a_{12}	a_{13}
α2	a_{21}	a_{22}	a_{23}
α3	a_{31}	a_{32}	a_{33}

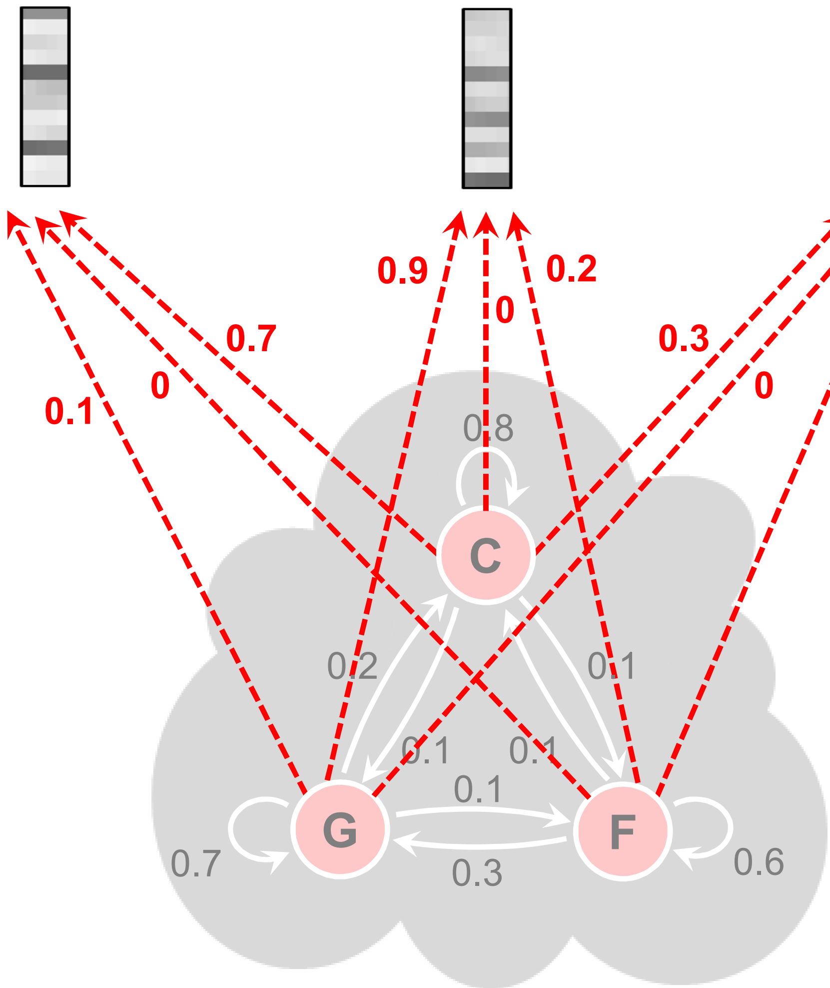
Initial state probabilities — c_i

C	α1	α2	α3
	c_1	c_2	c_3

Observation symbols — β_k for $k \in [1:K]$

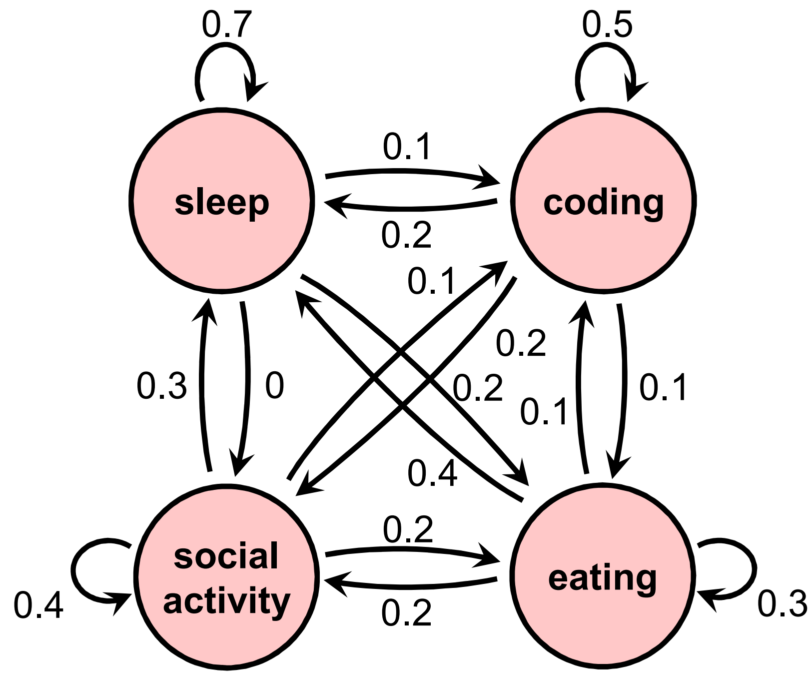
Emission probabilities — b_{ik}

B	β1	β2	β3
α1	b_{11}	b_{12}	b_{13}
α2	b_{21}	b_{22}	b_{23}
α3	b_{31}	b_{32}	b_{33}



Markov Chains

- Analogon: the student's life
  - Set of states (hidden)
  - State transition probabilities
  - *Initial state probabilities*



Hidden Markov Model

- Analogon: the student's life
- Consists of:
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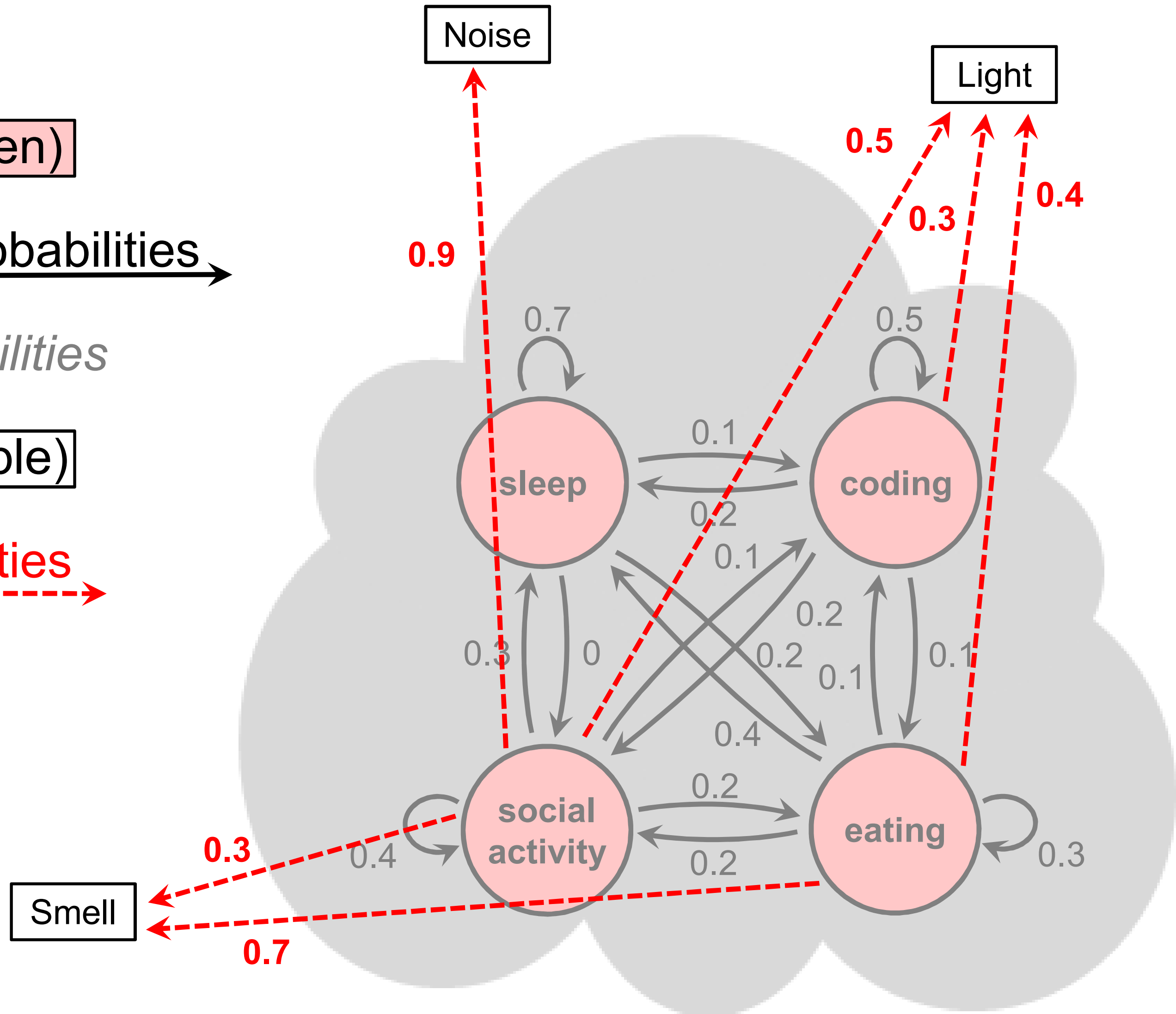
  - Observations (visible)
  - Emission probabilities



Hidden Markov Model

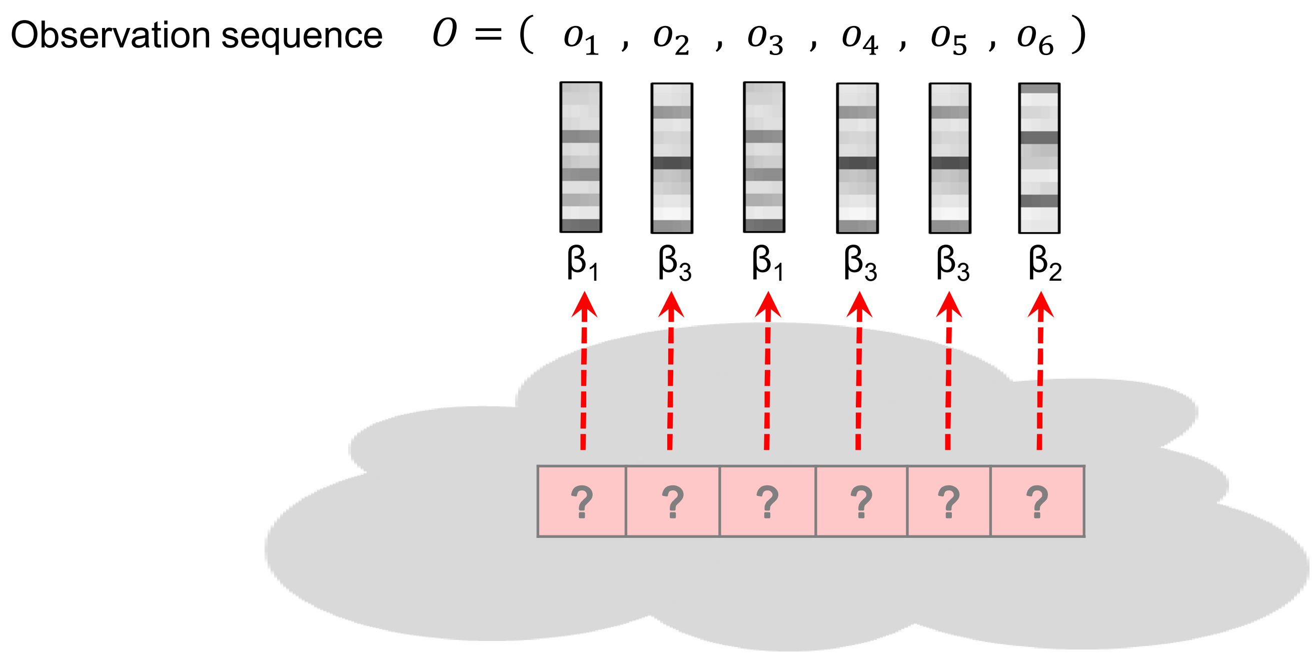
- Only observation sequence is visible!

Different algorithmic problems:

- **Evaluation problem**
  - *Given:* observation sequence and model
  - *Find:* fitness (how well the model matches the sequence)
- **Uncovering problem:**
  - *Given:* observation sequence and model
  - *Find:* optimal hidden state sequence
- **Estimation problem** („training“ the HMM):
  - *Given:* observation sequence
  - *Find:* model parameters
  - Baum-Welch algorithm (Expectation-Maximization)

Uncovering Problem

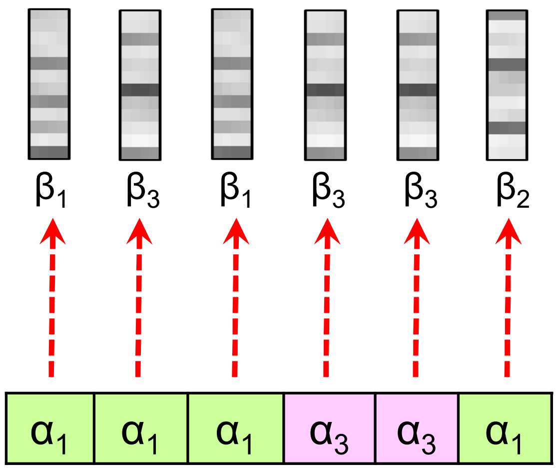
- *Given:* observation sequence $O = (o_1, \dots, o_N)$ of length $N \in \mathbb{N}$ and HMM Θ (model parameters)
- *Find:* optimal hidden state sequence $S^* = (s_1^*, \dots, s_N^*)$
- Corresponds to chord estimation task!



Uncovering Problem

- *Given:* observation sequence $O = (o_1, \ldots, o_N)$ of length $N \in \mathbb{N}$ and HMM Θ (model parameters)
- *Find:* optimal hidden state sequence $S^* = (s_1^*, \ldots, s_N^*)$
- Corresponds to chord estimation task!

Observation sequence $O = (\ o_1\ ,\ o_2\ ,\ o_3\ ,\ o_4\ ,\ o_5\ ,\ o_6\ )$



Hidden state sequence $S^* = (\ s_1^*\ ,\ s_2^*\ ,\ s_3^*\ ,\ s_4^*\ ,\ s_5^*\ ,\ s_6^*\ )$

Uncovering Problem

- *Given:* observation sequence $O = (o_1, \ldots, o_N)$ of length $N \in \mathbb{N}$ and HMM Θ (model parameters)
- *Find:* optimal hidden state sequence $S^* = (s_1^*, \ldots, s_N^*)$
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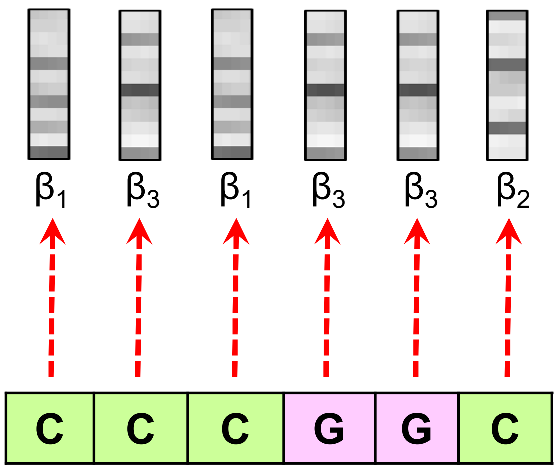
Observation sequence $O = (\ o_1\ ,\ o_2\ ,\ o_3\ ,\ o_4\ ,\ o_5\ ,\ o_6\ )$



Hidden state sequence $S^* = (\ s_1^*\ ,\ s_2^*\ ,\ s_3^*\ ,\ s_4^*\ ,\ s_5^*\ ,\ s_6^*\ )$

Uncovering Problem

- **Optimal** hidden state sequence?
  - "Best explains" given observation sequence O
  - Maximizes probability $P[O, S \mid \Theta]$

$$\text{Prob}^* = \max_S P[O, S \mid \Theta]$$

$$S^* = \underset{S}{\operatorname{argmax}} P[O, S \mid \Theta]$$

- Straight-forward computation (naive approach):
  - Compute probability for each possible sequence S
  - Number of possible sequences of length N (I = number of states):

$$\underbrace{I \cdot I \cdot \ldots \cdot I}_{N \text{ factors}} = I^N$$

computationally infeasible!

Viterbi Algorithm

- Based on dynamic programming (similar to DTW)
- Idea: Recursive computation from sub-problems
- Use truncated versions of observation sequence

$$O(1:n) \coloneqq (o_1, \ldots, o_n), \text{ length } n \in [1:N]$$

- Define $\mathbf{D}(i,n)$ as the highest probability along a single state sequence $(s_1, \ldots, s_n)$ that ends in state $s_n = \alpha_i$

$$\mathbf{D}(i,n) = \max_{(s_1,\ldots,s_n)} P[O(1:n), (s_1, \ldots, s_{n-1}, s_n = \alpha_i) \mid \Theta]$$

- Then, our solution is the state sequence yielding

$$\mathrm{Prob}^* = \max_{i \in [1:I]} \mathbf{D}(i,N)$$

Viterbi Algorithm

- $\mathbf{D}$: matrix of size $I \times N$
- Recursive computation of $\mathbf{D}(i, n)$ along the column index n
- Initialization:
  - $n = 1$
  - Truncated observation sequence: $O(1) = (o_1)$
  - Current observation: $o_1 = \beta_{k_1}$

$\mathbf{D}(i, 1) = c_i \cdot b_{ik_1}$ for some $i \in [1:I]$

Viterbi Algorithm

- $\mathbf{D}$: matrix of size $I \times N$
- Recursive computation of $\mathbf{D}(i,n)$ along the column index n
- Recursion:
  - $n \in [2:N]$
  - Truncated observation sequence: $O(1:n) = (o_1, \ldots, o_n)$
  - Last observation: $o_n = \beta_{k_n}$

$$\mathbf{D}(i,n) = b_{ik_n} \cdot a_{j^*i} \cdot \underbrace{P\big[O(1:n-1), (s_1, \ldots, s_{n-1} = \alpha_{j^*}) \,\big|\, \Theta\big]}_{\text{must be maximal!}} \qquad \text{for } i \in [1:I]$$

$$\mathbf{D}(i,n) = b_{ik_n} \cdot a_{j^*i} \cdot \mathbf{D}(j^*, n-1)$$

Viterbi Algorithm

- $\mathbf{D}$: matrix of size $I \times N$
- Recursive computation of $\mathbf{D}(i,n)$ along the column index n
- Recursion:
  - $n \in [2:N]$
  - Truncated observation sequence: $O(1:n) = (o_1, \ldots, o_n)$
  - Last observation: $o_n = \beta_{k_n}$

$$\mathbf{D}(i,n) = b_{ik_n} \cdot a_{j^*i} \cdot \underbrace{P\big[O(1:n-1), (s_1, \ldots, s_{n-1} = \alpha_{j^*}) \,\big|\, \Theta\big]}_{\text{must be maximal!}} \qquad \text{for } i \in [1:I]$$

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \underbrace{a_{j^*i} \cdot \mathbf{D}(j^*, n-1)}_{\text{must be maximal (best index } j^*)}$$

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \max_{j \in [1:I]} \big(a_{ji} \cdot \mathbf{D}(j, n-1)\big)$$

Viterbi Algorithm

- $\mathbf{D}$ given – find optimal state sequence $S^* = (s_1^*, \ldots, s_N^*) := \left(\alpha_{i_1}, \ldots, \alpha_{i_N}\right)$
- Backtracking procedure (reverse order)
- Last element:
  - $n = N$
  - Optimal state: α_{i_N}

$$i_N = \underset{j \in [1:I]}{\operatorname{argmax}} \, \mathbf{D}(j, N)$$

Viterbi Algorithm

- $\mathbf{D}$ given – find optimal state sequence $S^* = (s_1^*, \ldots, s_N^*) := \left(\alpha_{i_1}, \ldots, \alpha_{i_N}\right)$
- Backtracking procedure (reverse order)
- Further elements:
  - $n = N-1, N-2, \ldots, 1$
  - Optimal state: α_{i_n}

$$i_n = \underset{j \in [1:I]}{\operatorname{argmax}} \left(a_{j i_{n+1}} \cdot \mathbf{D}(j, n)\right)$$

Viterbi Algorithm

- $\mathbf{D}$ given – find optimal state sequence $S^* = (s_1^*, \ldots, s_N^*) := \left(\alpha_{i_1}, \ldots, \alpha_{i_N}\right)$
- Backtracking procedure (reverse order)
- Further elements:
  - $n = N-1, N-2, \ldots, 1$
  - Optimal state: α_{i_n}

$$i_n = \underset{j\in[1:I]}{\operatorname{argmax}}\left(a_{j i_{n+1}} \cdot \mathbf{D}(j,n)\right)$$

- Simplification of backtracking: Keep track of maximizing index j in

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \max_{j\in[1:I]}\left(a_{ji} \cdot \mathbf{D}(j,n-1)\right)$$

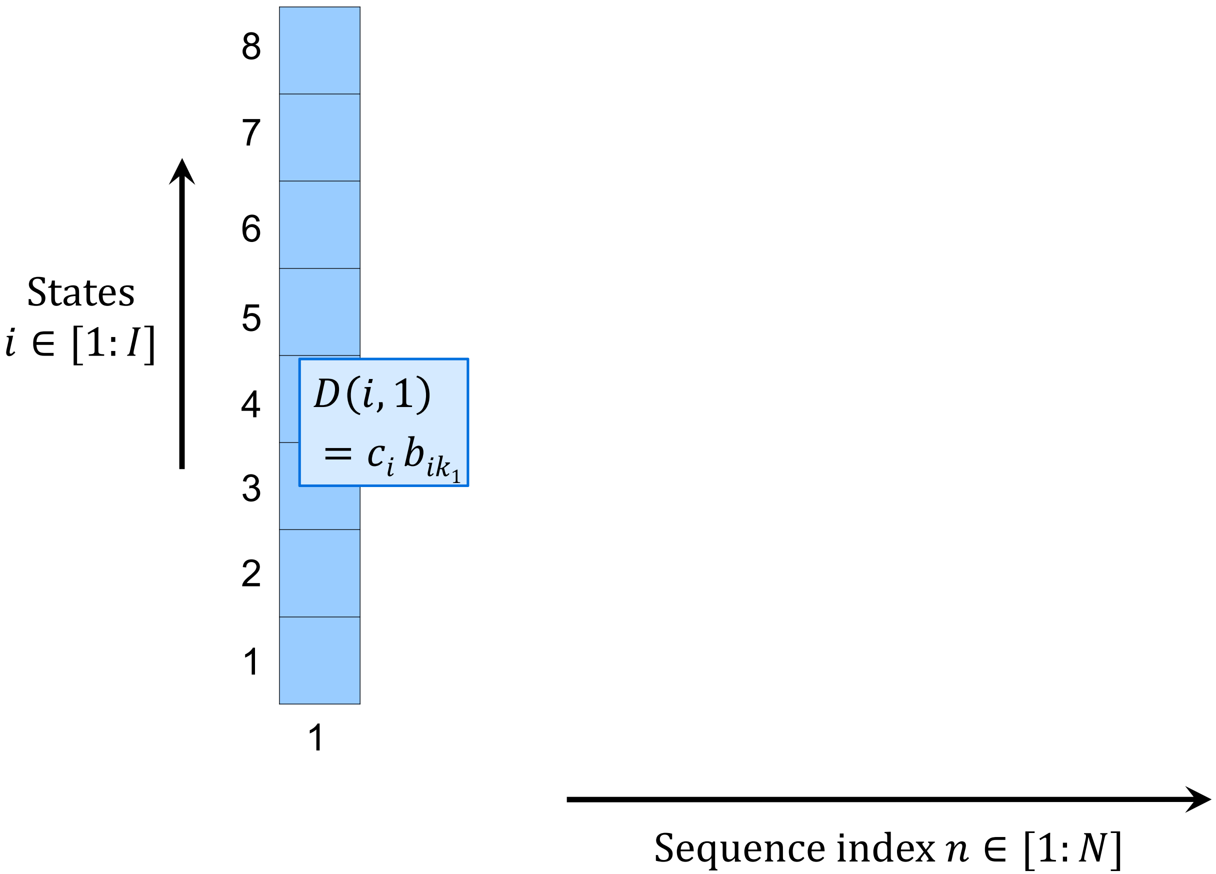
- Define $(I \times (N-1))$ matrix $\mathbf{E}$:

$$\mathbf{E}(i,n-1) = \underset{j\in[1:I]}{\operatorname{argmax}}\left(a_{ji} \cdot \mathbf{D}(j,n-1)\right)$$

Viterbi Algorithm

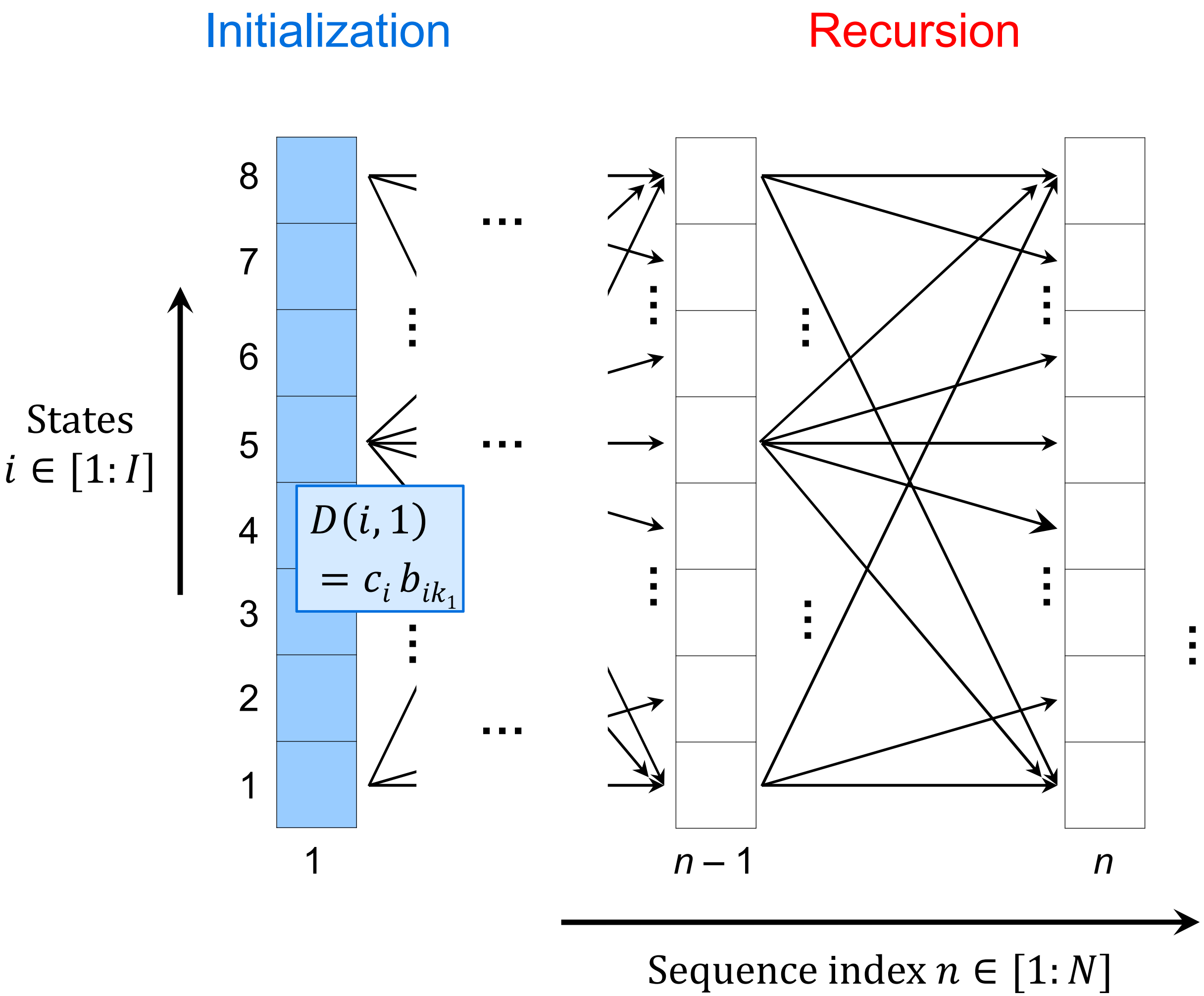
Summary

Initialization



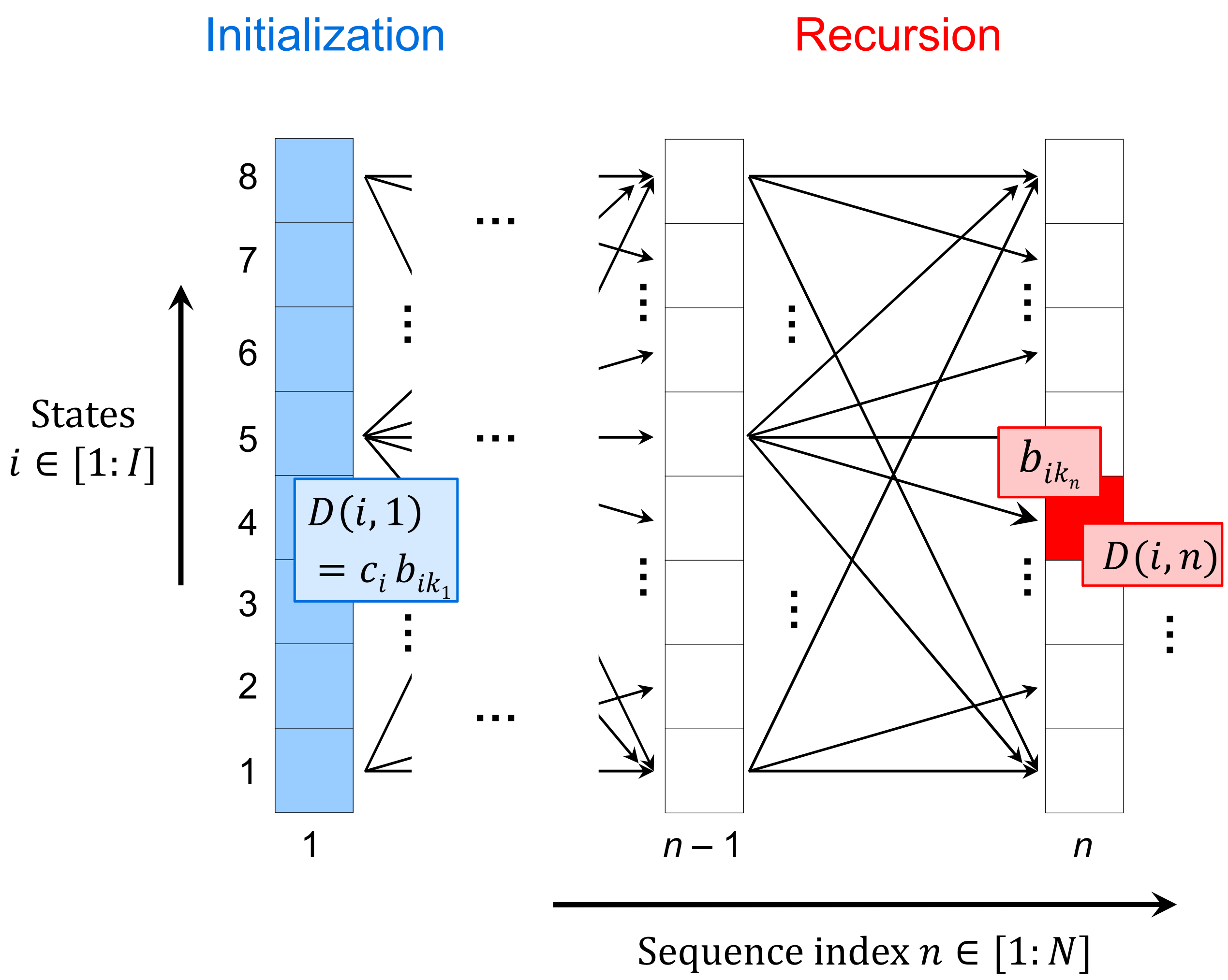
Viterbi Algorithm

Summary



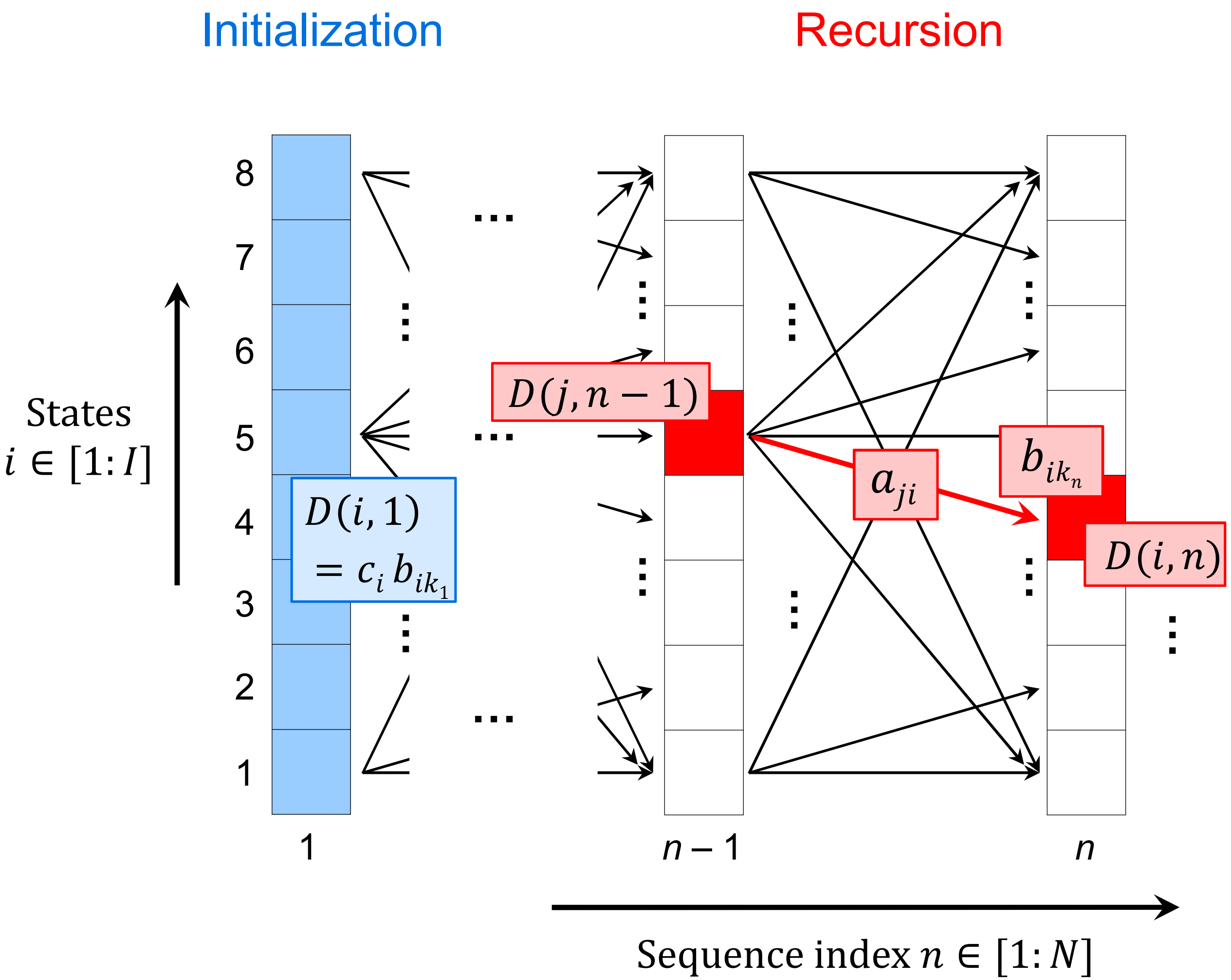
Viterbi Algorithm

Summary



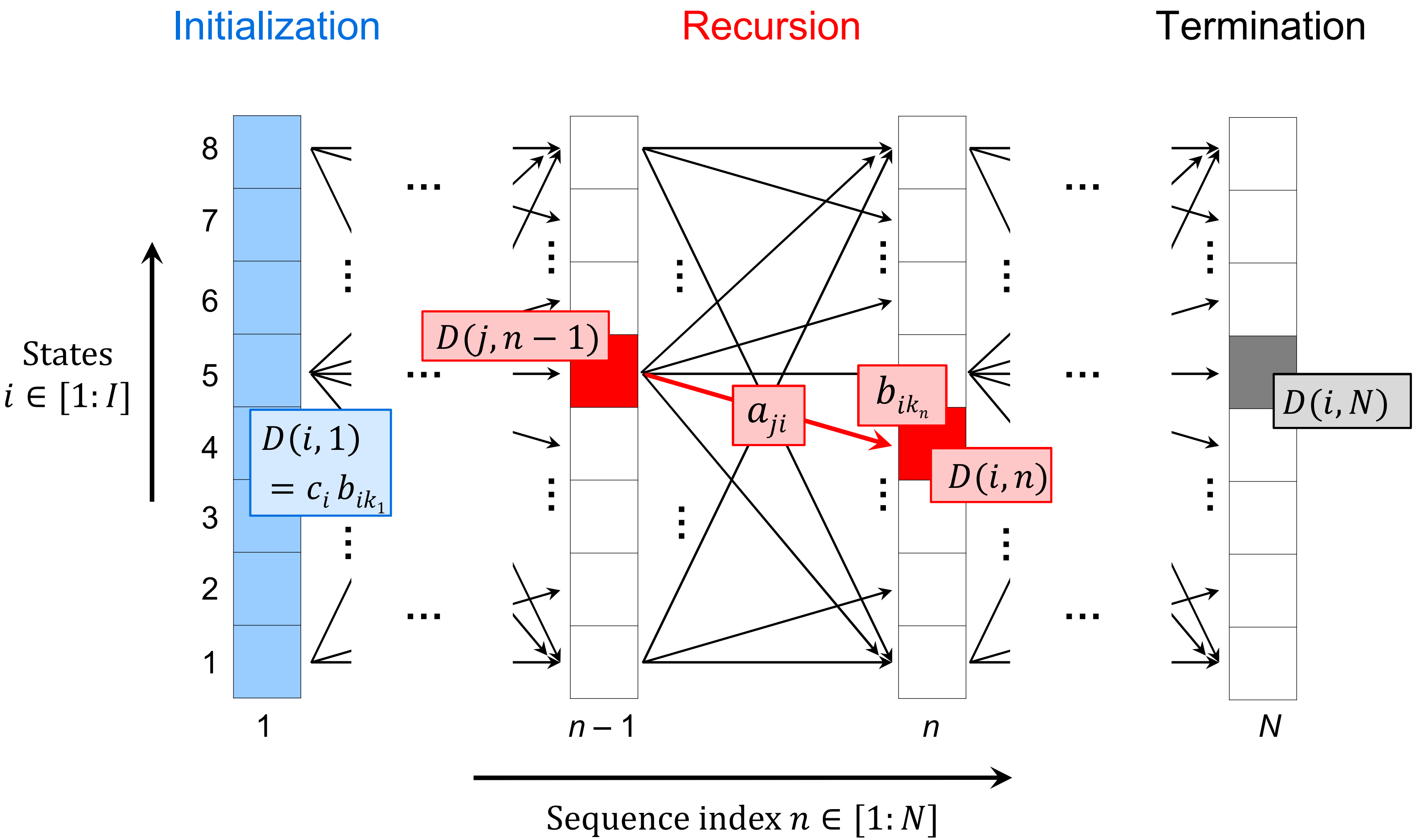
Viterbi Algorithm

Summary



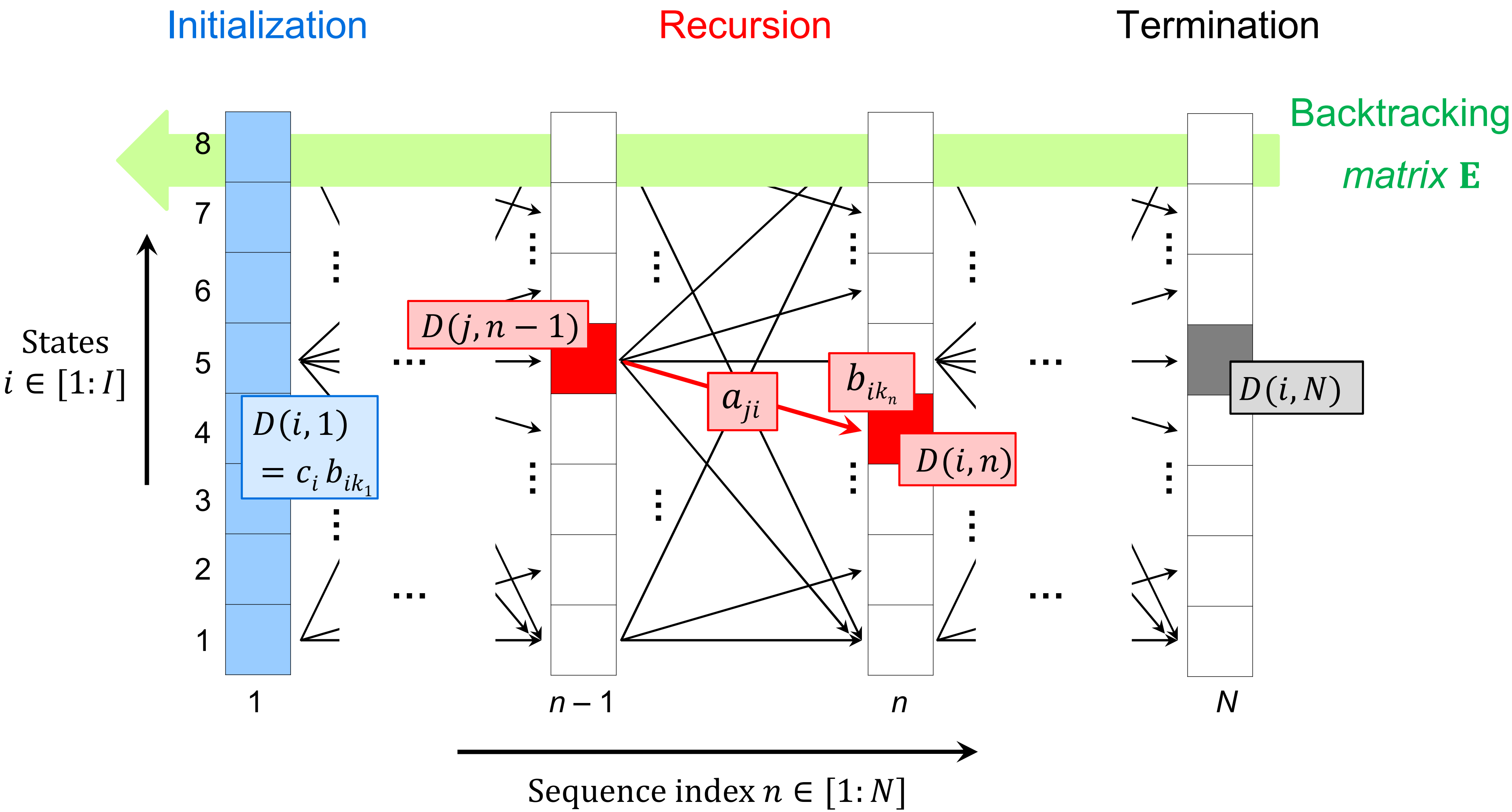
Viterbi Algorithm

Summary



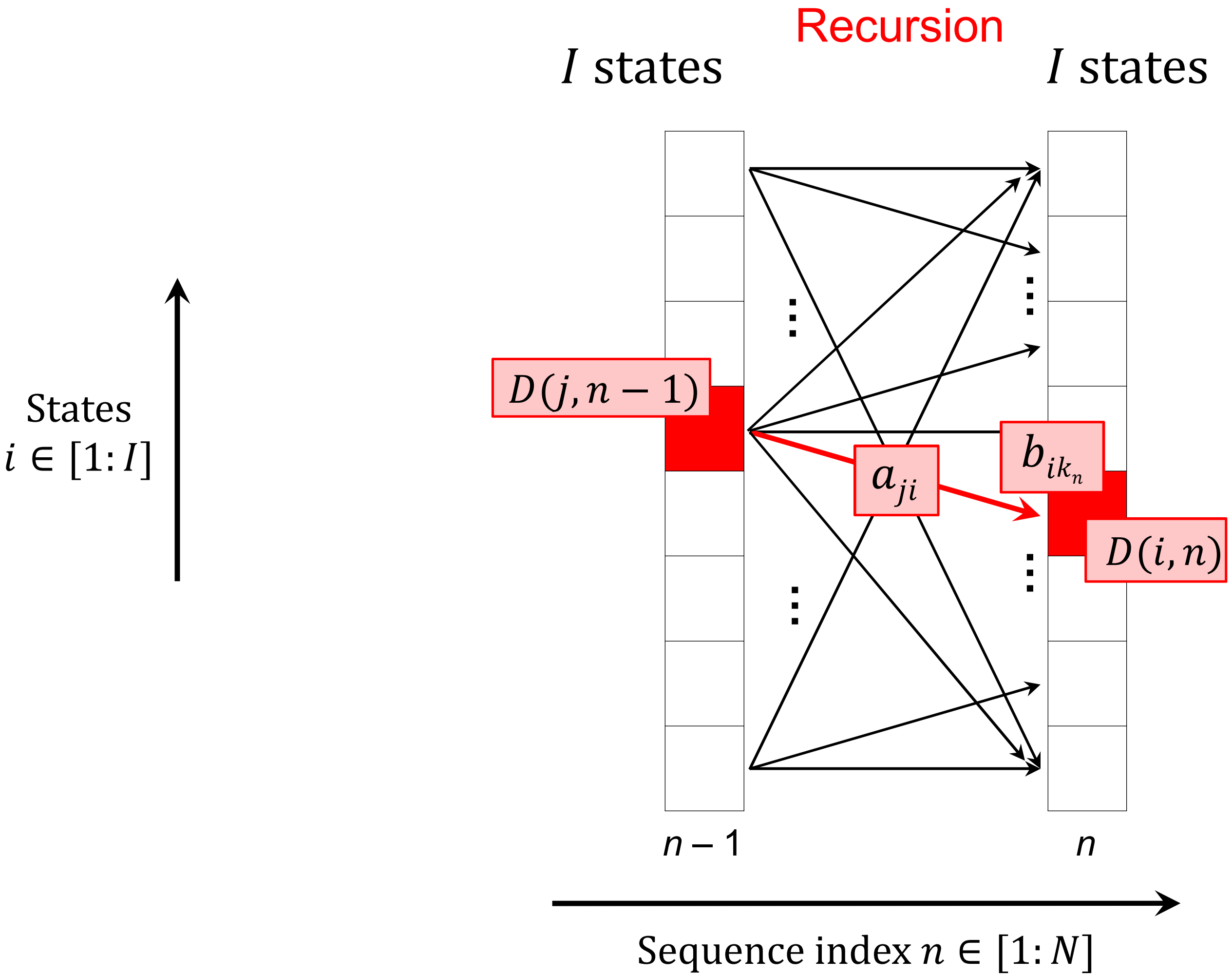
Viterbi Algorithm

Summary



Viterbi Algorithm

Computational Complexity



Per recursion step:
$I \cdot I$

Total recursion:
$I^2 \cdot N$

Viterbi Algorithm

Summary

Algorithm: VITERBI

Input: HMM specified by $\Theta = (\mathcal{A}, A, C, \mathcal{B}, B)$
Observation sequence $O = (o_1 = \beta_{k_1}, o_2 = \beta_{k_2}, \ldots, o_N = \beta_{k_N})$
Output: Optimal state sequence $S^* = (s_1^*, s_2^*, \ldots, s_N^*)$

Procedure: Initialize the $(I \times N)$ matrix $\mathbf{D}$ by $\mathbf{D}(i,1) = c_i b_{ik_1}$ for $i \in [1:I]$. Then compute in a nested loop for $n = 2, \ldots, N$ and $i = 1, \ldots, I$:

$$\begin{aligned} \mathbf{D}(i,n) &= \max_{j \in [1:I]} \left(a_{ji} \cdot \mathbf{D}(j, n-1) \right) \cdot b_{ik_n} \\ \mathbf{E}(i, n-1) &= \operatorname{argmax}_{j \in [1:I]} \left(a_{ji} \cdot \mathbf{D}(j, n-1) \right) \end{aligned}$$

Set $i_N = \operatorname{argmax}_{j \in [1:I]} \mathbf{D}(j, N)$ and compute for decreasing $n = N-1, \ldots, 1$ the maximizing indices

$$i_n = \operatorname{argmax}_{j \in [1:I]} \left(a_{ji_{n+1}} \cdot \mathbf{D}(j,n) \right) = \mathbf{E}(i_{n+1}, n).$$

The optimal state sequence $S^* = (s_1^*, \ldots, s_N^*)$ is defined by $s_n^* = \alpha_{i_n}$ for $n \in [1:N]$.

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α1	α2	α3
α1	a_{11}	a_{12}	a_{13}
α2	a_{21}	a_{22}	a_{23}
α3	a_{31}	a_{32}	a_{33}

Emission probabilities

b_{ik}

B	β1	β2	β3
α1	b_{11}	b_{12}	b_{13}
α2	b_{21}	b_{22}	b_{23}
α3	b_{31}	b_{32}	b_{33}

Initial state probabilities

c_i

C	α1	α2	α3
	c_1	c_2	c_3

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities

b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
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α_3	0.1	0.3	0.6

Emission probabilities

b_{ik}

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α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

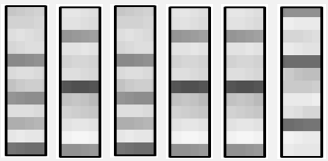
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

Observation sequence

$O = (o_1, o_2, o_3, o_4, o_5, o_6)$



β_1 β_3 β_1 β_3 β_3 β_2

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

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α_3	0.1	0.3	0.6

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α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

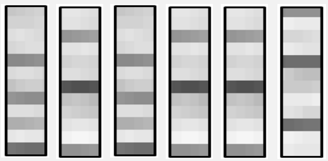
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

Observation sequence

$O = (o_1, o_2, o_3, o_4, o_5, o_6)$



β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$	$o_6 = \beta_2$
α_1						
α_2						
α_3						

E	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$
α_1					
α_2					
α_3					

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

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Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6

β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$	$o_6 = \beta_2$
α_1						
α_2						
α_3						

E	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$
α_1					
α_2					
α_3					

Initialization

$$\mathbf{D}(i,1) = c_i \cdot b_{ik_1}$$

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

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β_k for $k \in [1:K]$

State transition probabilities

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α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6

β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$	$o_6 = \beta_2$
α_1	0.4200					
α_2	0.0200					
α_3	0					

E	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$
α_1					
α_2					
α_3					

Initialization

$$\mathbf{D}(i,1) = c_i \cdot b_{ik_1}$$

Recursion

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \max_{j \in [1:I]} \left(a_{ji} \cdot \mathbf{D}(j, n-1) \right)$$

$$\mathbf{E}(i,n-1) = \underset{j \in [1:I]}{\operatorname{argmax}} \left(a_{ji} \cdot \mathbf{D}(j, n-1) \right)$$

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

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α_1	0.8	0.1	0.1
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α_3	0.1	0.3	0.6

Emission probabilities

b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6

β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$	$o_6 = \beta_2$
α_1	0.4200	0.1008				
α_2	0.0200	0				
α_3	0	0.0336				

E	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$
α_1	1				
α_2	1				
α_3	1				

Initialization

$$\mathbf{D}(i,1) = c_i \cdot b_{ik_1}$$

Recursion

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \max_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1) \right)$$

$$\mathbf{E}(i,n-1) = \operatorname*{argmax}_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1) \right)$$

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities

b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6

β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$	$o_6 = \beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$
α_1	1	1	1	1	1
α_2	1	1	1	1	3
α_3	1	3	1	3	3

Backtracking

$$i_N = \underset{j \in [1:I]}{\operatorname{argmax}} \mathbf{D}(j, n)$$

$$i_n = \mathbf{E}(i_{n+1}, n)$$

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α_1	α_2	α_3
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Emission probabilities

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B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6

β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	1	1	1	1	1
α_2	1	1	1	1	3
α_3	1	3	1	3	3

Backtracking

$$i_N = \operatorname*{argmax}_{j \in [1:I]} \mathbf{D}(j, n)$$

$$i_n = \mathbf{E}(i_{n+1}, n)$$

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities

b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6

β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$	$o_6 = \beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1 = \beta_1$	$o_2 = \beta_3$	$o_3 = \beta_1$	$o_4 = \beta_3$	$o_5 = \beta_3$
α_1	$\boxed{1}$	$\boxed{1}$	1	1	1
α_2	1	1	1	1	$\boxed{3}$
α_3	1	3	$\boxed{1}$	$\boxed{3}$	3

$\boxed{i_6 = 2}$

Backtracking

$$i_N = \underset{j \in [1:I]}{\operatorname{argmax}} \mathbf{D}(j, n)$$

$$i_n = \mathbf{E}(i_{n+1}, n)$$

Viterbi Algorithm: Example

HMM:

States

α_i for $i \in [1:I]$

Observation symbols

β_k for $k \in [1:K]$

State transition probabilities

a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities

b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities

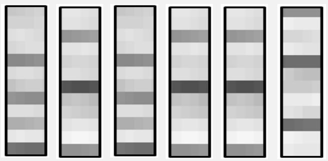
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

Observation sequence

$O = (o_1,o_2,o_3,o_4,o_5,o_6)$



Viterbi algorithm

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α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	$\boxed{1}$	$\boxed{1}$	1	1	1
α_2	1	1	1	1	$\boxed{3}$
α_3	1	3	$\boxed{1}$	$\boxed{3}$	3

$\boxed{i_6 = 2}$

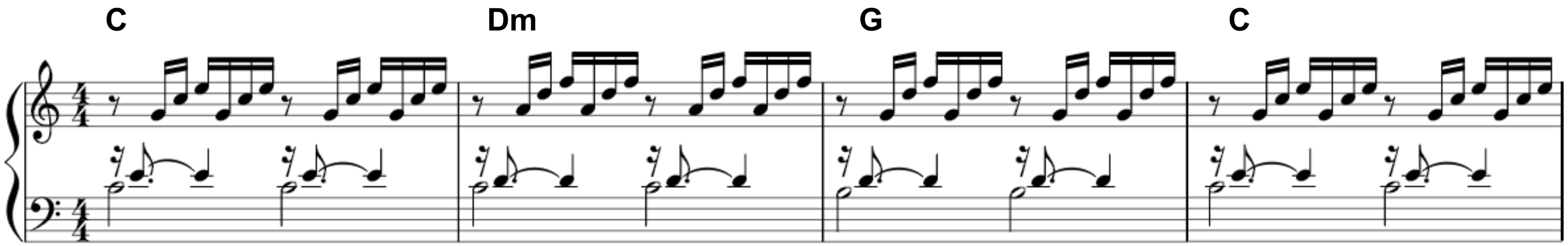
Output

Optimal state sequence

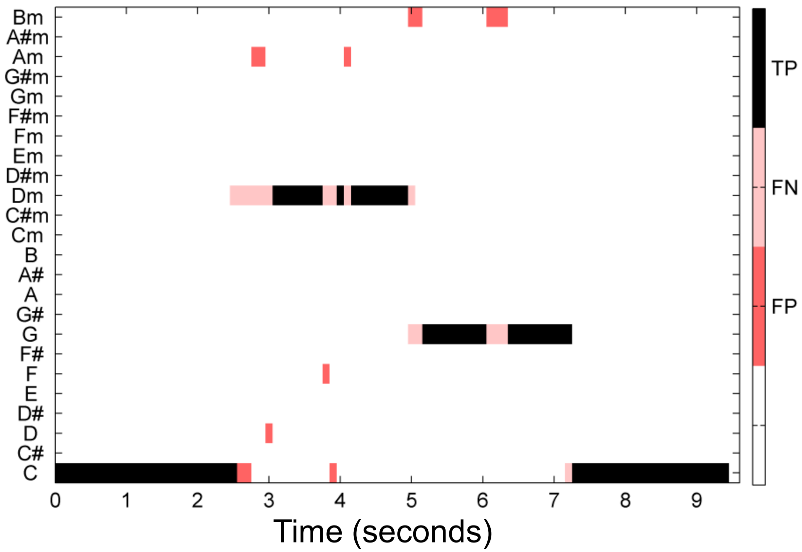
$S^* = (\alpha_1,\alpha_1,\alpha_1,\alpha_3,\alpha_3,\alpha_2)$

HMM: Application to Chord Recognition

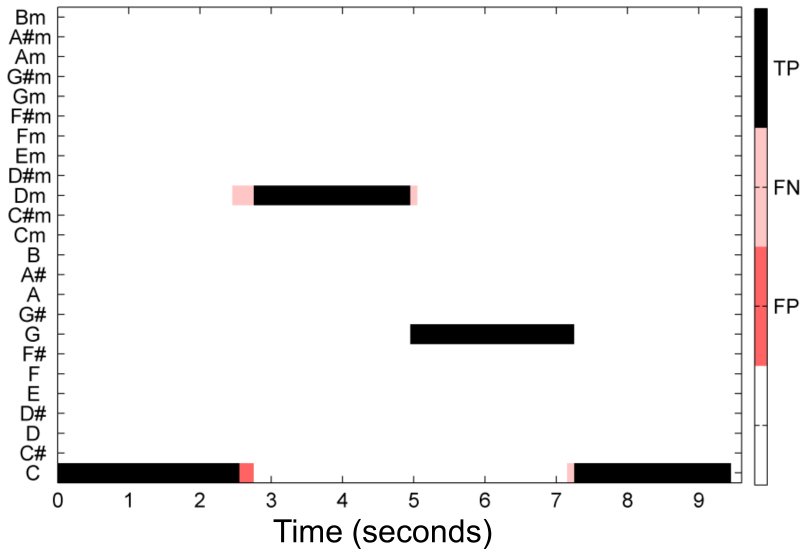
- Effect of HMM-based chord estimation and smoothing:



(a) Template Matching (frame-wise)

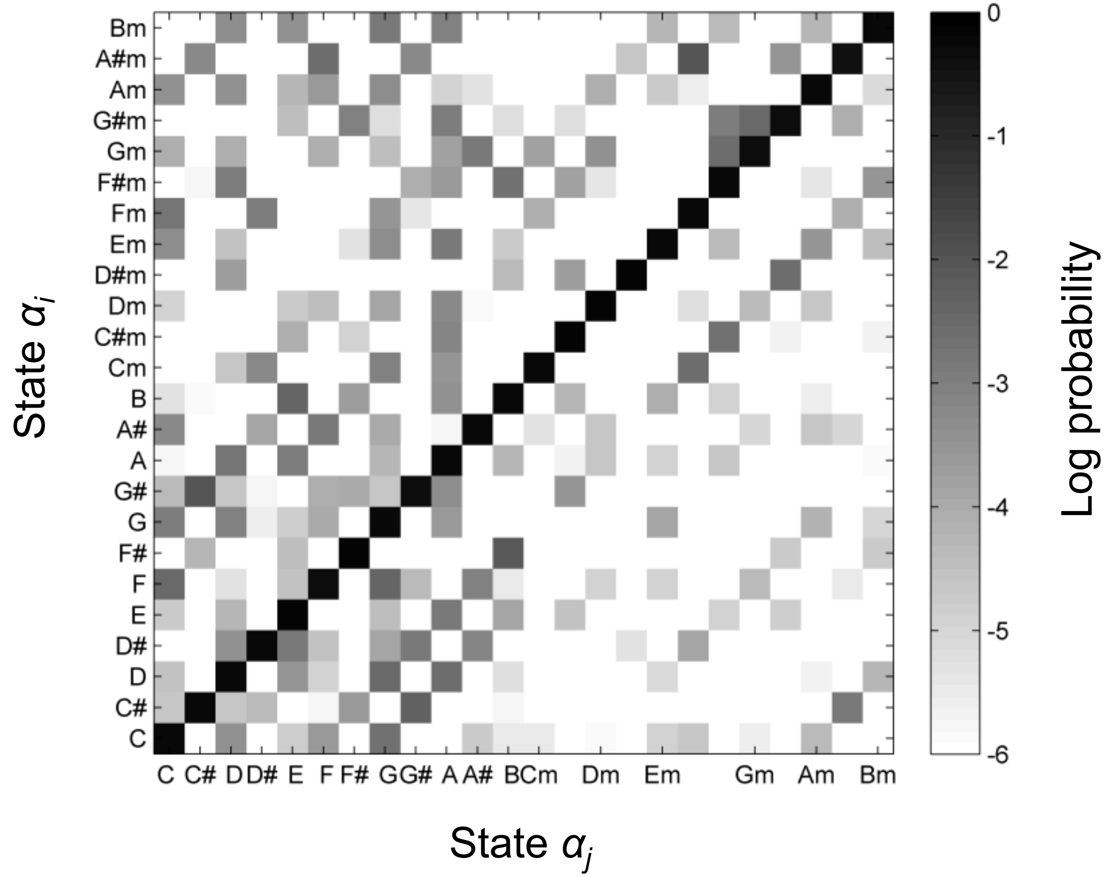


(b) HMM



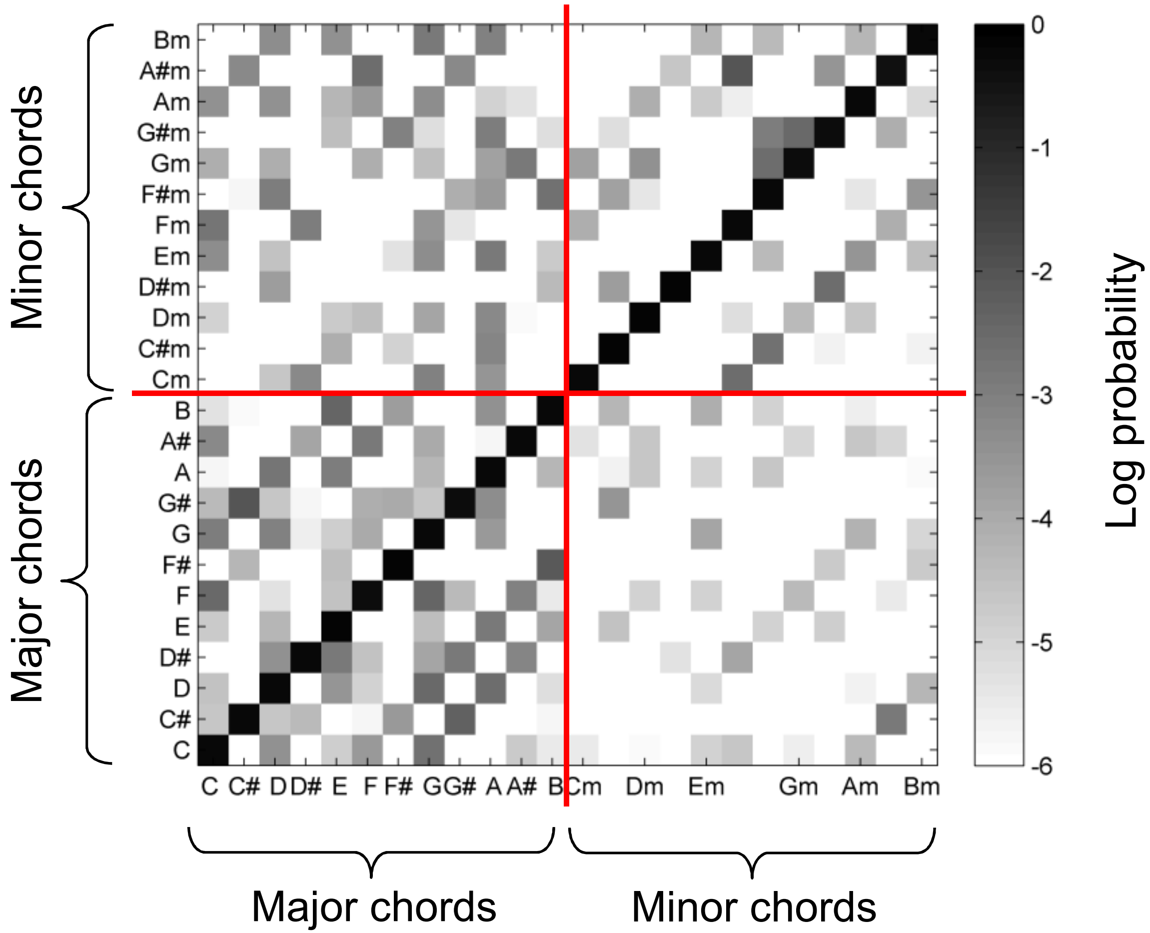
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Estimated from data



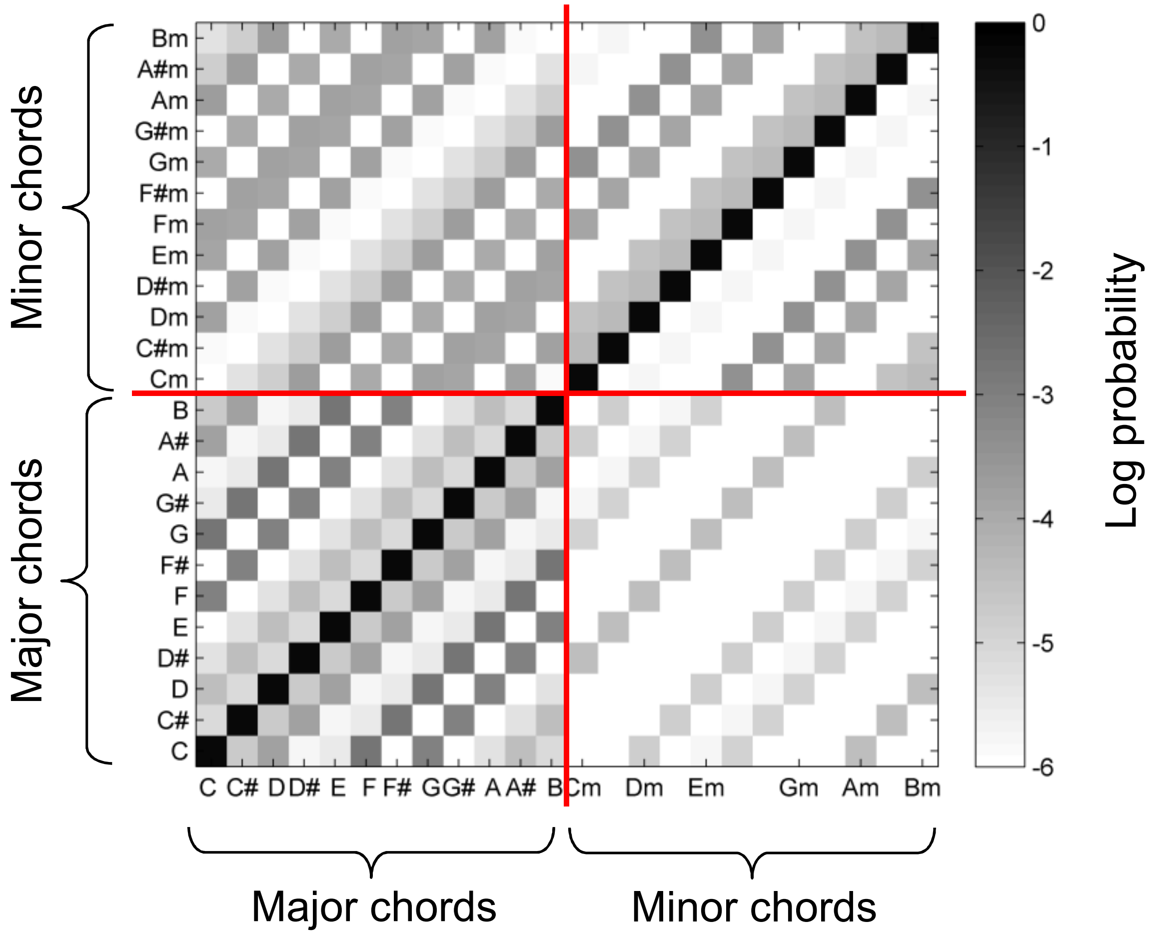
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Estimated from data



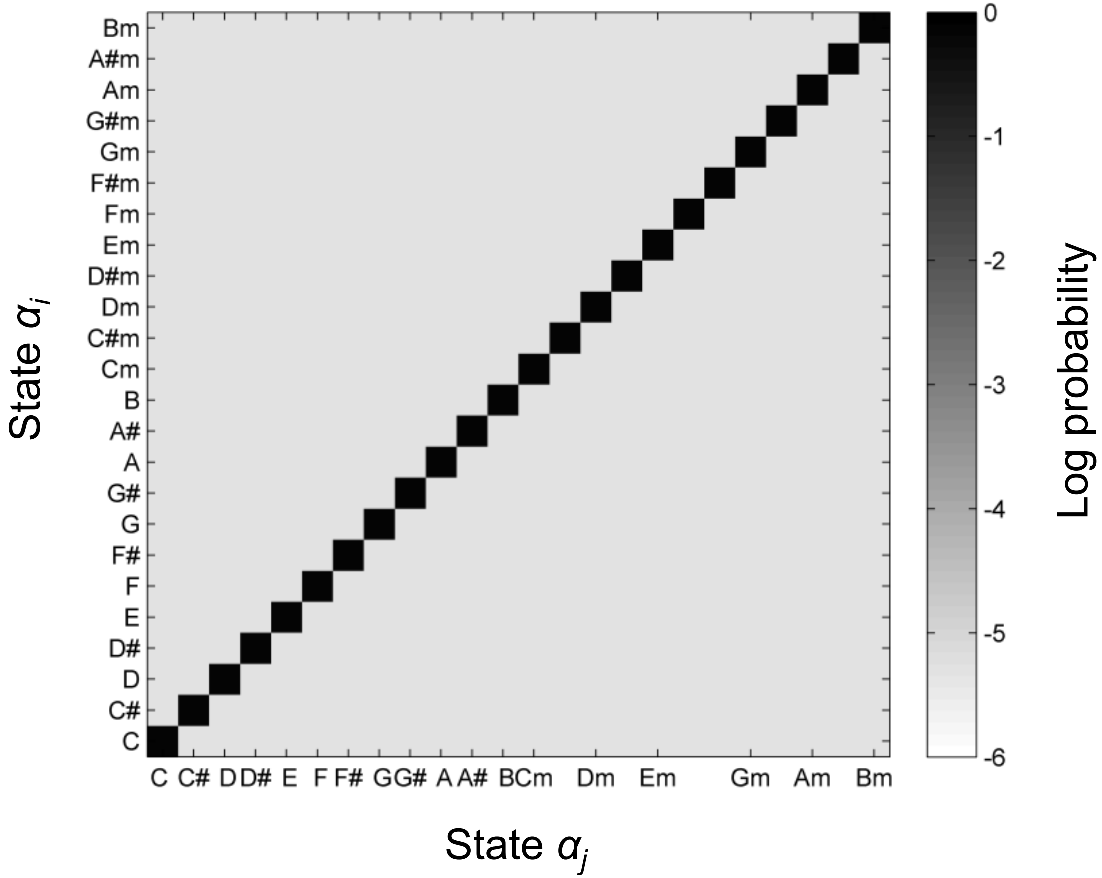
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Transposition-invariant



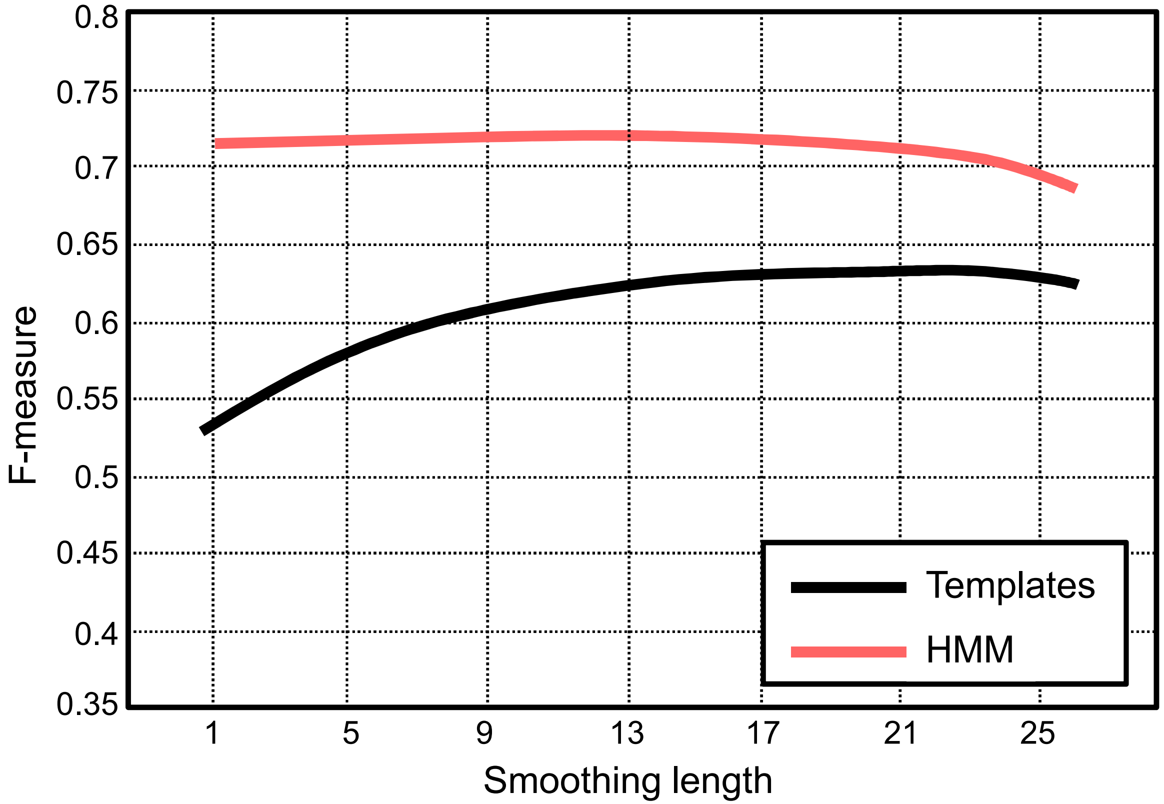
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Uniform, diagonal-enhanced transition matrix (only smoothing)



HMM: Application to Chord Recognition

- Evaluation on all Beatles songs

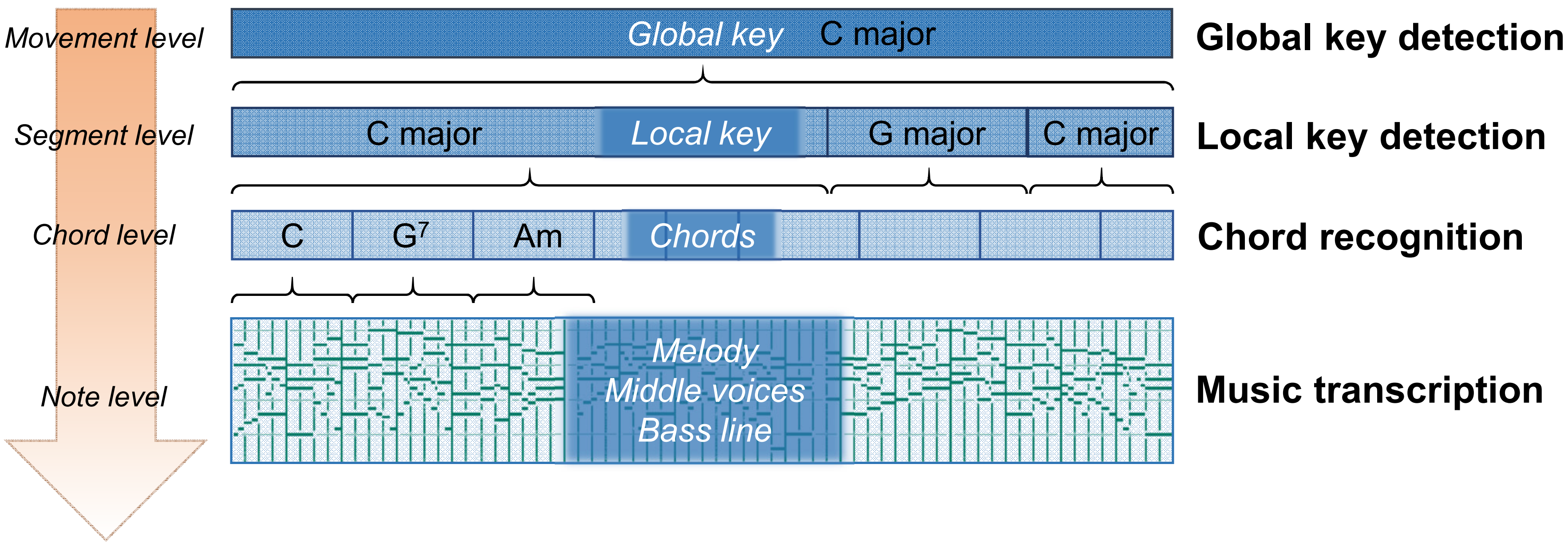


Harmony Analysis

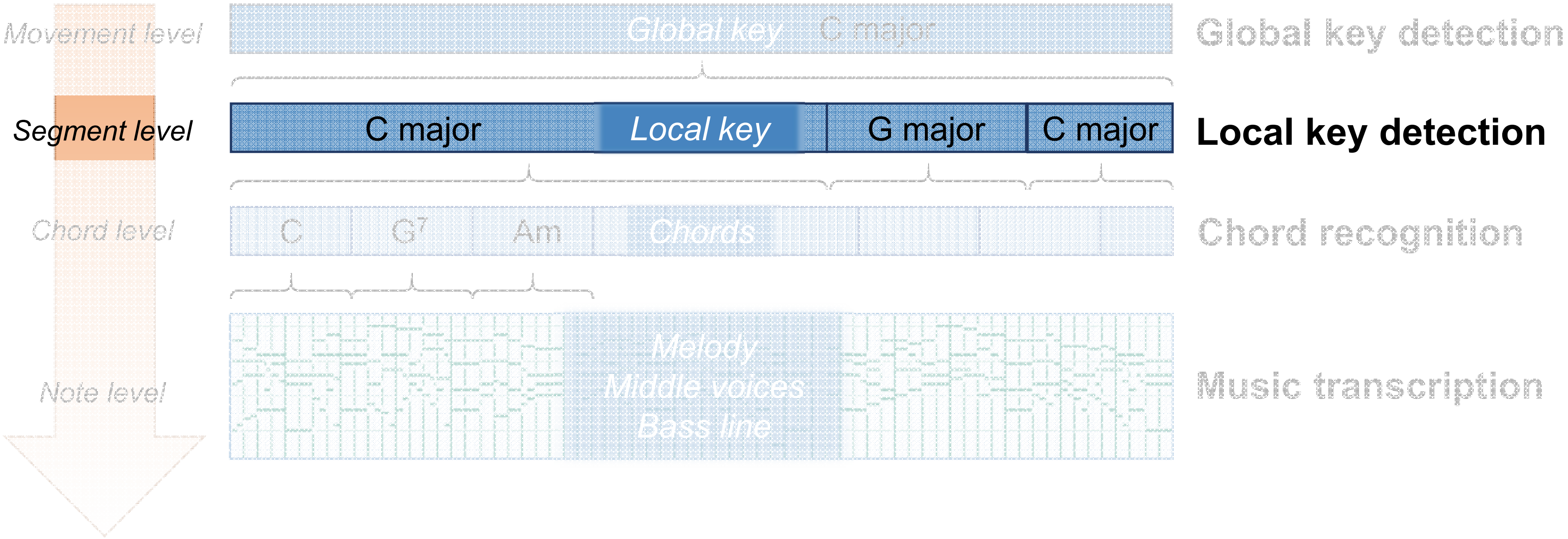
Overview

- Template-based Chord Recognition
- HMM-based Chord Recognition
- Local Key Detection
- Application for Musicology

Harmony Analysis: Overview

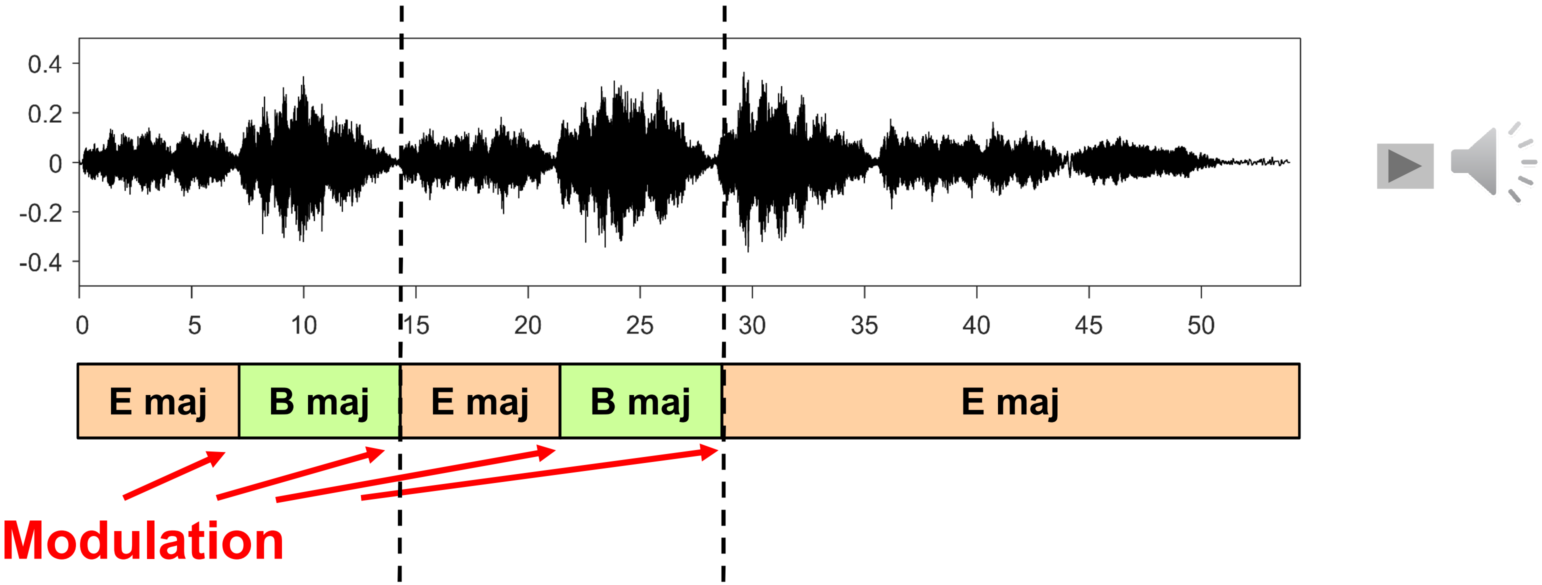


Harmony Analysis: Overview

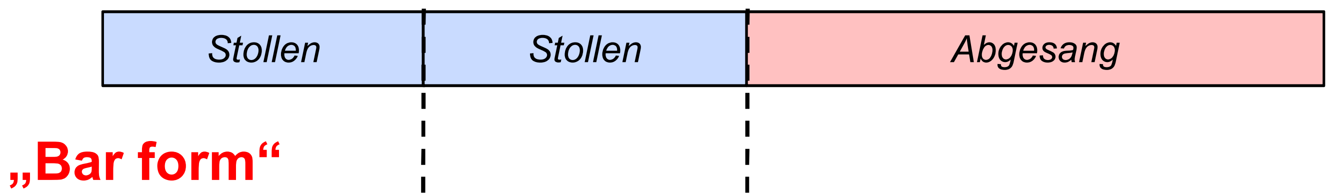


Local Key Detection

- Johann Sebastian Bach, Choral “Durch Dein Gefängnis” (*St. John’s Passion*) – **Local keys**

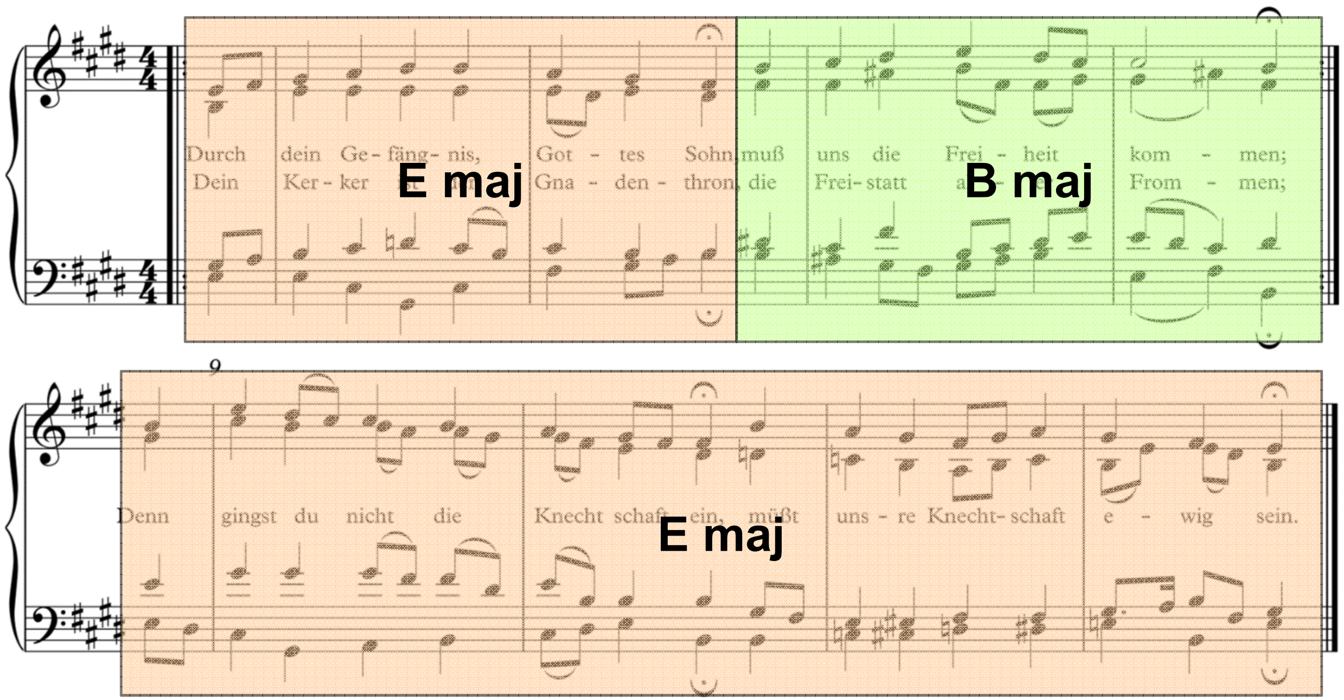


- Musical form:



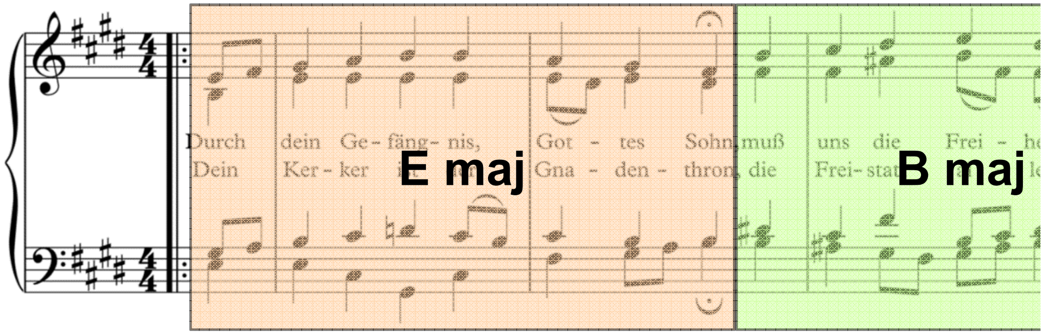
Local Key Detection

- Johann Sebastian Bach, Choral “Durch Dein Gefängnis” (*St. John’s Passion*) – **Local keys**

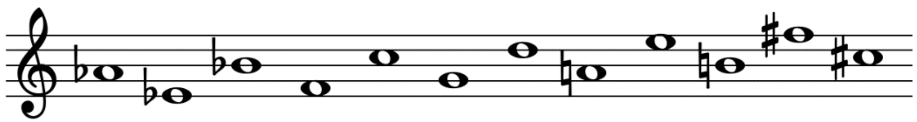


Local Key Detection: Diatonic Scales

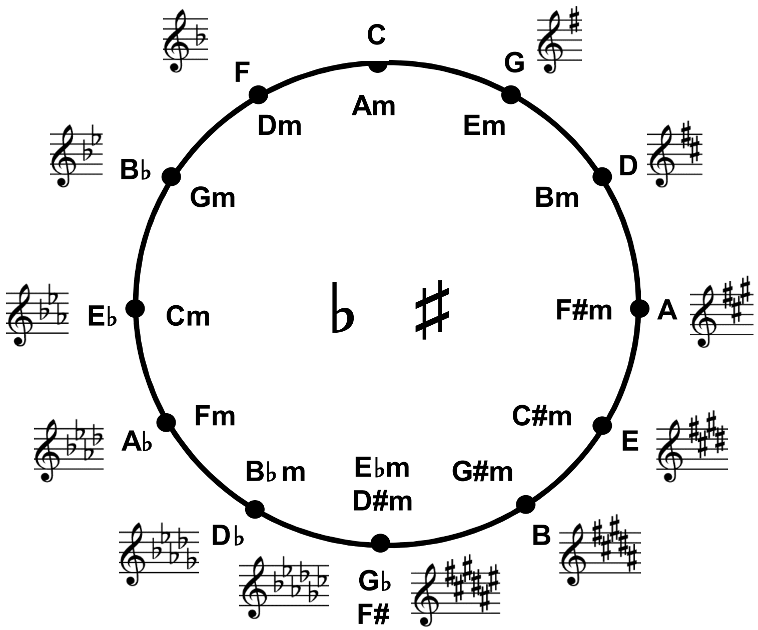
- Johann Sebastian Bach, Choral “Durch Dein Gefängnis” (*St. John’s Passion*) – **Local keys**



Series of fifths



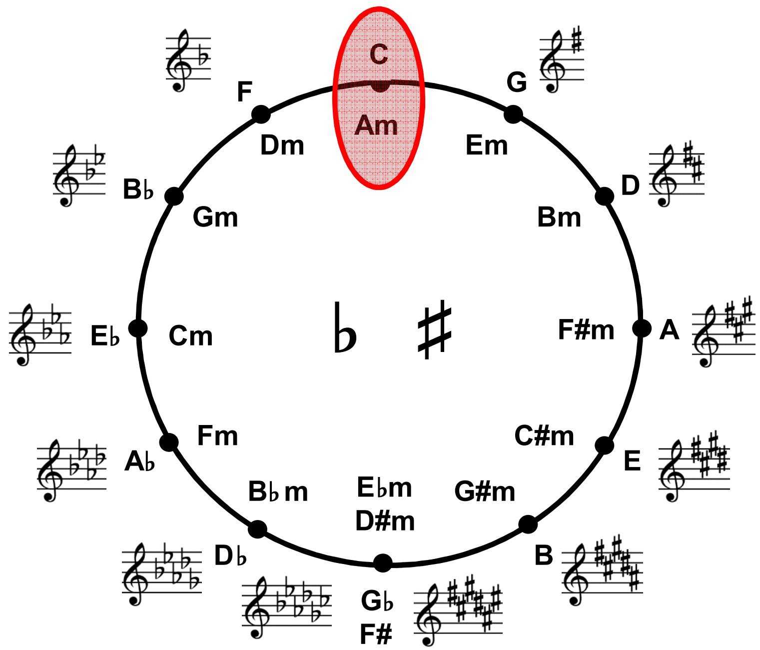
Circle of fifths



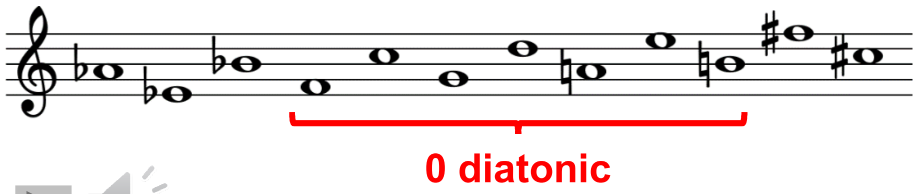
Local Key Detection: Diatonic Scales

- Johann Sebastian Bach, Choral “Durch Dein Gefängnis” (*St. John’s Passion*) – **Local keys**

Circle of fifths



Series of fifths



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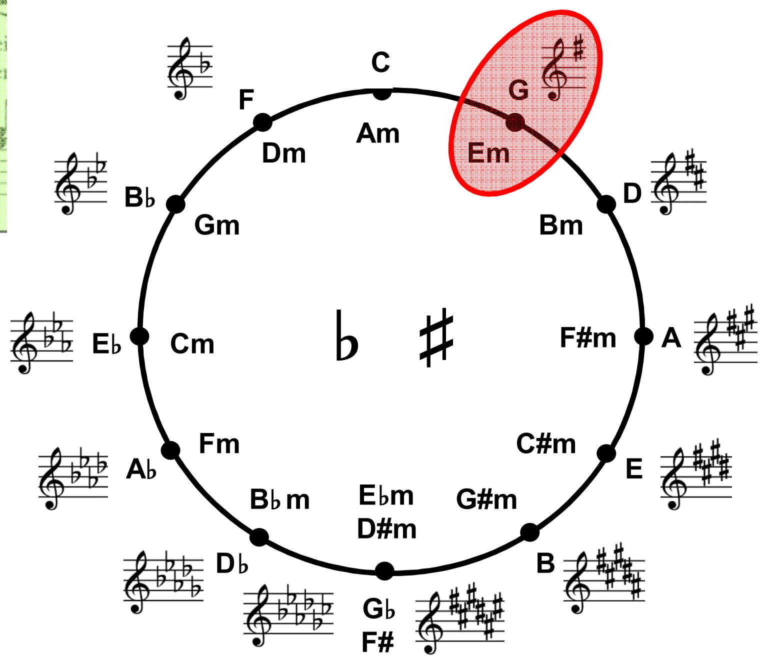
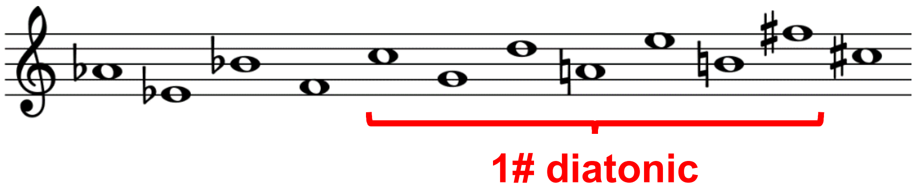


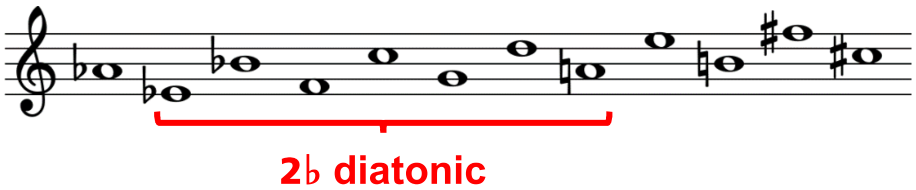
Series of fifths



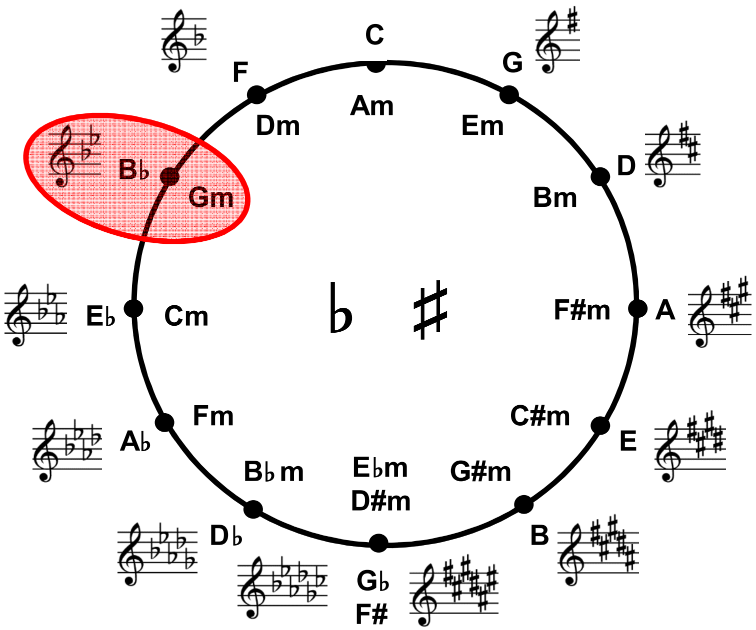
Local Key Detection: Diatonic Scales

- Johann Sebastian Bach, Choral "Durch Dein Gefängnis" (*St. John's Passion*) – **Local keys**

Series of fifths



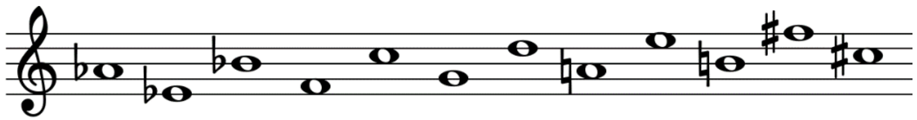
Circle of fifths



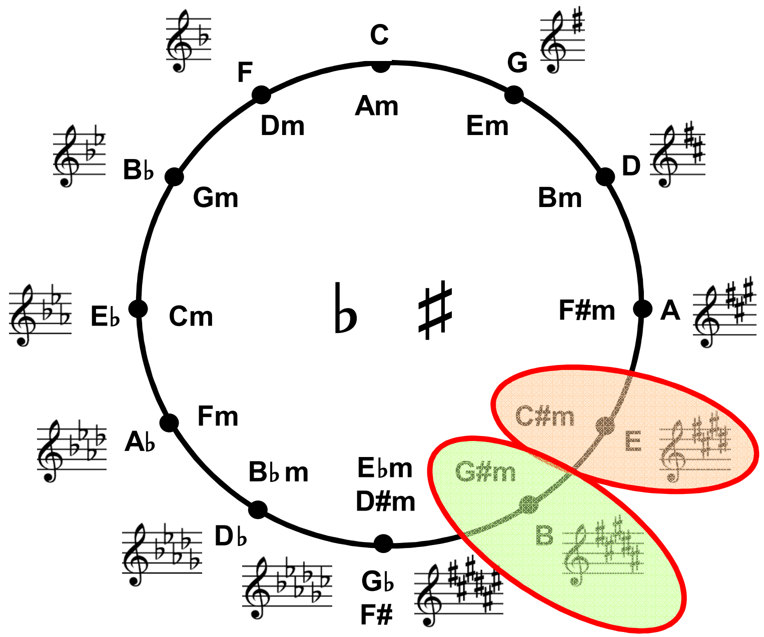
Local Key Detection: Diatonic Scales

- Johann Sebastian Bach, Choral “Durch Dein Gefängnis” (*St. John’s Passion*) – **Local keys**

Series of fifths

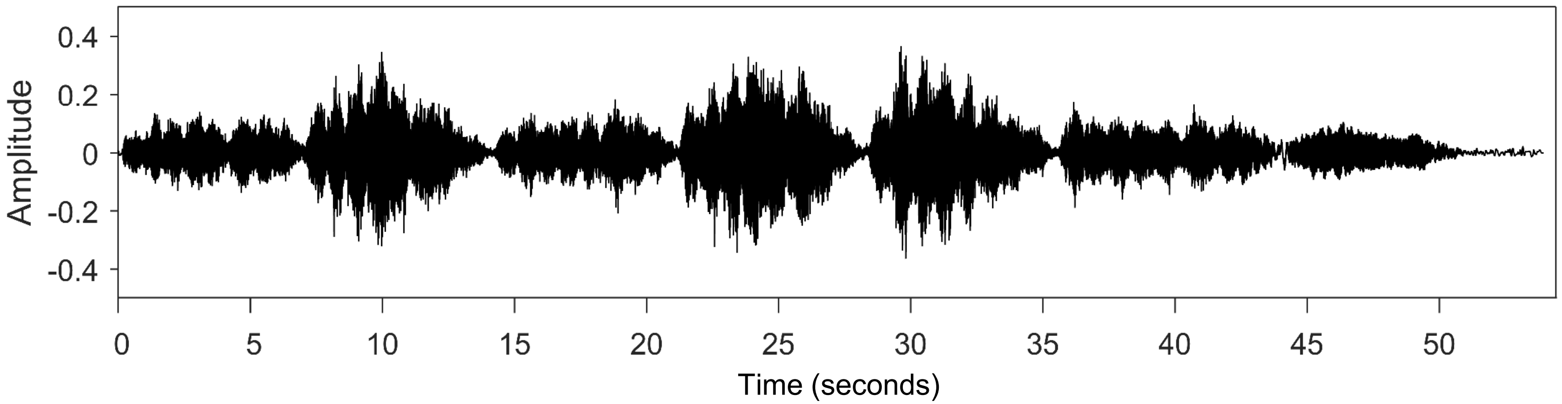


Circle of fifths



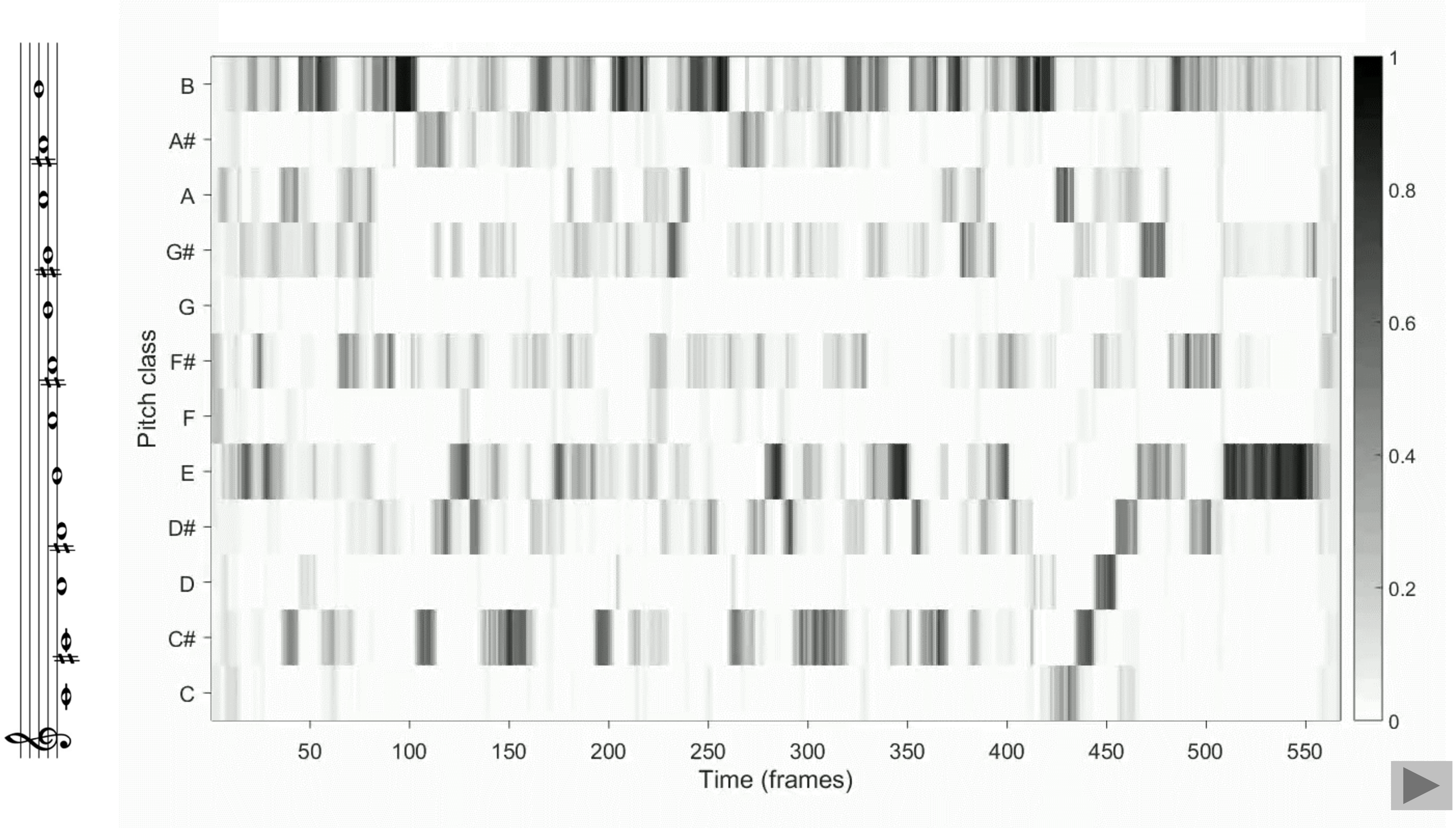
Local Key Detection

- Example: J.S. Bach, Choral "Durch Dein Gefängnis"
- **Audio** – Waveform (Scholars Baroque Ensemble, Naxos 1994)



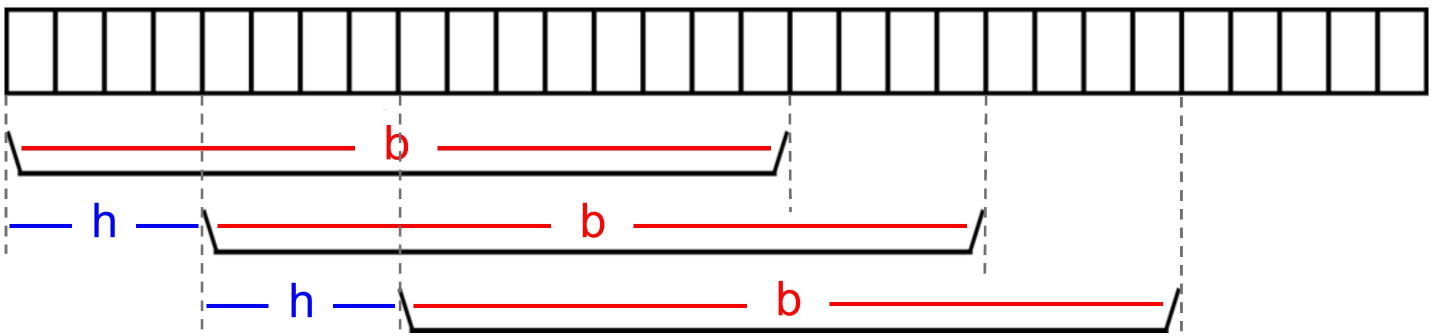
Local Key Detection: Chroma Features

- Example: J.S. Bach, Choral "Durch Dein Gefängnis"
- **Audio** – Chroma features (Scholars Baroque Ensemble, Naxos 1994)



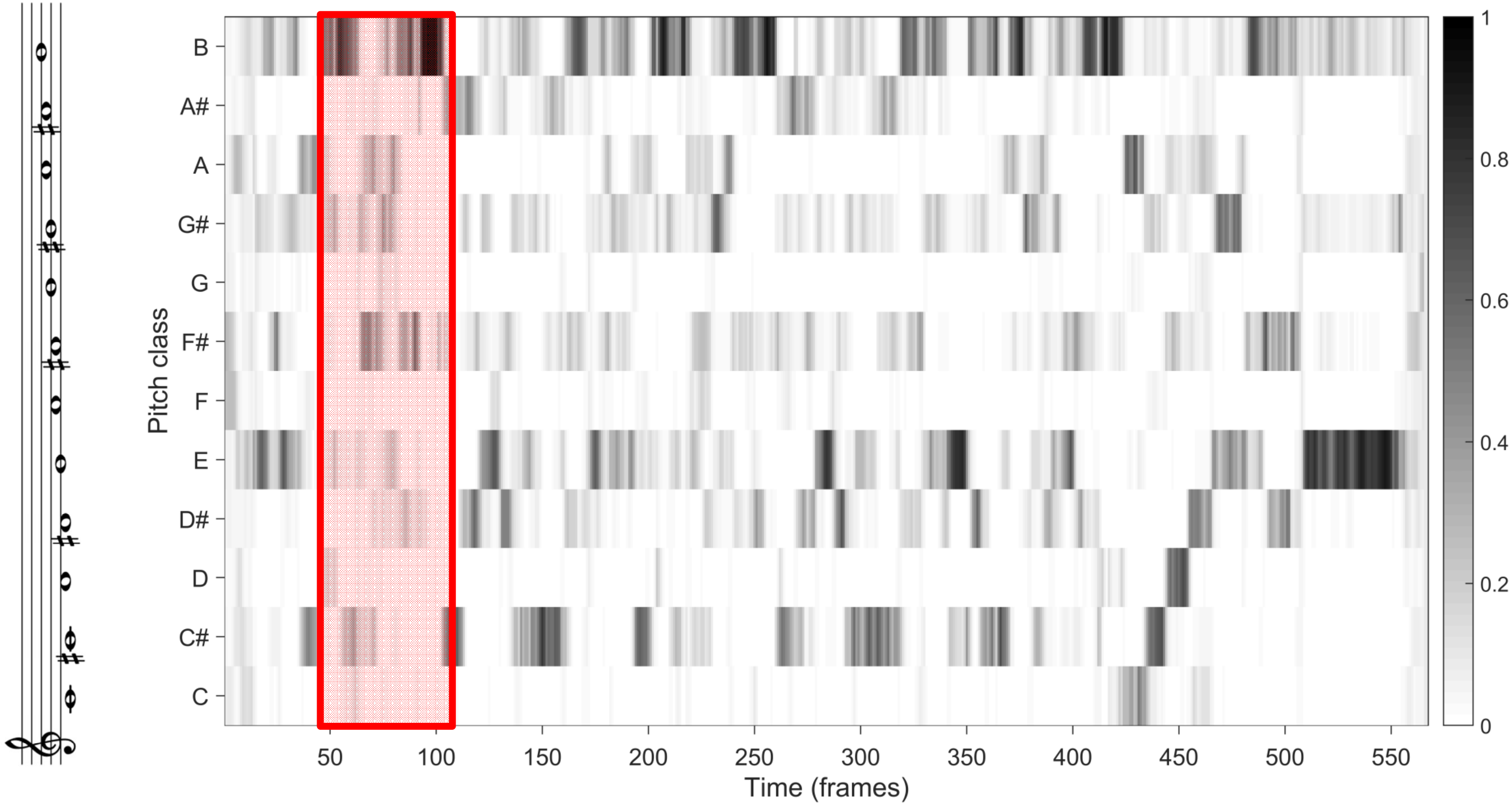
Local Key Detection: Chroma Smoothing

- Summarize pitch classes over a certain time
  - **Chroma smoothing** (mean filter)
  - Parameters: blocksize b and hopsize h



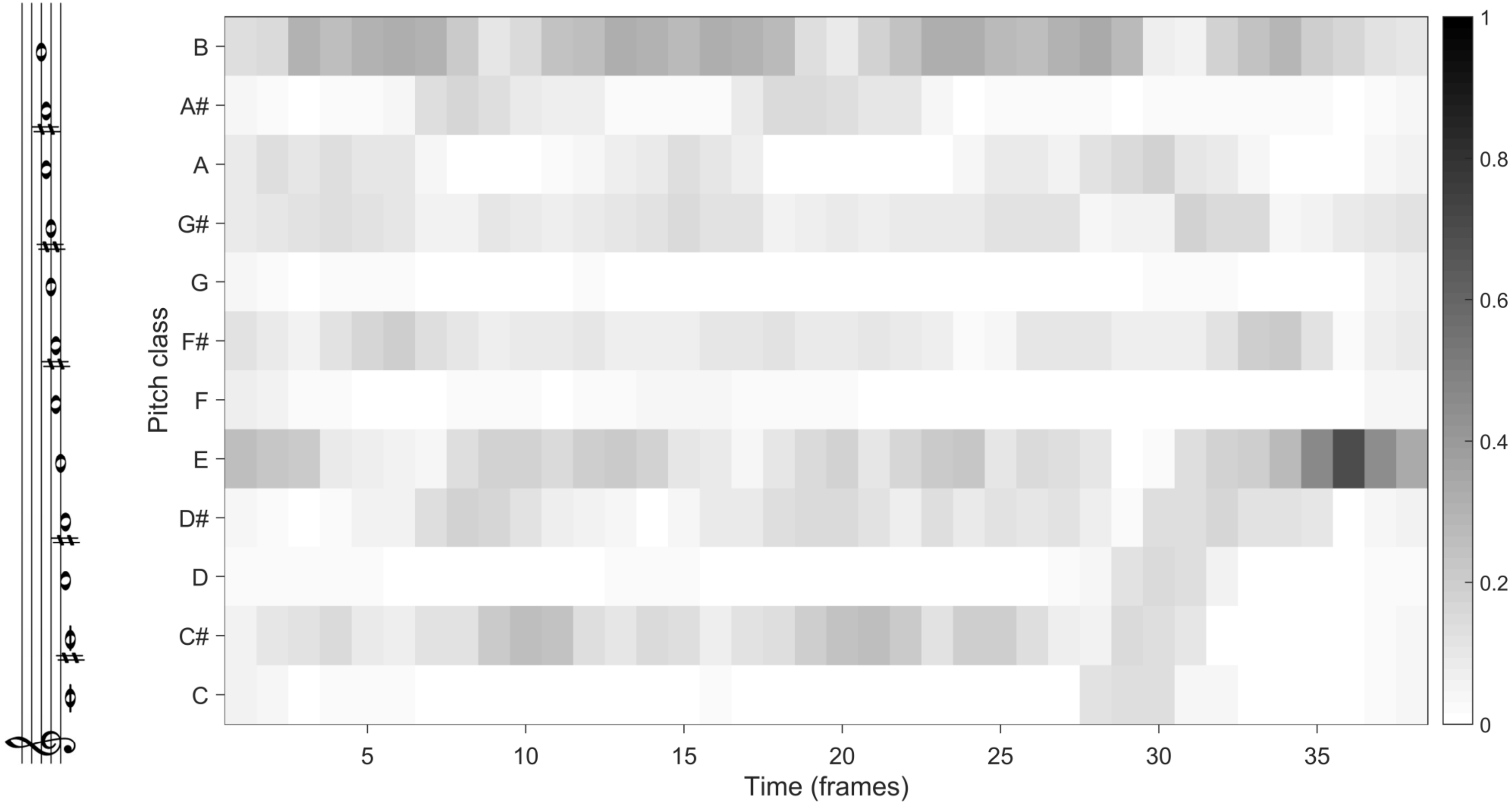
Local Key Detection: Chroma Smoothing

- Example: J.S. Bach, Choral "Durch Dein Gefängnis"
- Chroma features – smoothing



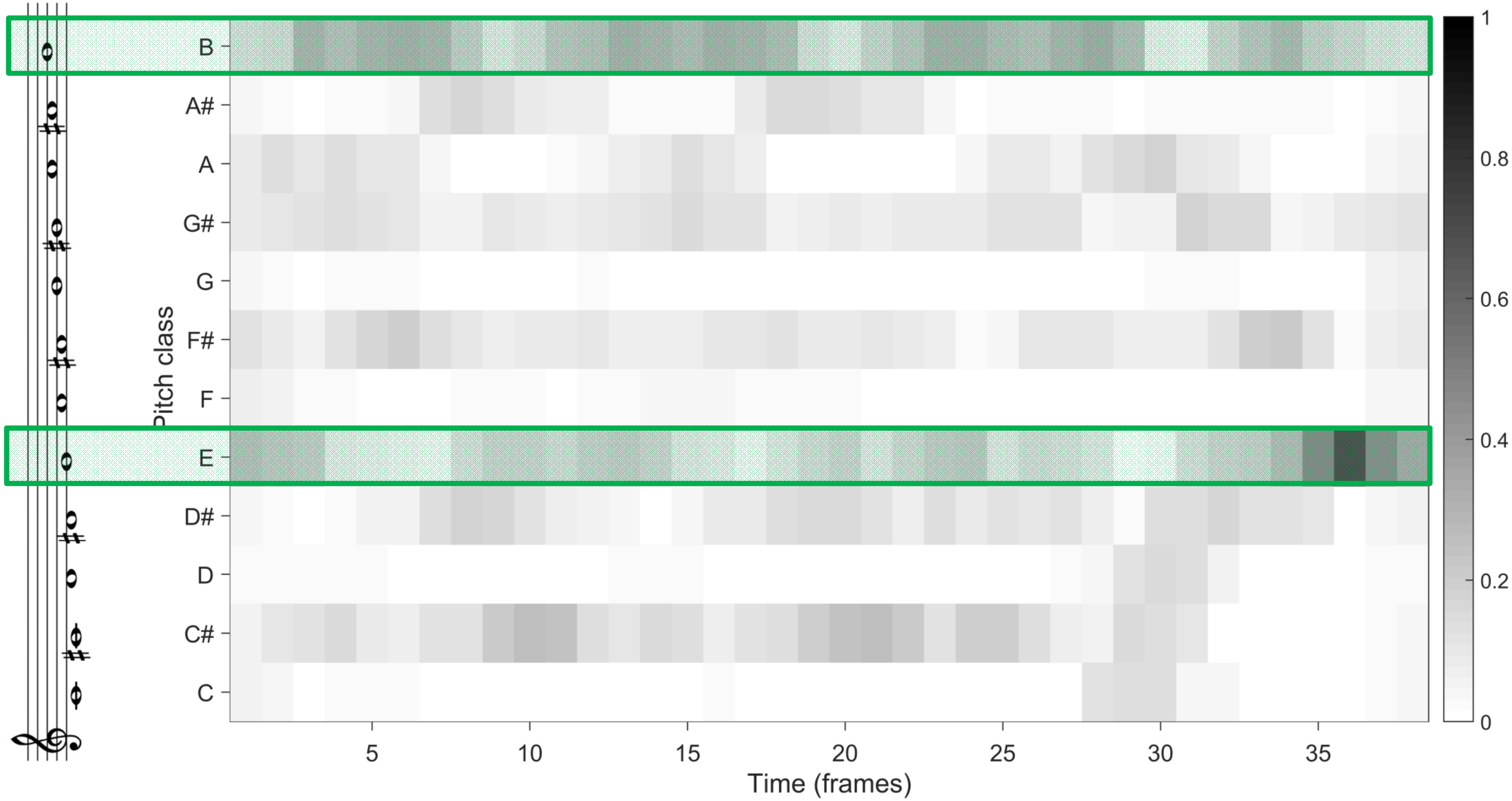
Local Key Detection: Chroma Smoothing

- Example: J.S. Bach, Choral "Durch Dein Gefängnis"
- Chroma features – smoothing (b = 42 frames and h = 15 frames)



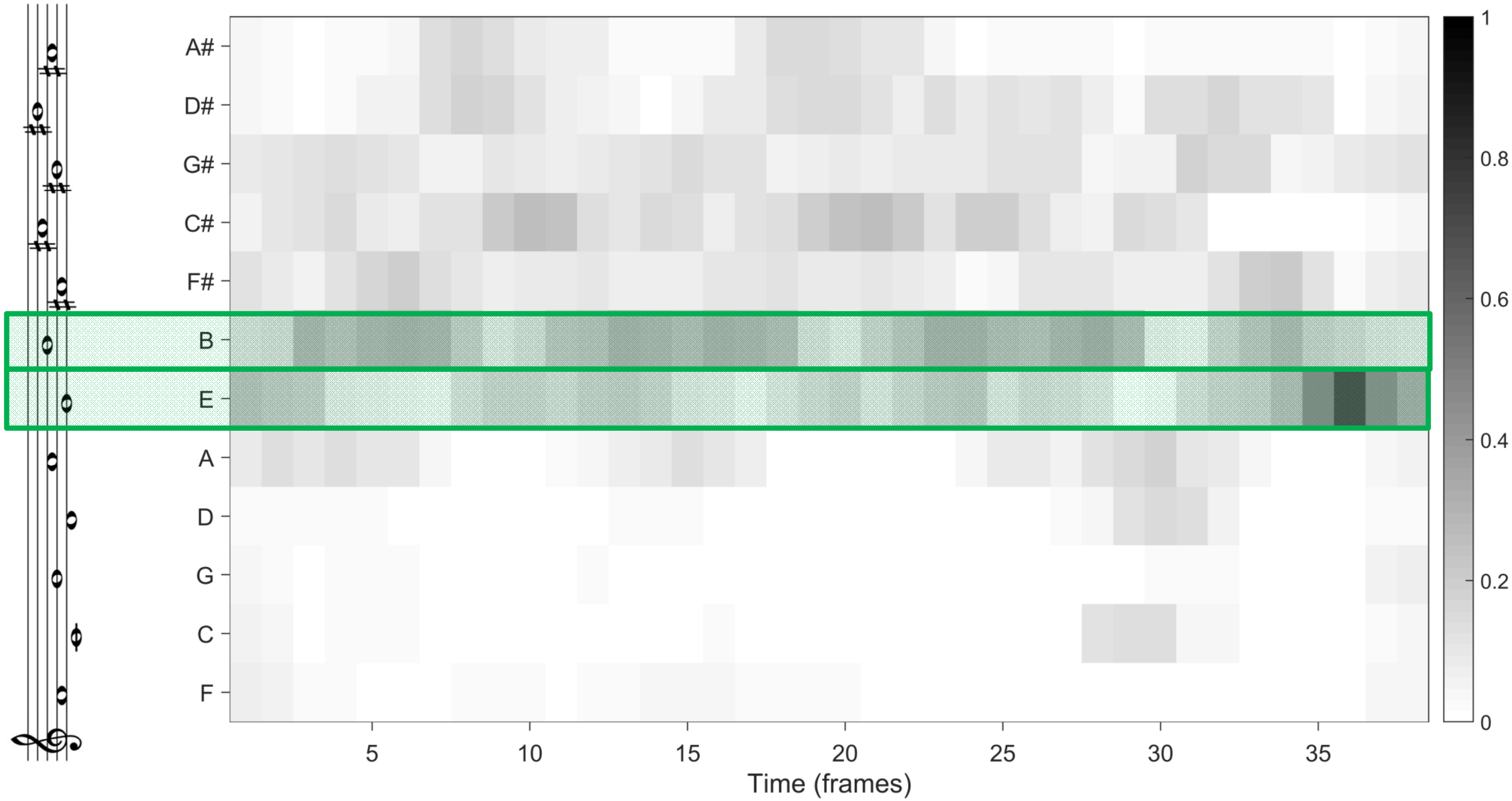
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Re-ordering to **perfect fifth** series



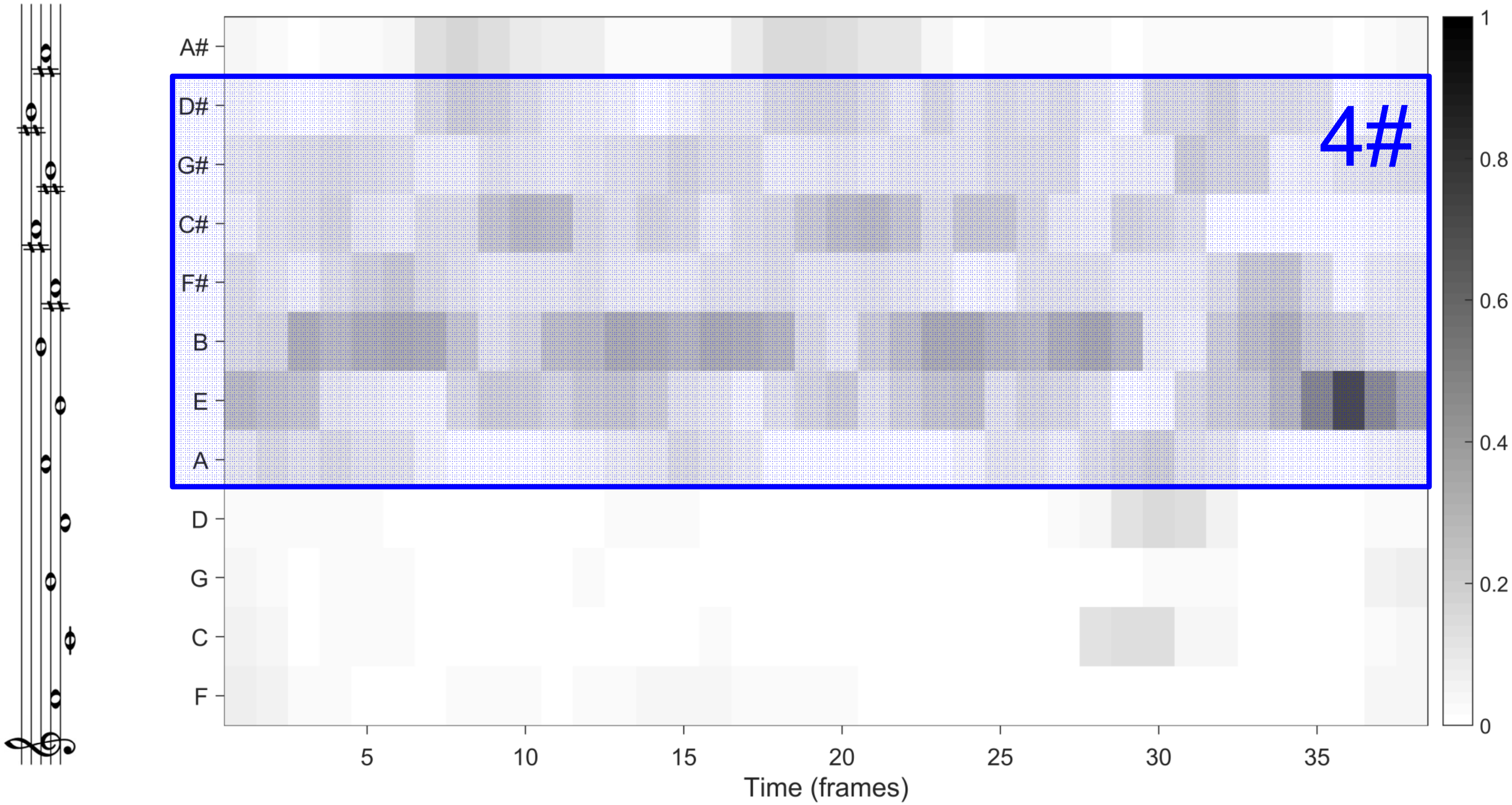
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Re-ordering to **perfect fifth** series



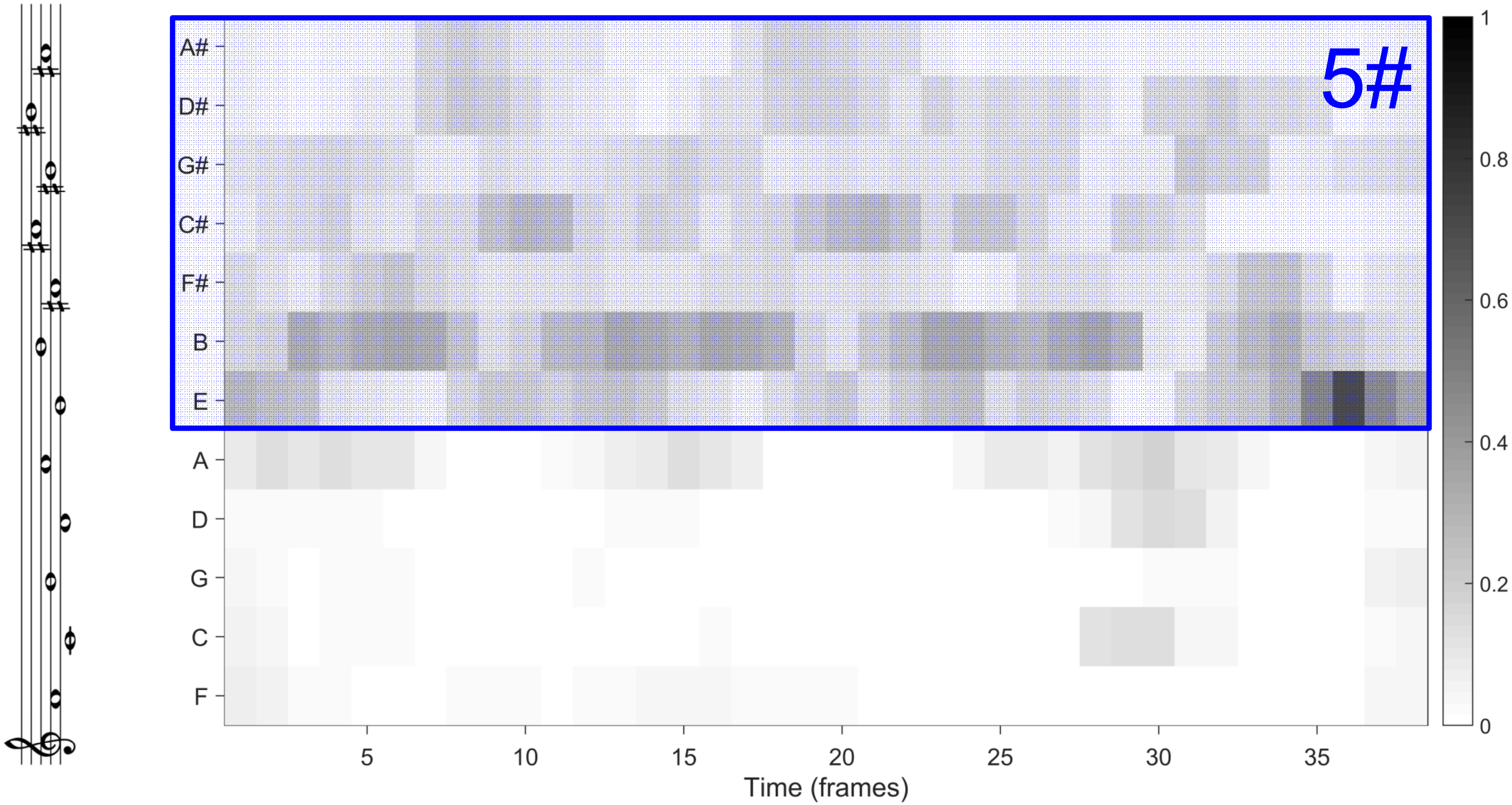
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales (7 fifths)



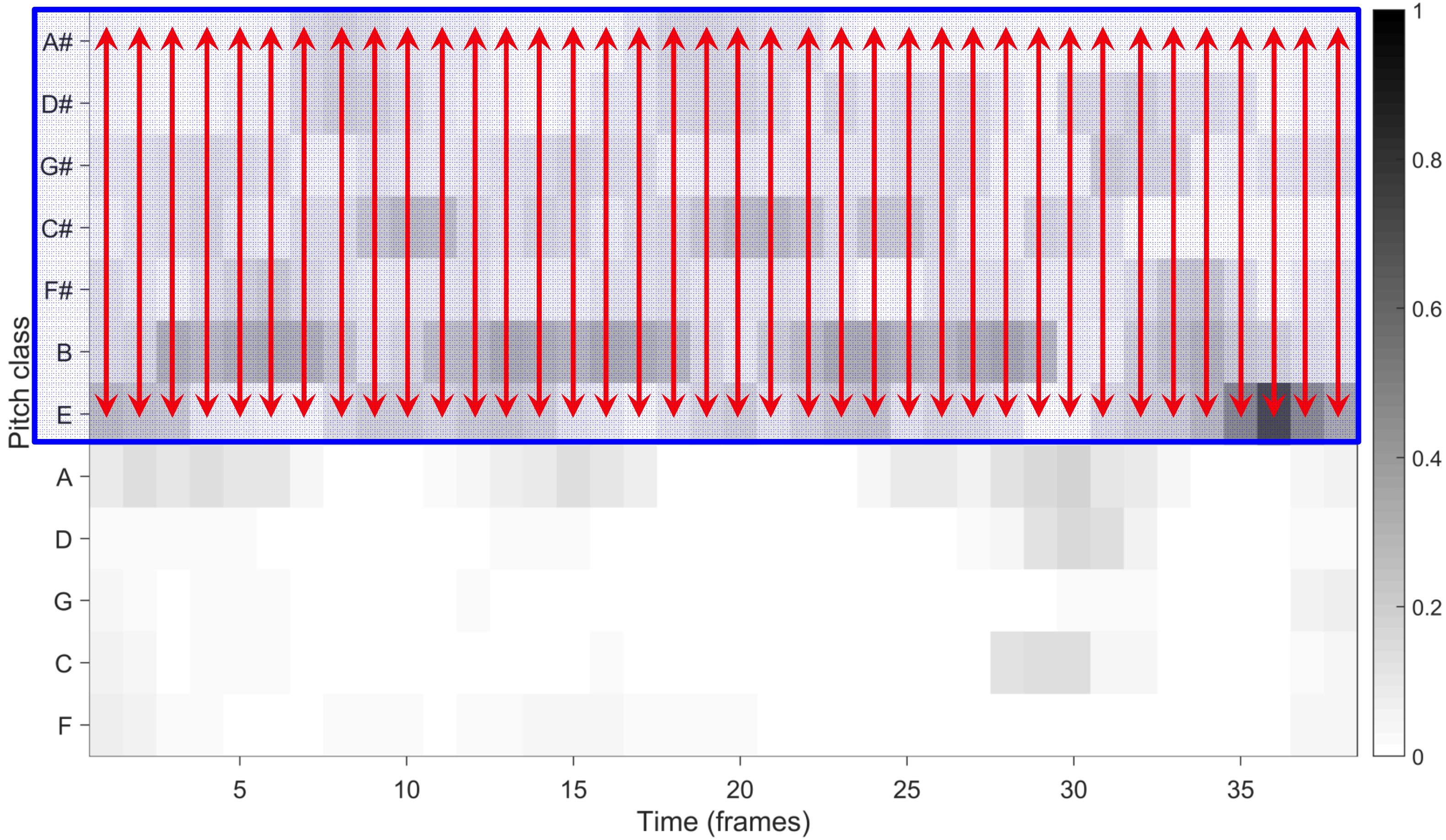
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales (7 fifths)



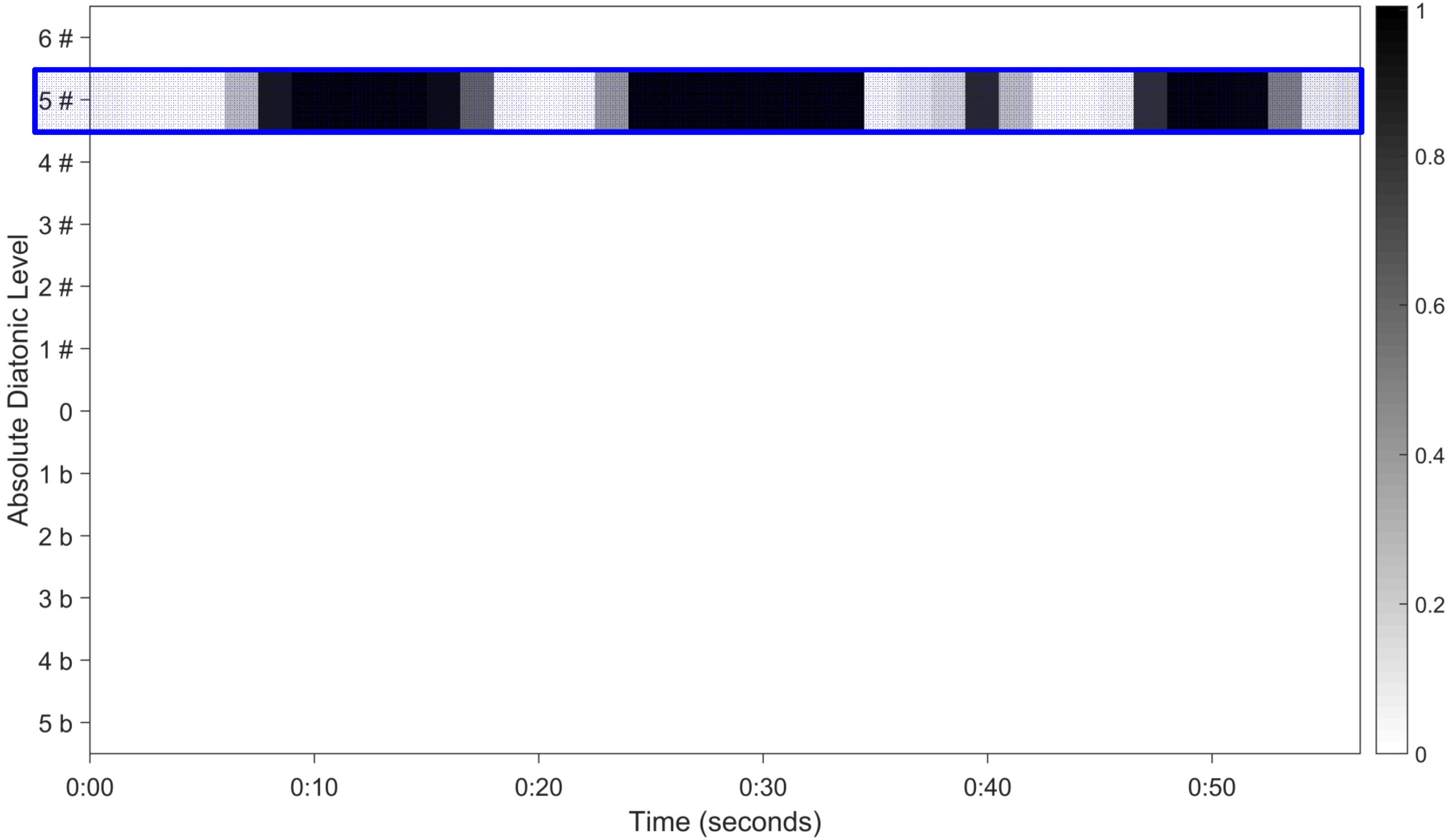
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – multiplication



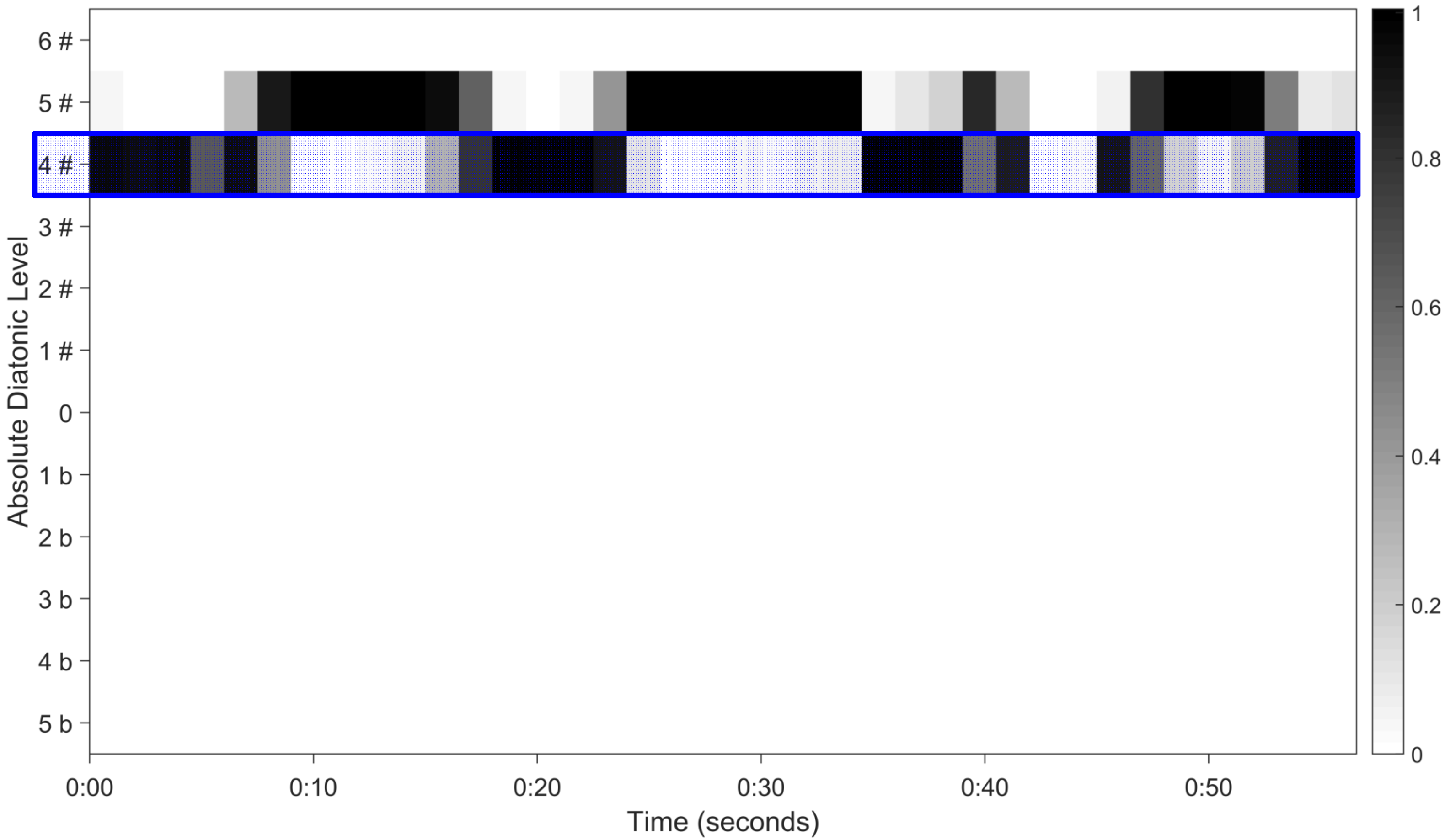
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – multiplication



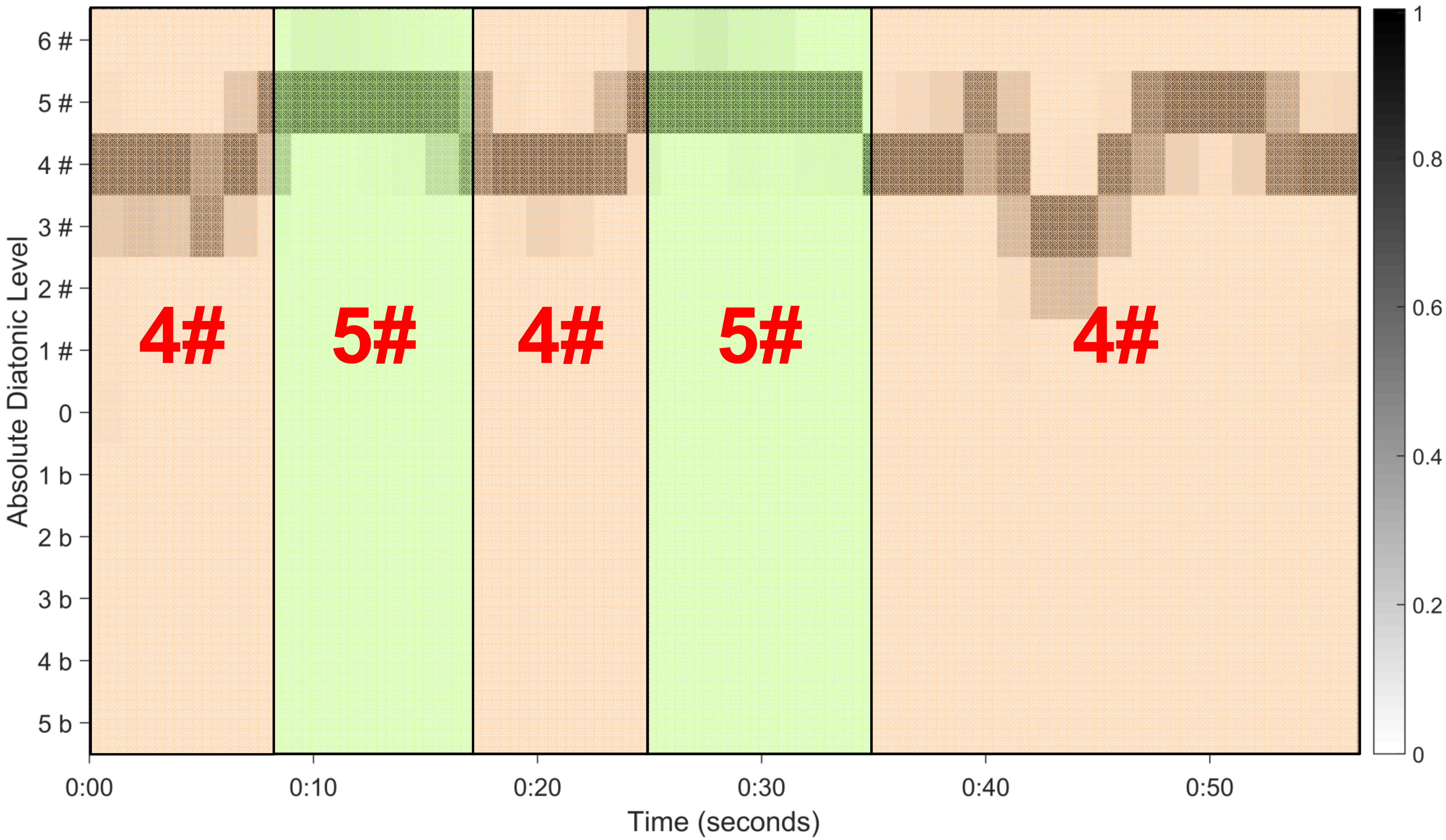
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – multiplication



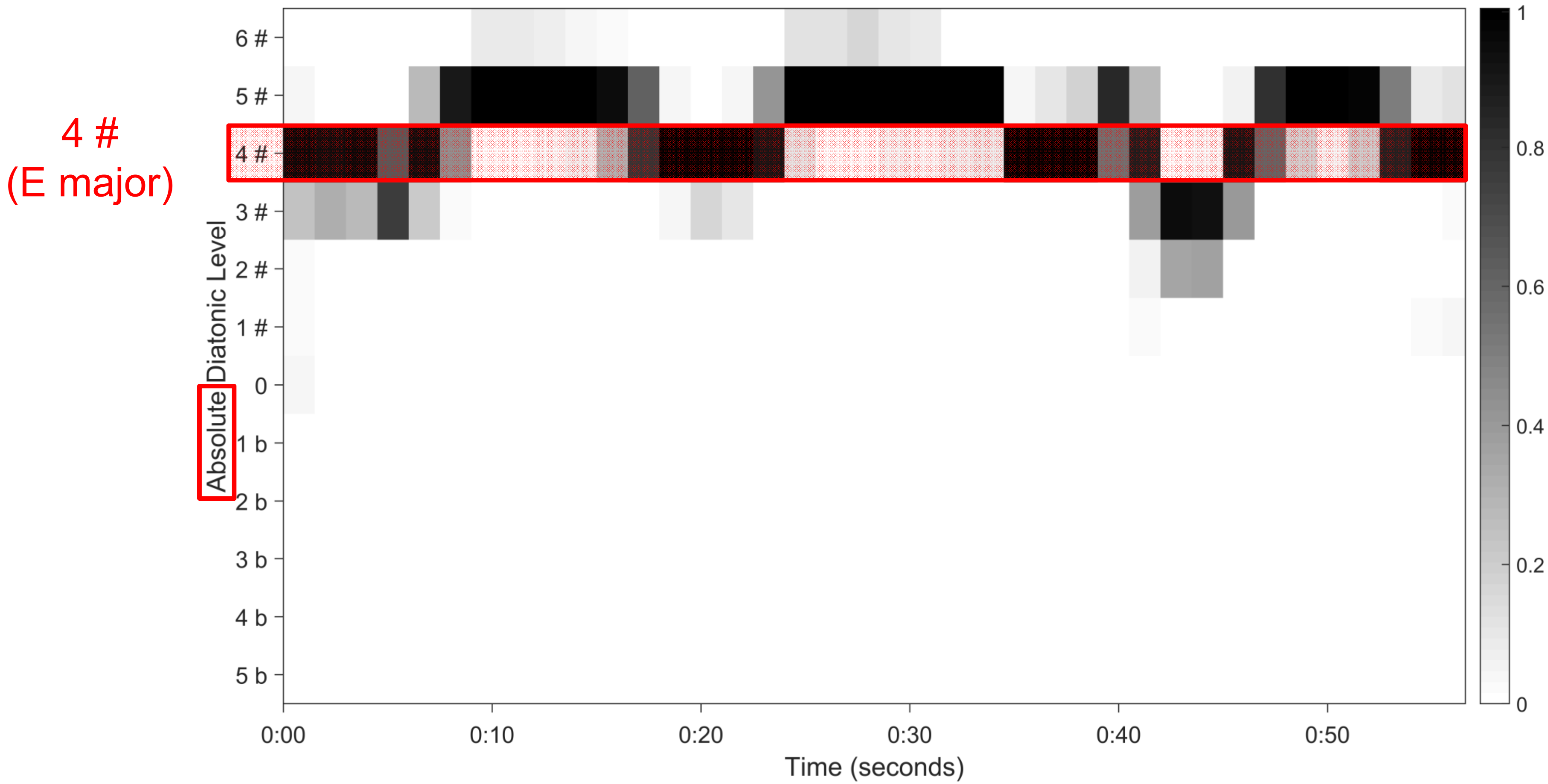
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – multiplication



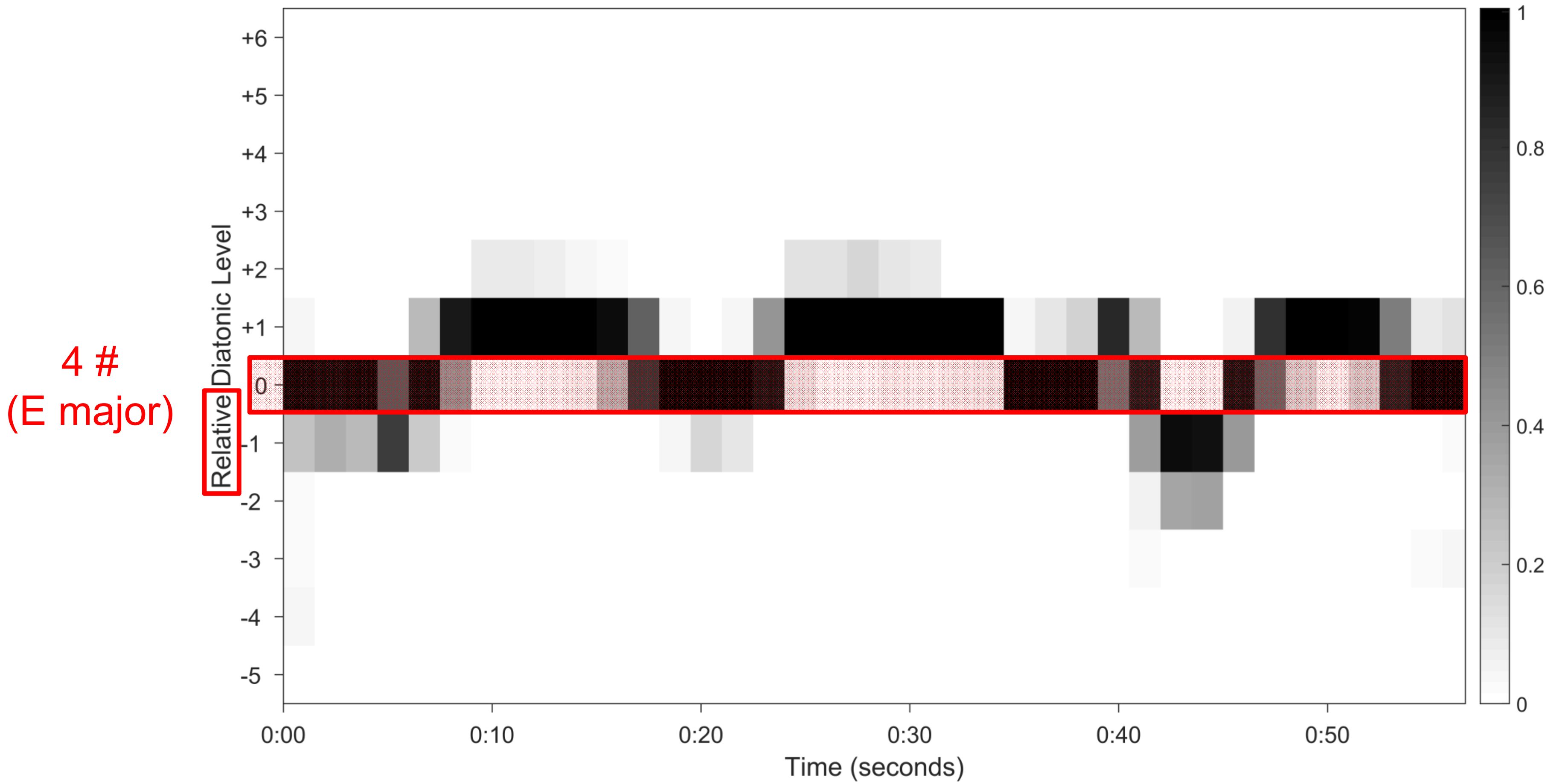
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – shift to **global key**



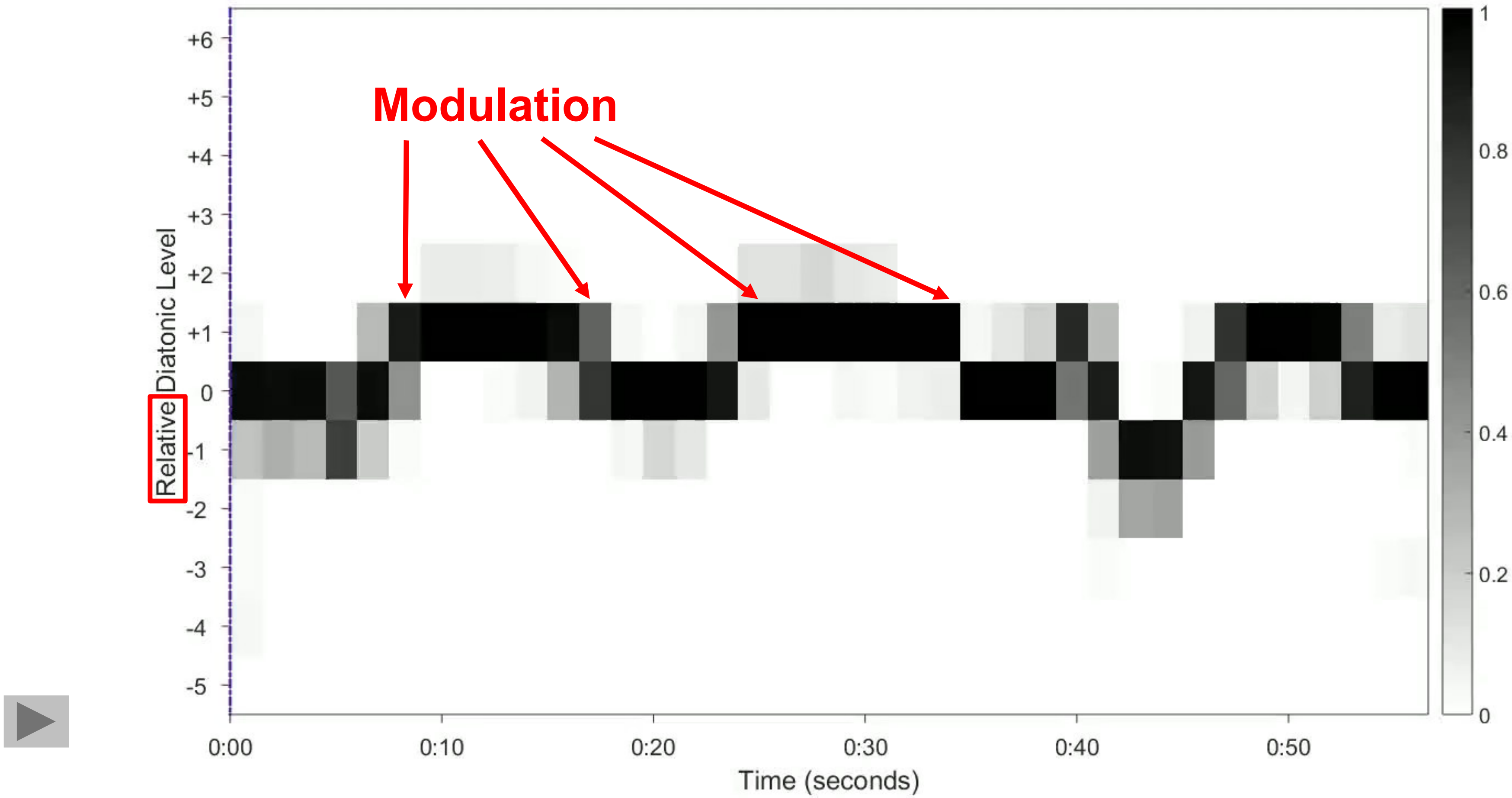
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – shift to **global key**



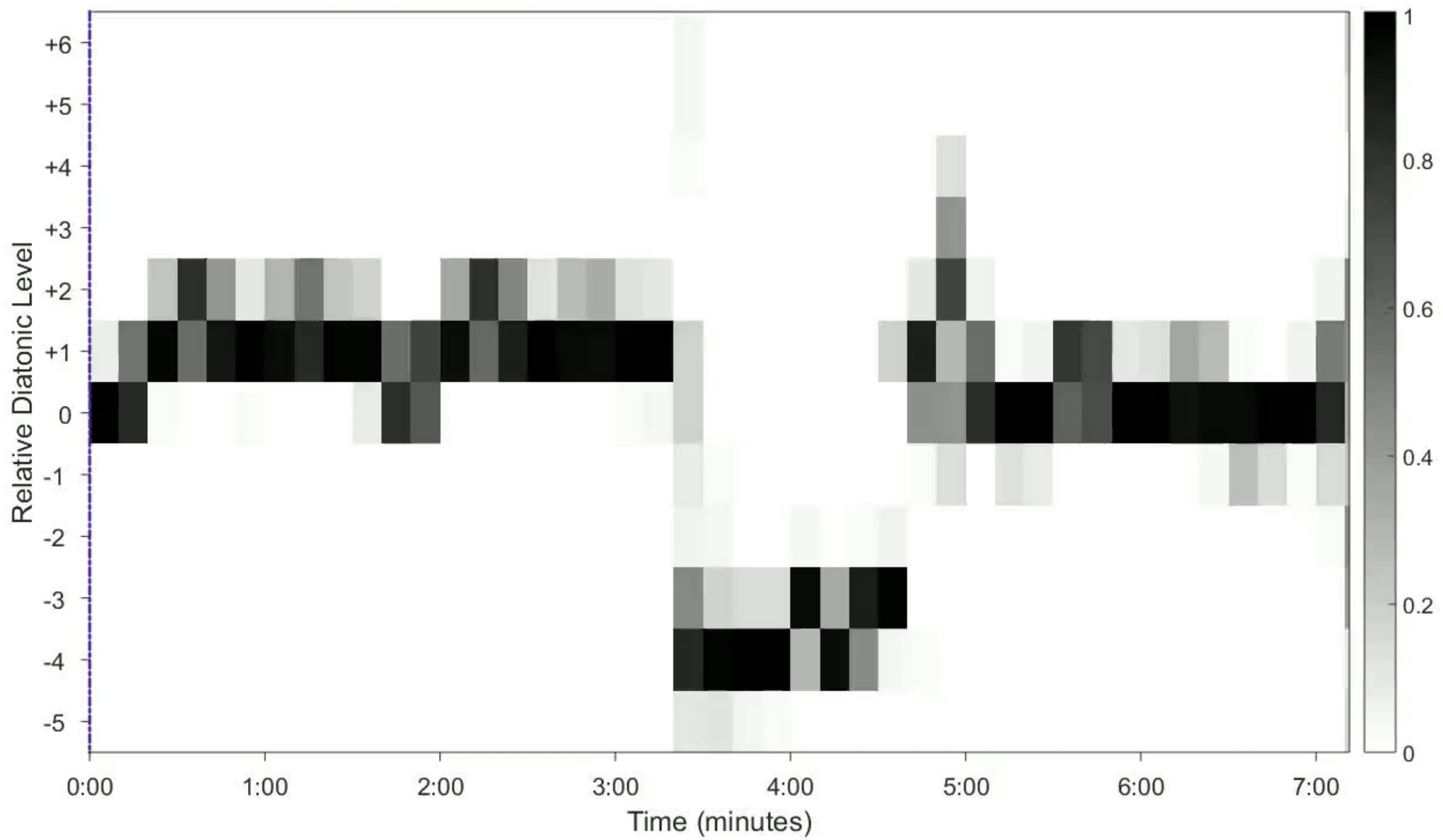
Local Key Detection: Diatonic Scales

- Example: J.S. Bach, Choral “Durch Dein Gefängnis”
- Diatonic Scales – relative (0 ≙ 4#)



Local Key Detection: Examples

- L. v. Beethoven – Sonata No. 10 op. 14 Nr. 2, 1. Allegro (0 ≙ 1)
  (Barenboim, EMI 1998)



Local Key Detection: Examples

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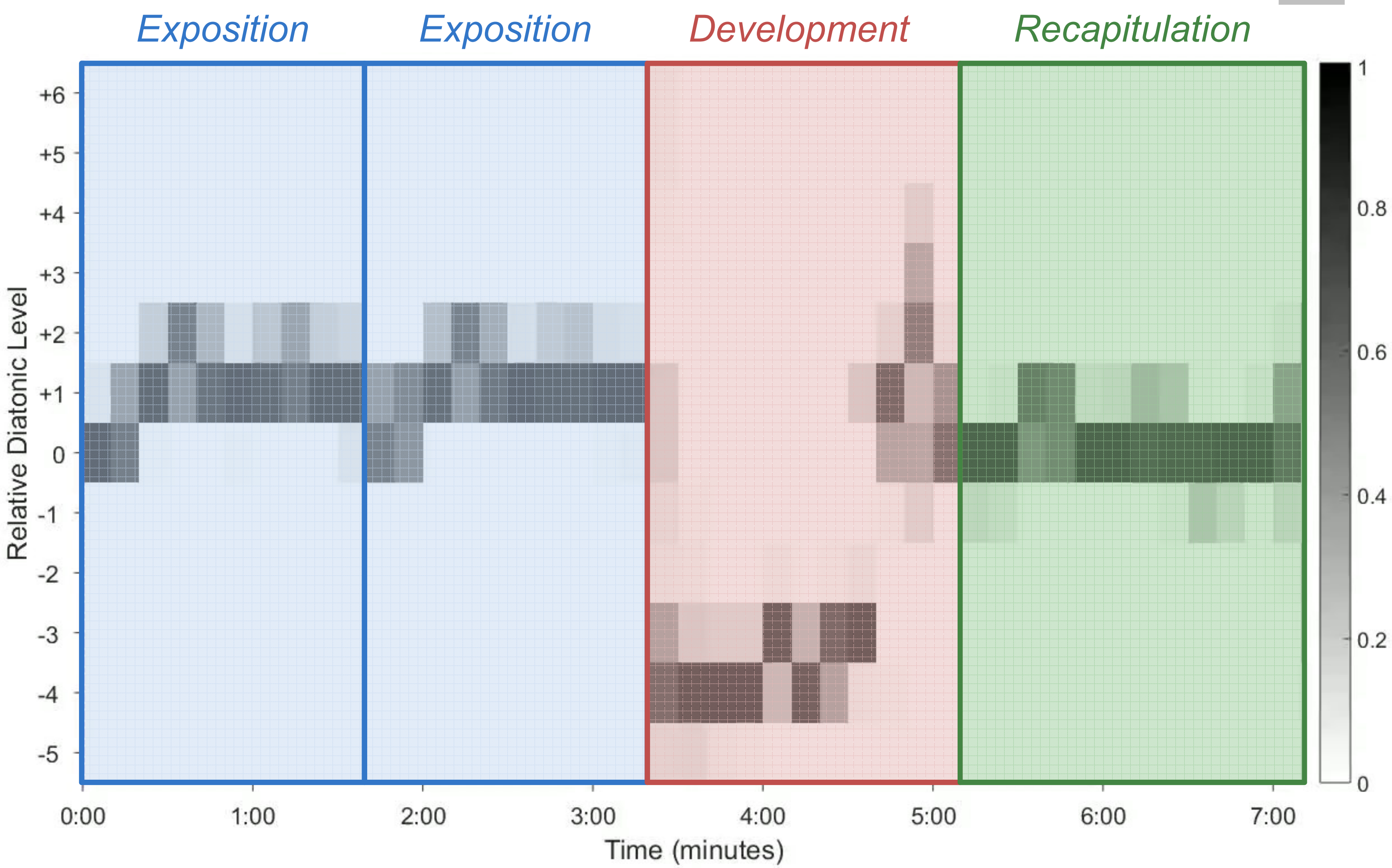
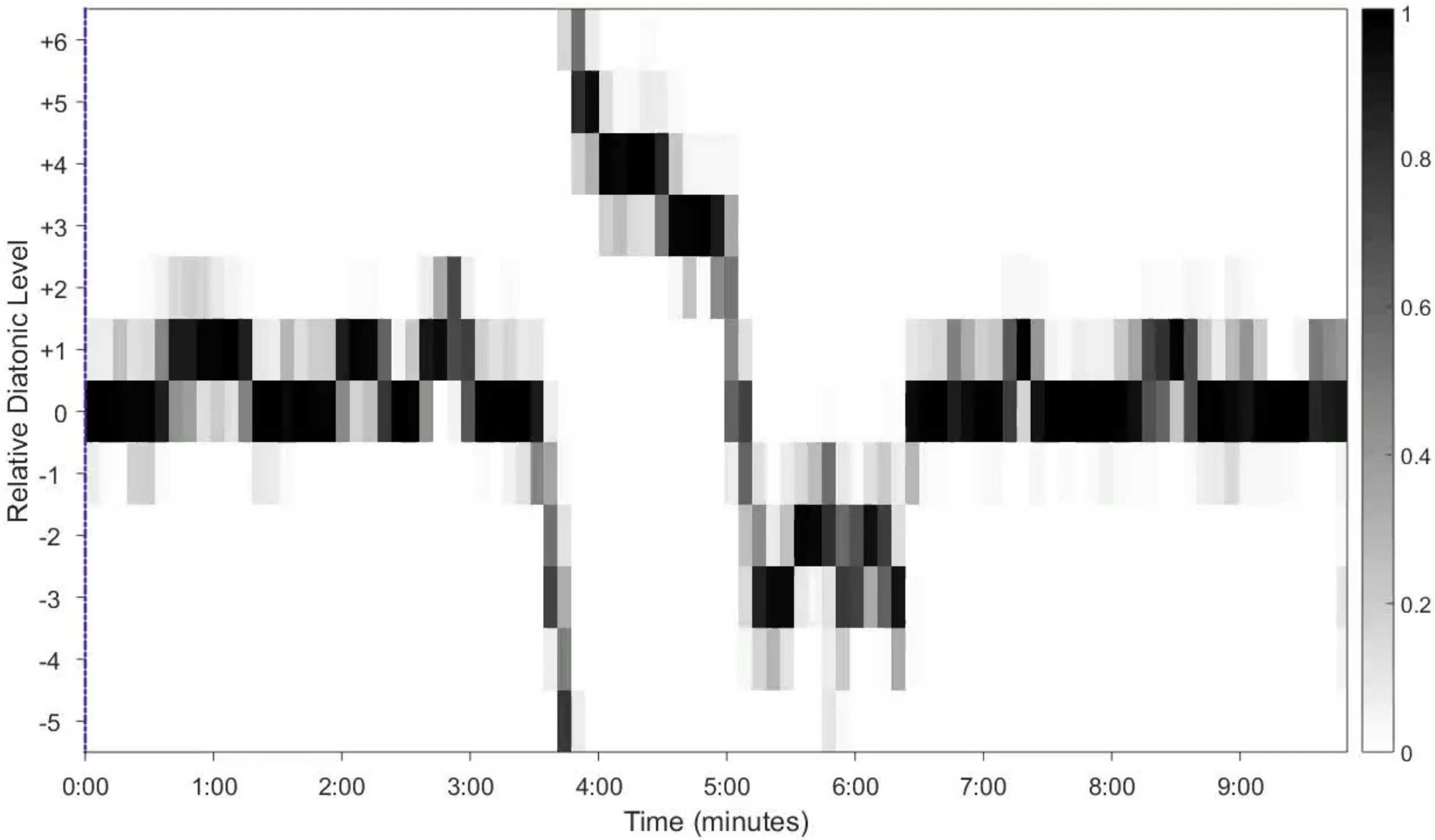


sonata form

Local Key Detection: Examples

- R. Wagner, *Die Meistersinger von Nürnberg*, Vorspiel ($0 \triangleq 0$)
  (Polish National Radio Symphony Orchestra, Naxos 1993)



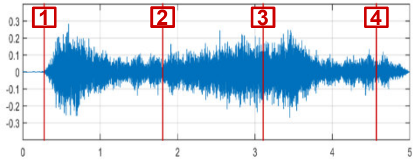
Harmony Analysis

Overview

- Template-based Chord Recognition
- HMM-based Chord Recognition
- Local Key Detection
- Application for Musicology

Local Key Detection: Examples

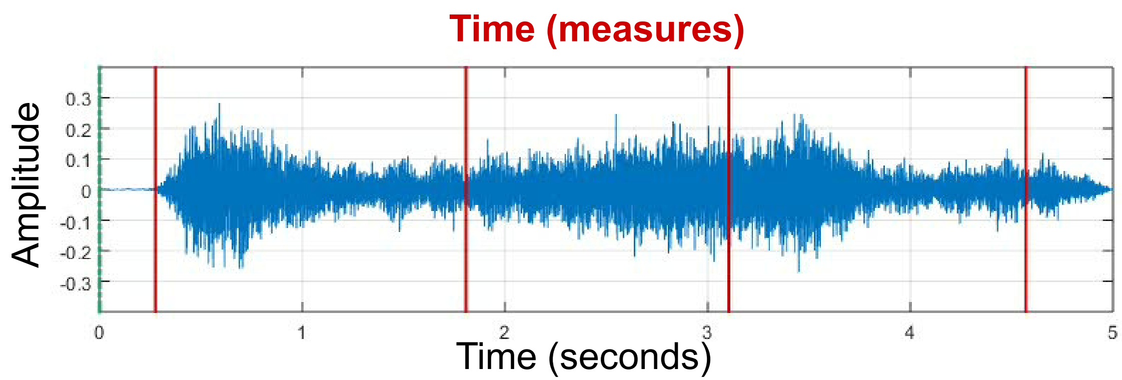
- R. Wagner, *Der Ring des Nibelungen* (four-opera cycle)
- 16 different performances (*versions*)
- Manual **measure annotations** for 3 versions



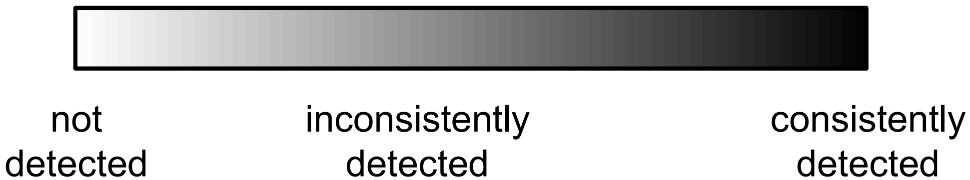
No.	Conductor	Recording	hh:mm:ss
1	Barenboim	1991–92	14:54:55
2	Boulez	1980–81	13:44:38
3	Böhm	1967–71	13:39:28
4	Furtwängler	1953	15:04:22
5	Haitink	1988–91	14:27:10
6	Janowski	1980–83	14:08:34
7	Karajan	1967–70	14:58:08
8	Keilberth/Furtwängler	1952–54	14:19:56
9	Krauss	1953	14:12:27
10	Levine	1987–89	15:21:52
11	Neuhold	1993–95	14:04:35
12	Sawallisch	1989	14:06:50
13	Solti	1958–65	14:36:58
14	Swarowsky	1968	14:56:34
15	Thielemann	2011	14:31:13
16	Weigle	2010–12	14:48:46

Local Key Detection: Cross-Version Analysis

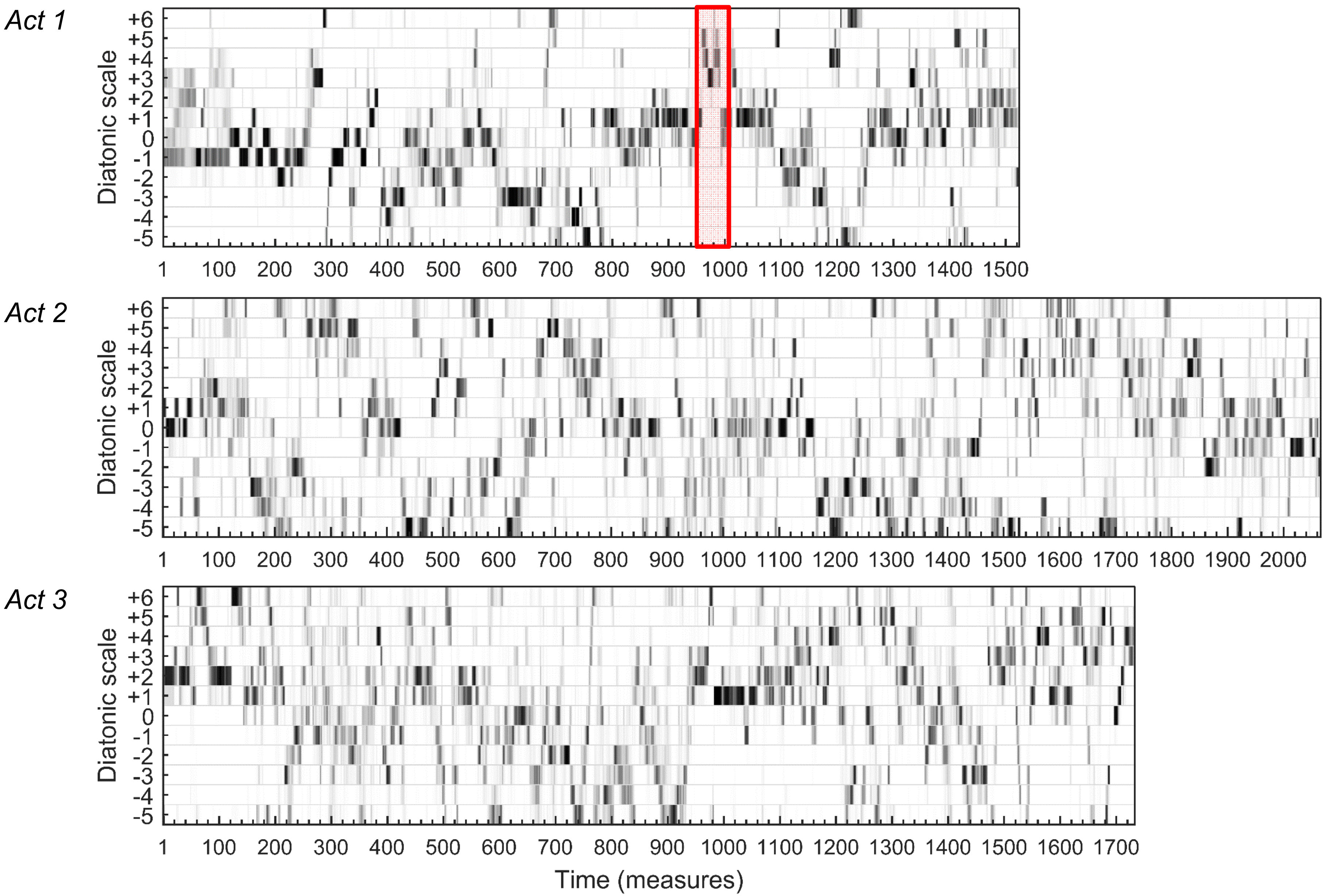
- R. Wagner, *Der Ring des Nibelungen* (four-opera cycle)
- 16 different performances (*versions*)
- Manual **measure annotations** for 3 versions



- Visualize cross-version consistency with gray scale

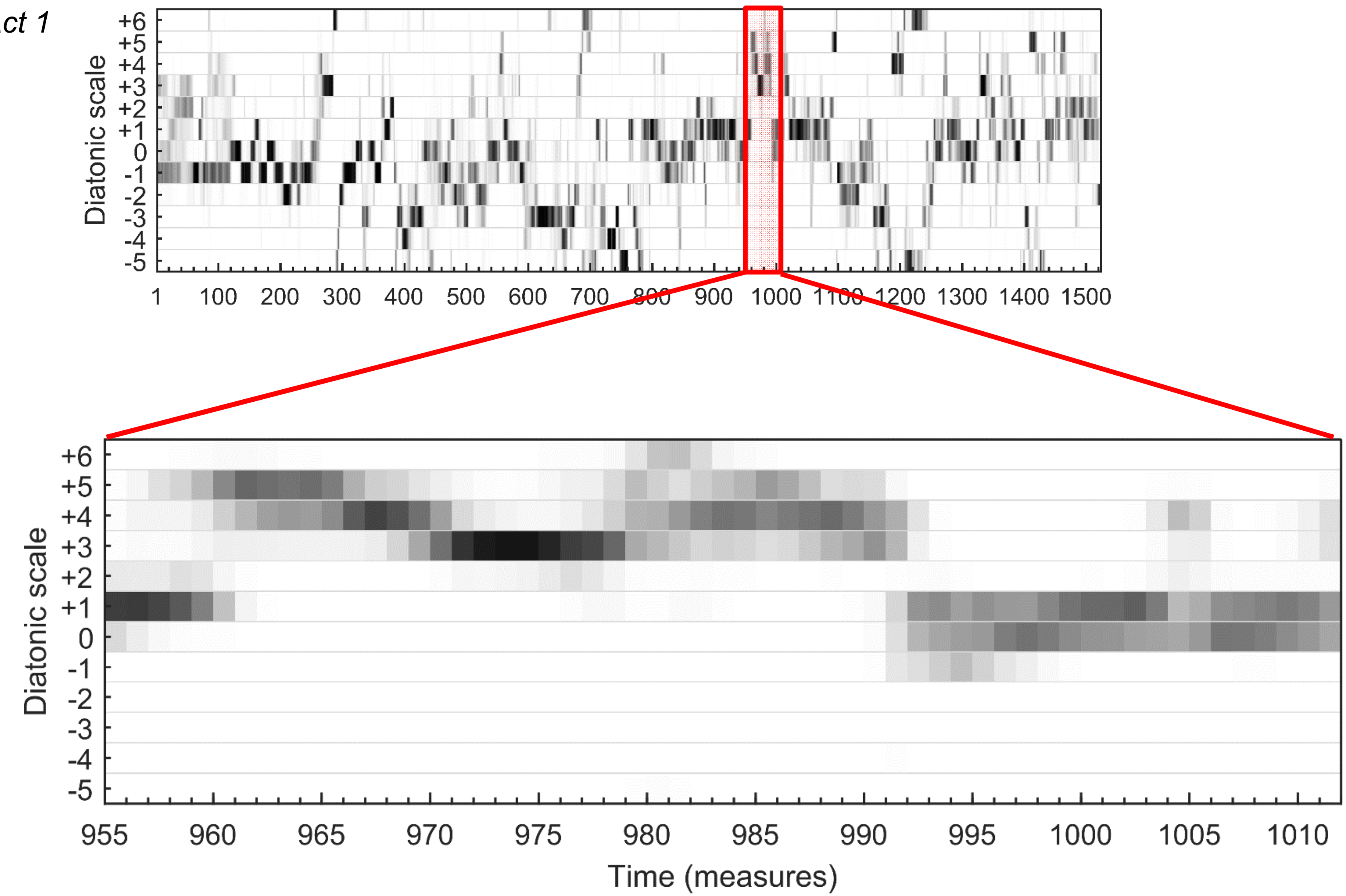


Die Walküre WWV 86 B



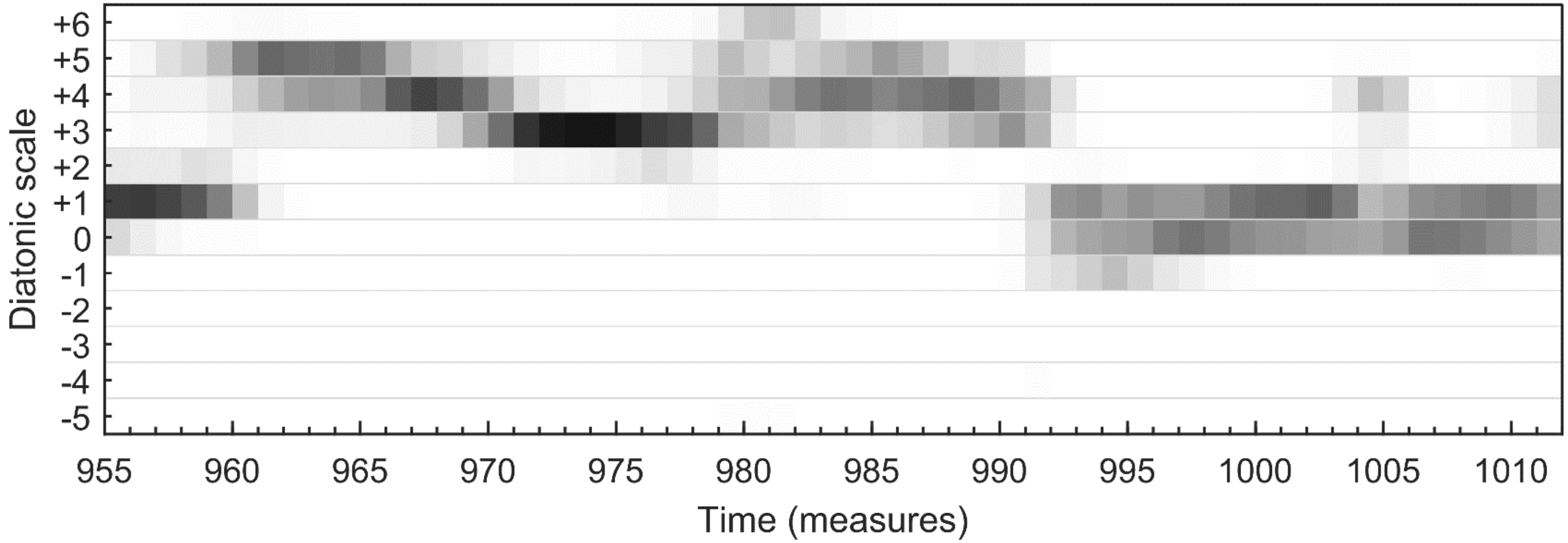
Die Walküre WWV 86 B

Act 1

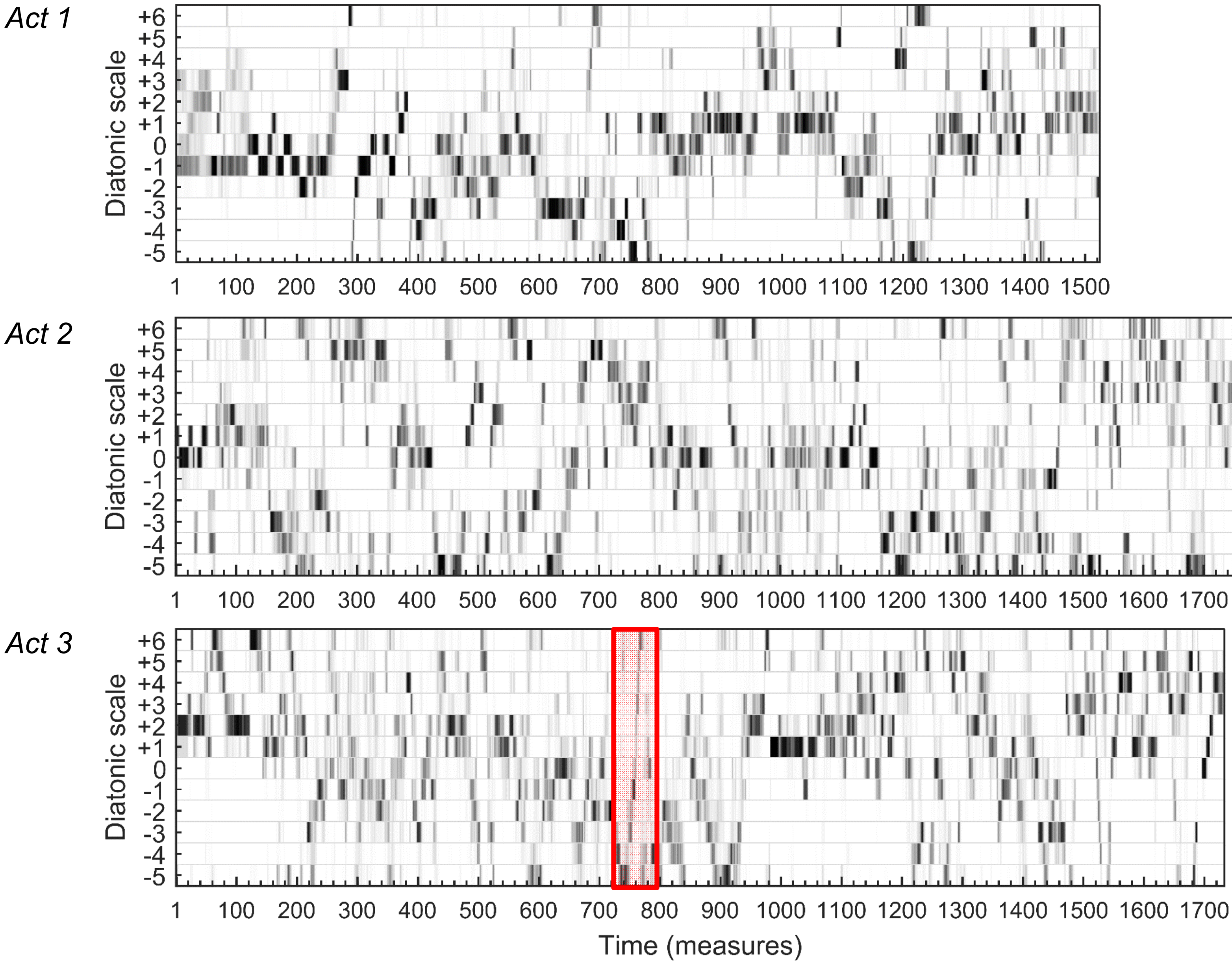


Die Walküre WWV 86 B

- Act 1, measures 955–1012
- Sieglinde’s narration

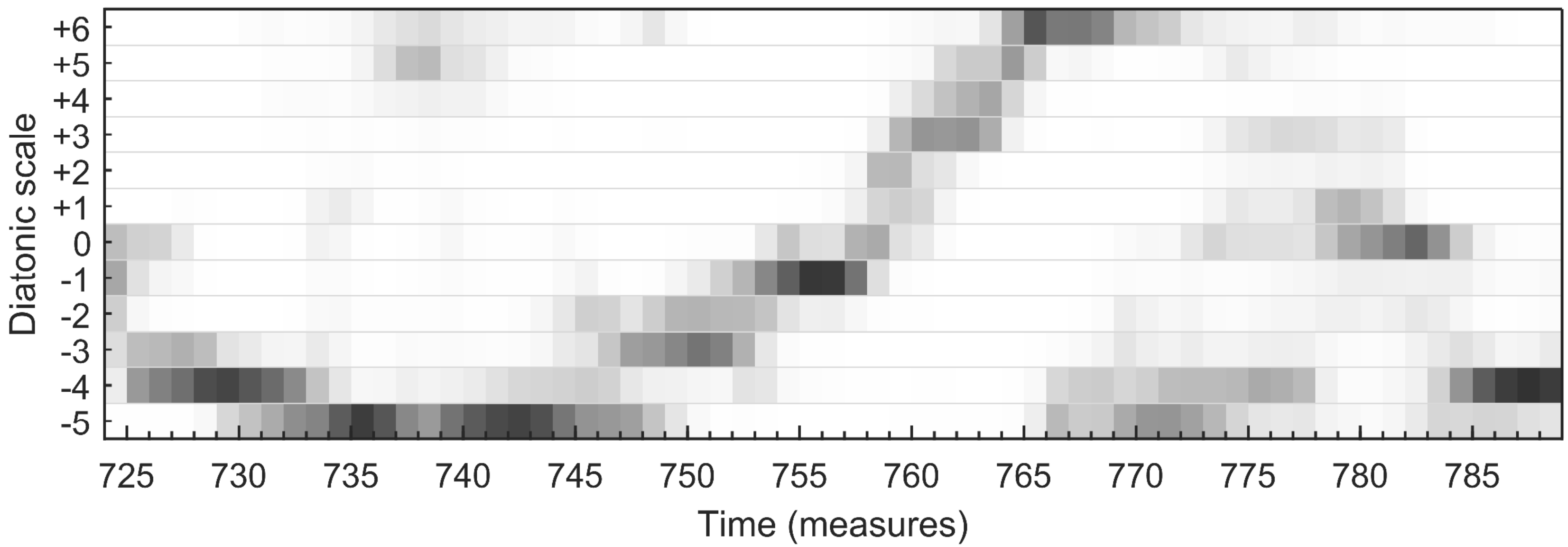


Die Walküre WWV 86 B



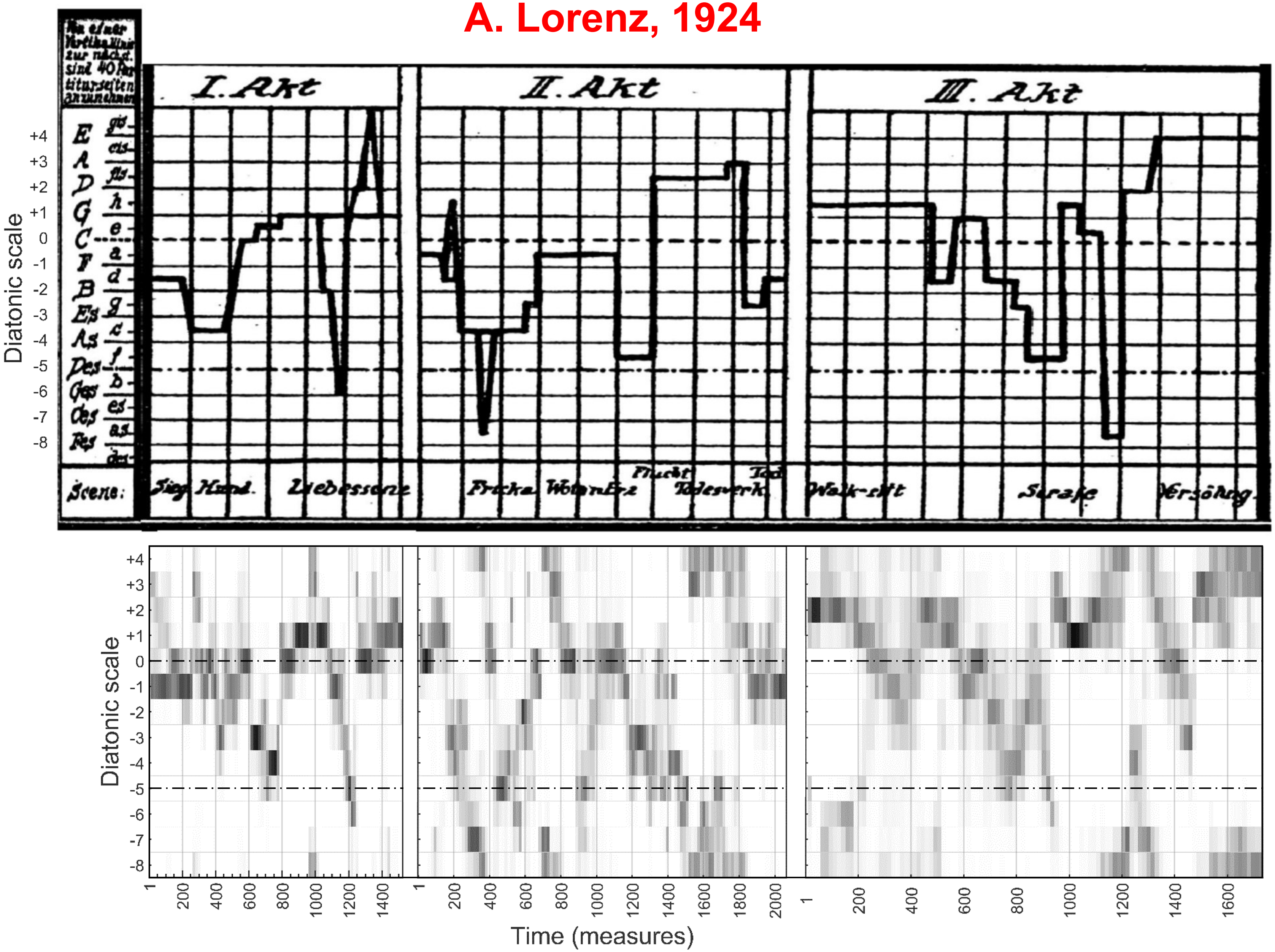
Die Walküre WWV 86 B

- Act 3, measures 724–789
- Wotan's punishment



Die Walküre WWV 86 B

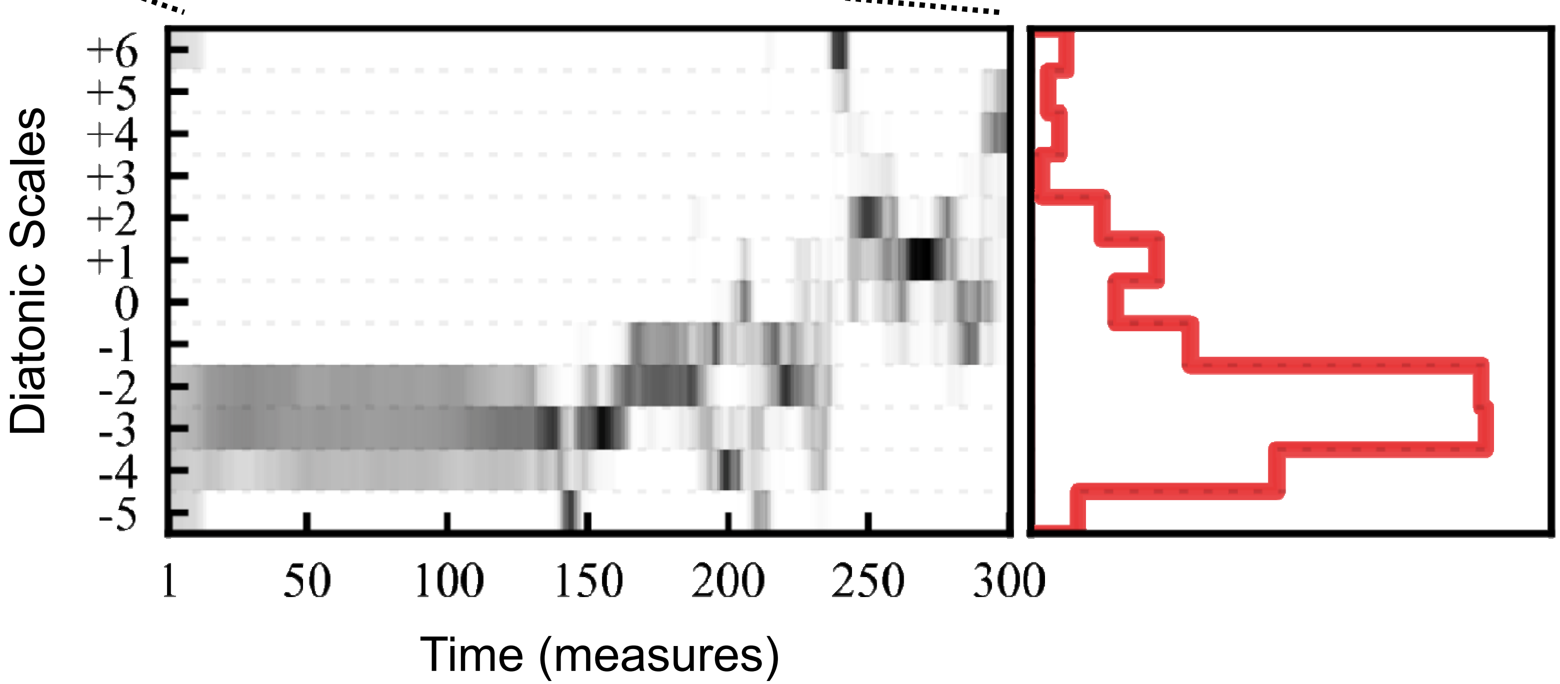
A. Lorenz, 1924



Exploring Tonal-Dramatic Relationships

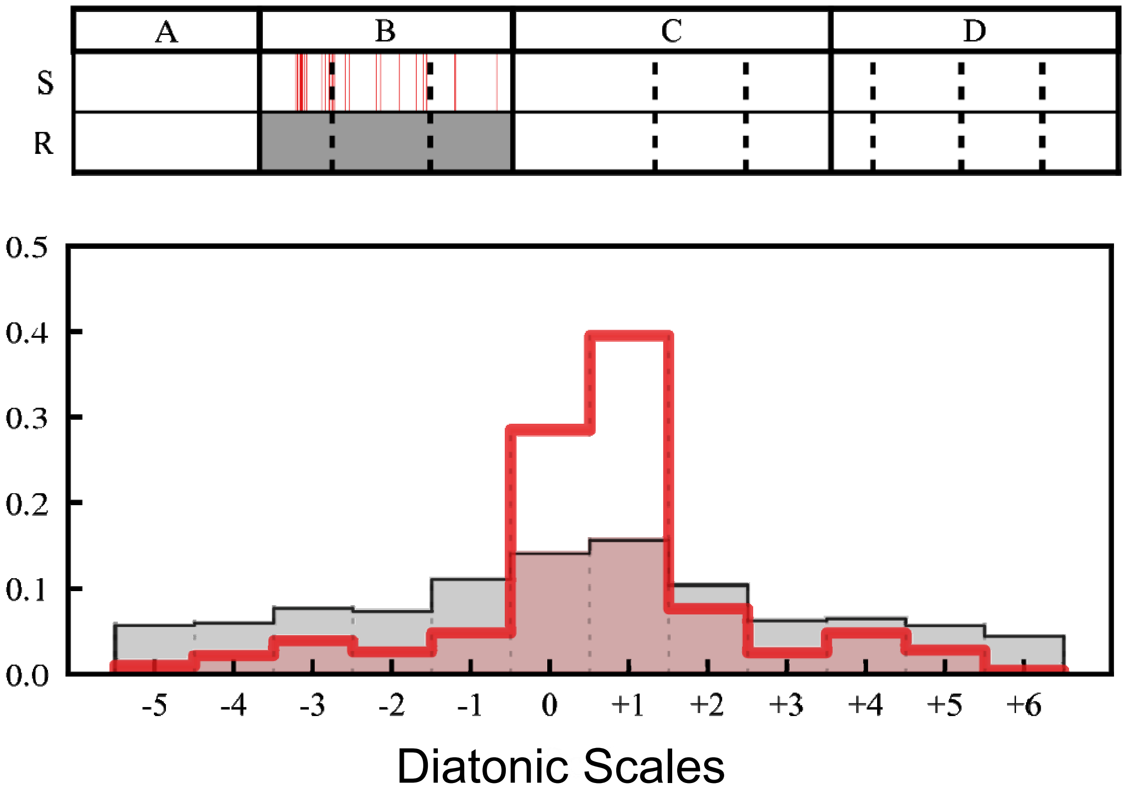
- Histograms of Analysis over time

Das Rheingold WWV 86 A 3897 measures	Die Walküre WWV 86 B 5322 measures	Siegfried WWV 86 C 6682 measures	Götterdämmerung WWV 86 D 6040 measures



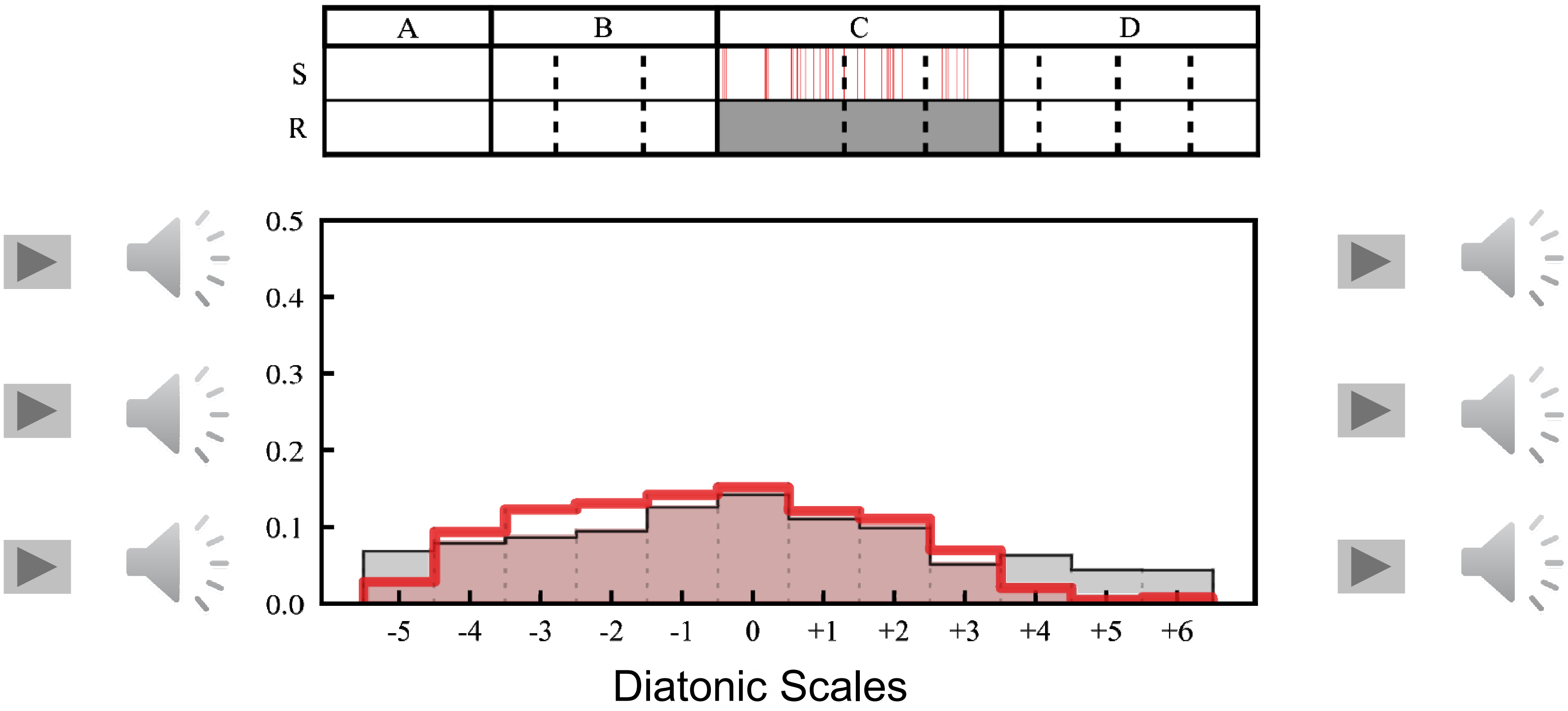
Exploring Tonal-Dramatic Relationships

Die Walküre – Sword motif



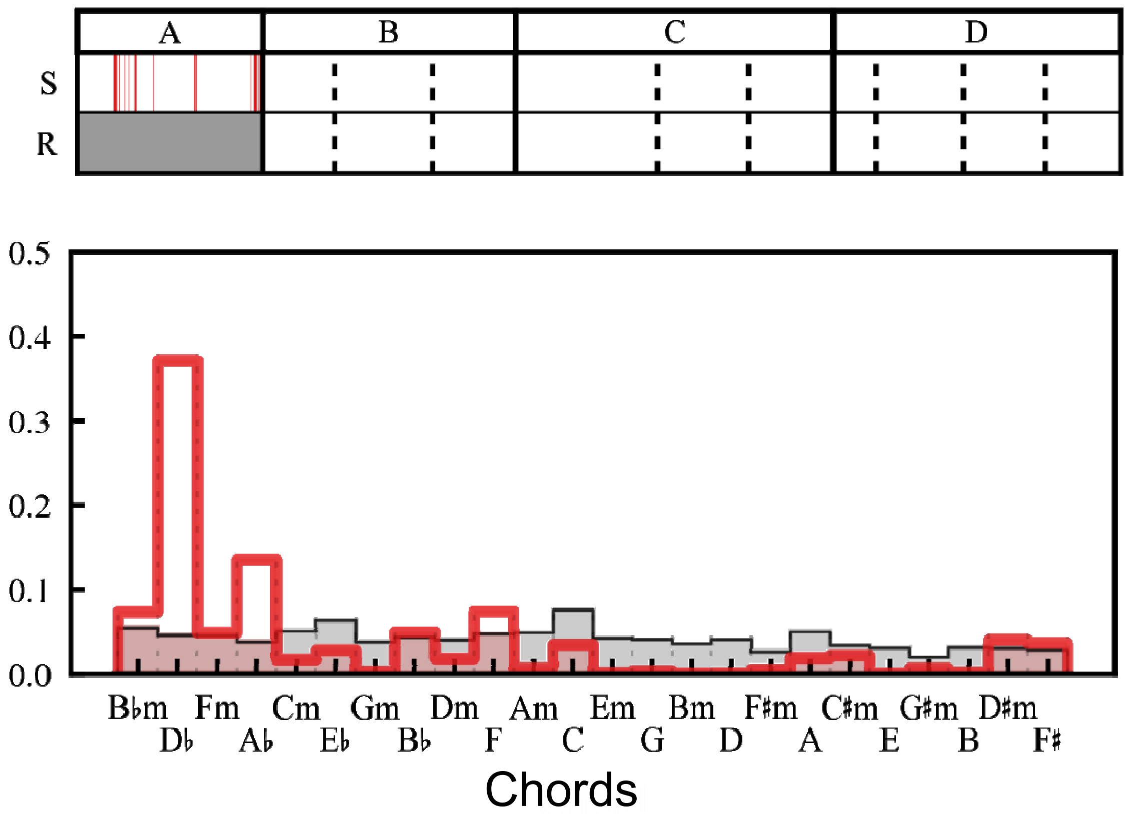
Exploring Tonal-Dramatic Relationships

Siegfried – Sword motif



Exploring Tonal-Dramatic Relationships

Das Rheingold – Valhalla motif



Exploring Tonal-Dramatic Relationships

Die Walküre – Valhalla motif