

Hochschule für Musik Karlsruhe

Blockvorlesung

## Advanced Audio-Based Music Processing

## 5. Harmonic Feature Design

Christof Weiß, Frank Zalkow, Meinard Müller

International Audio Laboratories Erlangen

christof.weiss@audiolabs-erlangen.de  
frank.zalkow@audiolabs-erlangen.de  
meinard.mueller@audiolabs-erlangen.de

## Dissertation: Tonality-Based Style Analysis

Christof Weiß

Computational Methods for Tonality-Based Style Analysis of  
Classical Music Audio Recordings

PhD thesis, Ilmenau University of Technology, 2017

[https://www.db-thueringen.de/receive/dbt\\_mods\\_00032890](https://www.db-thueringen.de/receive/dbt_mods_00032890)

Chapter 6: Design of Tonal Features

## Harmonic Feature Design

## Motivation

- Harmony analysis:
  - Recognition of **specific** chords, keys, ...
  - Resulting features / analyses characterize a **specific** piece
  - Allows for playing / accompaniment
  - Sensitive to transpositions, **key-dependent**
- Harmonic features:
  - Describe more **general** properties of a piece
  - Characterize musical „language“ or **style**
  - Allow for historical analysis / style classification
  - Independent of key, **transposition-invariant**
  - Quantification of chord **types**, **relative** chord transitions, ...

## Harmonic Feature Design

## Overview

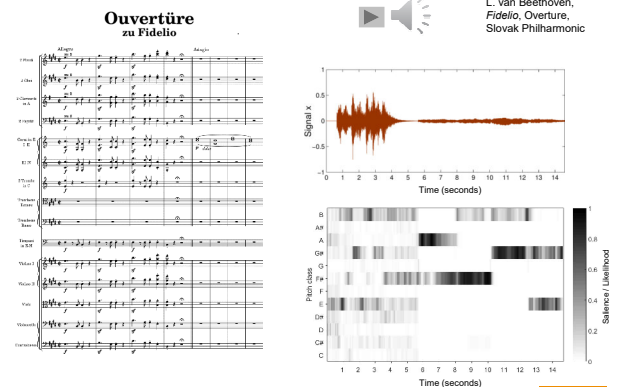
- Measuring Interval and Chord Categories
- Measuring Tonal Complexity

## Harmonic Feature Design

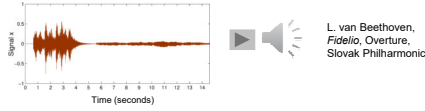
## Overview

- Measuring Interval and Chord Categories
- Measuring Tonal Complexity

## Signal Processing: Chroma Features

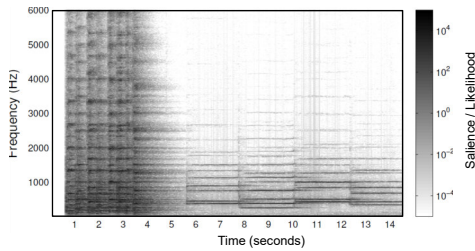


## Signal Processing: Chroma Features



L. van Beethoven,  
*Fidelio*, Overture,  
Slovak Philharmonic

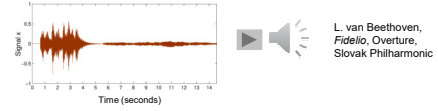
Spectrogram: Time – Frequency



7

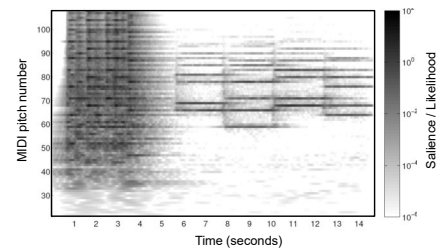
AUDIO  
LABS

## Signal Processing: Chroma Features



L. van Beethoven,  
*Fidelio*, Overture,  
Slovak Philharmonic

Log-Frequency-Spectrogram: Time – Pitch



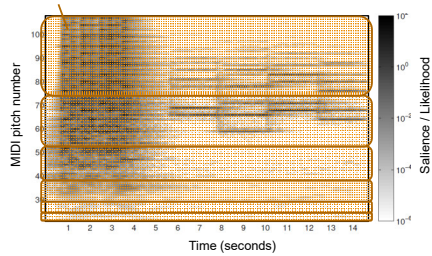
8

AUDIO  
LABS

## Signal Processing: Spectral Features

- independent of exact pitches
- describe **timbral** properties (sound color)
- „standard features“ for genre classification

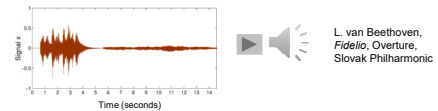
Frequency bands: Loudness, Spectral Flatness, Spectral Centroid



9

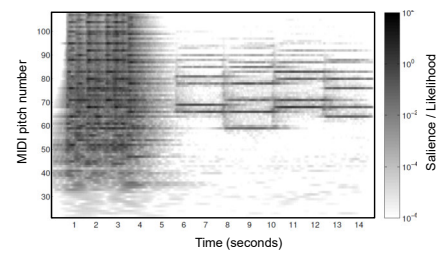
AUDIO  
LABS

## Signal Processing: Chroma Features



L. van Beethoven,  
*Fidelio*, Overture,  
Slovak Philharmonic

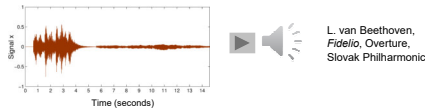
Log-Frequency-Spectrogram: Time – Pitch



10

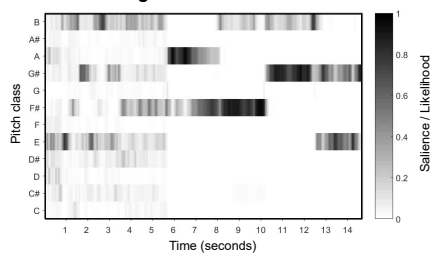
AUDIO  
LABS

## Signal Processing: Chroma Features



L. van Beethoven,  
*Fidelio*, Overture,  
Slovak Philharmonic

Chromagram: Time – Pitch class



11

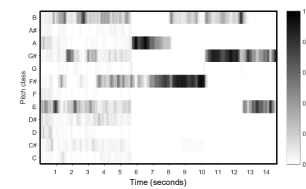
AUDIO  
LABS

## Signal Processing: Chroma Features

### Orchestra



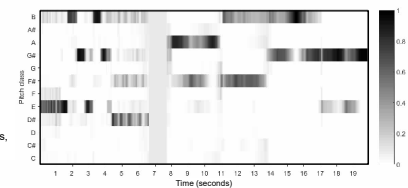
L. van Beethoven,  
*Fidelio*, Overture,  
Slovak Philharmonic



### Piano



*Fidelio*, Overture,  
arr. Alexander Zemlinsky  
M. Namekawa, D.R. Davies,  
piano four hands

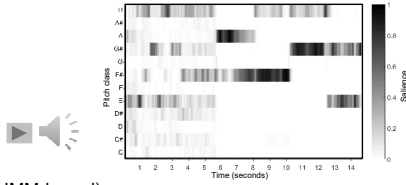


12

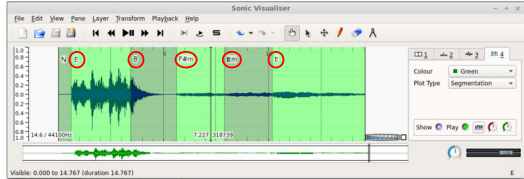
AUDIO  
LABS

## Design of Harmonic Features

- Chromagram



- Chord Recognition (HMM-based)



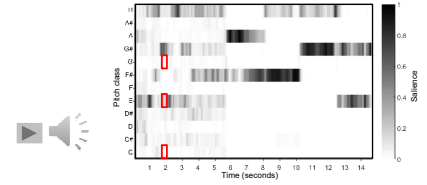
Sonic Visualizer,  
Chordino Plugin  
(Queen Mary  
Univ., London)

AUDIO  
LABS

13

## Design of Harmonic Features

- Chromagram



- Chord types

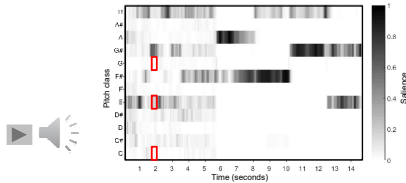
- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**

AUDIO  
LABS

14

## Design of Harmonic Features

- Chromagram



- Chord types

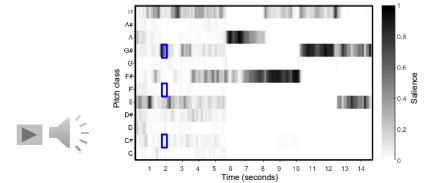
- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**

AUDIO  
LABS

15

## Design of Harmonic Features

- Chromagram



- Chord types

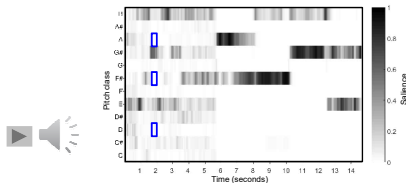
- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**

AUDIO  
LABS

16

## Design of Harmonic Features

- Chromagram



- Chord types

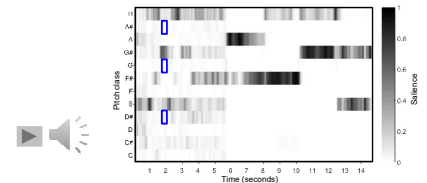
- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**

AUDIO  
LABS

17

## Design of Harmonic Features

- Chromagram



- Chord types

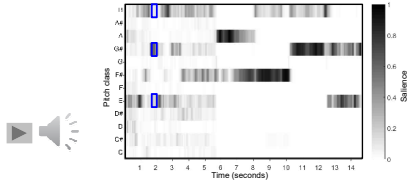
- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**

AUDIO  
LABS

18

## Design of Harmonic Features

- Chromagram

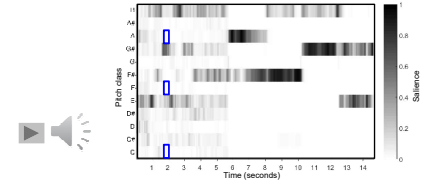


- Chord types

- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**

## Design of Harmonic Features

- Chromagram

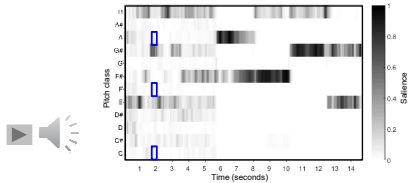


- Chord types

- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**

## Design of Harmonic Features

- Chromagram

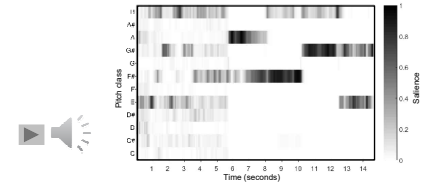


- Chord types

- Major / minor / diminished / augmented?
- Probabilities / likelihoods (*continuous*) instead of recognition (*discrete*)
- Multiply chroma values that form a **certain chord**
- Repeat this for all **cyclic shifts**
- Sum up all similarity values (products) → **transposition-invariance**

## Design of Harmonic Features

- Chromagram

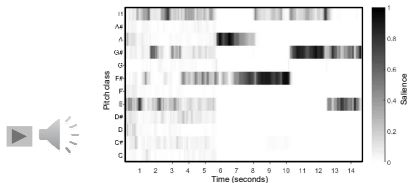


- Chord types

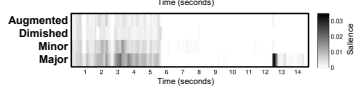
- Major:  $T^{CM} = (1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0)^T$
- Minor:  $T^{Cm} = (1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0)^T$
- Diminished:  $T^{Co} = (1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0)^T$
- Augmented:  $T^{C+} = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0)^T$

## Design of Harmonic Features

- Chromagram

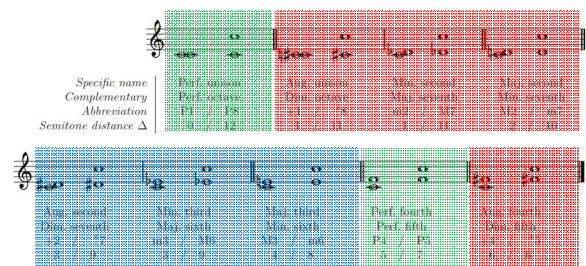


- Chord types



## Design of Harmonic Features

### Consonant & Dissonant Intervals



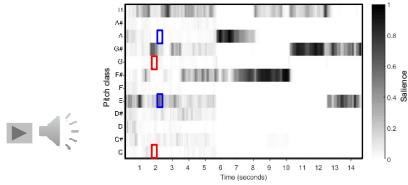
- Perfect consonances
- Imperfect consonances
- Dissonances





## Design of Harmonic Features

### Chromagram

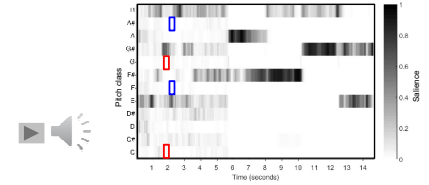


### Interval categories

- $\Delta = 1 / 11$ :  $T^{IC1} = (1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 2 / 10$ :  $T^{IC2} = (1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 3 / 9$ :  $T^{IC3} = (1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 4 / 8$ :  $T^{IC4} = (1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 5 / 7$ :  $T^{IC5} = (1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 6 / 6$ :  $T^{IC6} = (1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0)^T$

## Design of Harmonic Features

### Chromagram

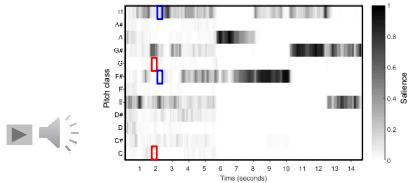


### Interval categories

- $\Delta = 1 / 11$ :  $T^{IC1} = (1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 2 / 10$ :  $T^{IC2} = (1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 3 / 9$ :  $T^{IC3} = (1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 4 / 8$ :  $T^{IC4} = (1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 5 / 7$ :  $T^{IC5} = (1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 6 / 6$ :  $T^{IC6} = (1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0)^T$

## Design of Harmonic Features

### Chromagram

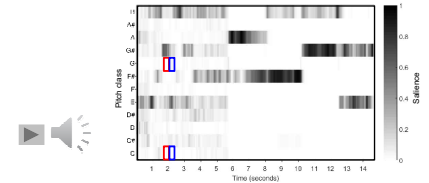


### Interval categories

- $\Delta = 1 / 11$ :  $T^{IC1} = (1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 2 / 10$ :  $T^{IC2} = (1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 3 / 9$ :  $T^{IC3} = (1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 4 / 8$ :  $T^{IC4} = (1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 5 / 7$ :  $T^{IC5} = (1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 6 / 6$ :  $T^{IC6} = (1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0)^T$

## Design of Harmonic Features

### Chromagram

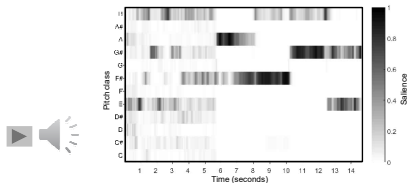


### Interval categories

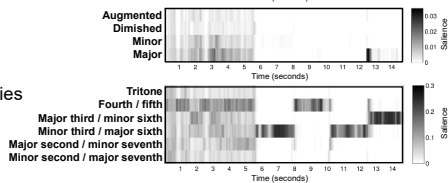
- $\Delta = 1 / 11$ :  $T^{IC1} = (1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 2 / 10$ :  $T^{IC2} = (1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 3 / 9$ :  $T^{IC3} = (1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 4 / 8$ :  $T^{IC4} = (1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 5 / 7$ :  $T^{IC5} = (1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0)^T$
- $\Delta = 6 / 6$ :  $T^{IC6} = (1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0)^T$

## Design of Harmonic Features

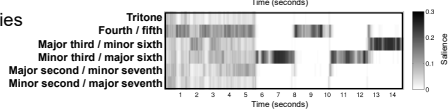
### Chromagram



### Chord types



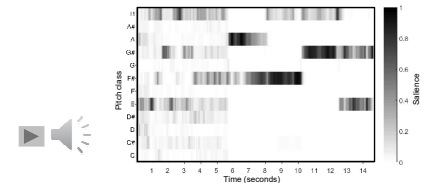
### Interval categories



→ transposition-invariant features!

## Design of Harmonic Features

### Chromagram



### Feature computation

$$\Psi^T(\mathbf{c}) = \sum_{q=0}^{11} \left( \prod_{k=0}^{11} \left( c_{(q+k) \bmod 12} T_k \right) \right)$$

sum      product      shift

## Harmonic Feature Design

### Overview

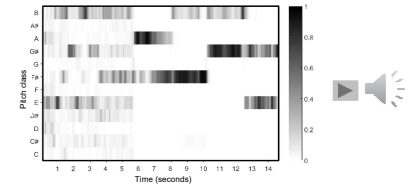
- Measuring Interval and Chord Categories
- Measuring Tonal Complexity

43



## Design of Harmonic Features

- Chromagram:

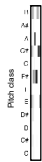


44



## Design of Harmonic Features

- Chromagram: Full piece

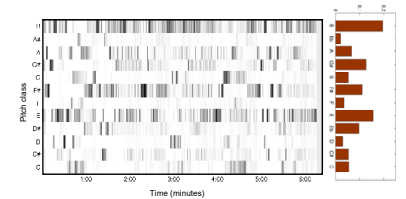


45

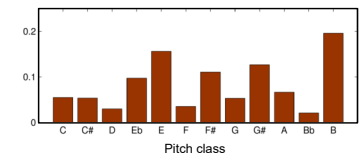


## Design of Harmonic Features

- Chromagram: Full piece



- Chroma statistics:

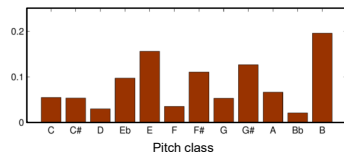


46

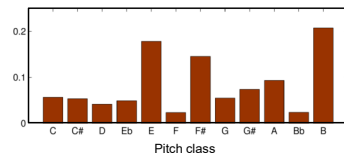


## Design of Harmonic Features

- Orchestra:



- Piano:

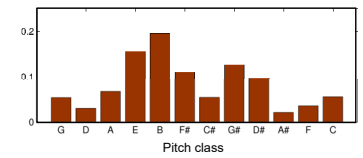


47



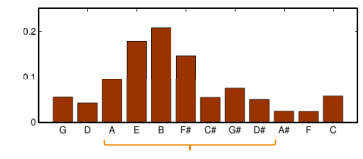
## Design of Harmonic Features

- Orchestra:



circle of fifths

- Piano:



E major key  
+4 diatonic scale

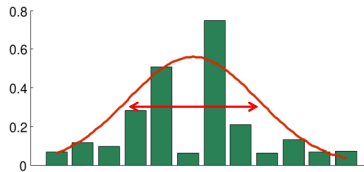
48





## Design of Harmonic Features

- What does a global chroma statistics tell us about a piece of music?
- Measure spread / flatness / centricity on the circle of fifths:

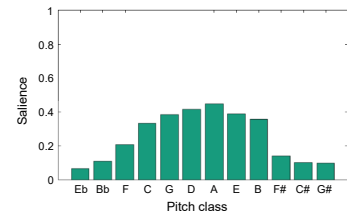


49

AUDIO  
LABS

## Tonal Complexity

- Global chroma statistics (audio)
- 1567 – G. P. da Palestrina, Missa de Beata Virgine, Credo



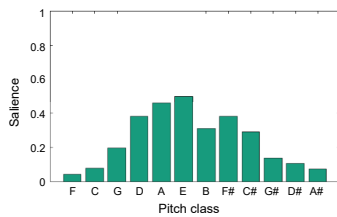
Circle of fifths →

50

AUDIO  
LABS

## Tonal Complexity

- Global chroma statistics (audio)
- 1725 – J. S. Bach, Suite No. 4 BWV 1069, 1. Overture (D major)



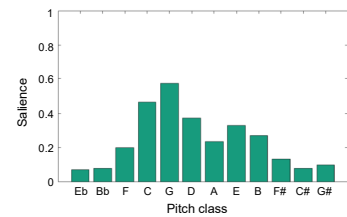
Circle of fifths →

51

AUDIO  
LABS

## Tonal Complexity

- Global chroma statistics (audio)
- 1783 – W. A. Mozart, „Linz“ symphony KV 425, 1. Adagio / Allegro



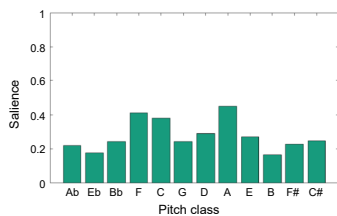
Circle of fifths →

52

AUDIO  
LABS

## Tonal Complexity

- Global chroma statistics (audio)
- 1883 – J. Brahms, Symphony No. 3, 1. Allegro con brio (F major)



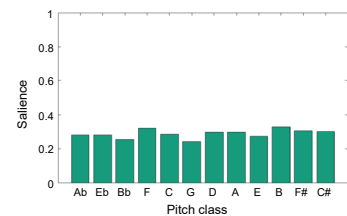
Circle of fifths →

53

AUDIO  
LABS

## Tonal Complexity

- Global chroma statistics (audio)
- 1940 – A. Webern, Variations for Orchestra op. 30



Circle of fifths →

54


AUDIO  
LABS


## Tonal Complexity

### Requirements

- Realization of complexity measure  $\Gamma$
- Input: L1-normalized chroma vector (entries sum to 1)

#### Boundary conditions:

- Sparse chroma vector:  $\mathbf{c}^{\text{sparse}} := (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$   
  $\rightarrow$  Complexity = 0

- Flat chroma vector:  $\mathbf{c}^{\text{flat}} := \frac{1}{12} \cdot (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)^T$   
  $\rightarrow$  Complexity = 1


- All other chroma vectors:  $\rightarrow 0 < \text{Complexity} < 1$   



## Tonal Complexity


### Entropy measure

- Compute **Shannon entropy of chroma vector**:

$$\Gamma_{\text{Entr}}(\mathbf{c}) := -\frac{1}{\log_2(12)} \left( \sum_{q=0}^{11} c_q \cdot \log_2(c_q) \right)$$

- Sparse chroma vector:  $\mathbf{c}^{\text{sparse}} := (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$   
  $\rightarrow$  Complexity = 0

- Flat chroma vector:  $\mathbf{c}^{\text{flat}} := \frac{1}{12} \cdot (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)^T$   
  $\rightarrow$  Complexity = 1


- All other chroma vectors:  $\rightarrow 0 < \text{Complexity} < 1$   



## Tonal Complexity

### Flatness measure

- Compute **Flatness measure (ratio of geometric and arithmetic mean)**:

$$\Gamma_{\text{Flat}}(\mathbf{c}) := \frac{\left( \prod_{q=0}^{11} c_q \right)^{1/12}}{\frac{1}{12} \sum_{q=0}^{11} c_q}$$

- Sparse chroma vector:  $\mathbf{c}^{\text{sparse}} := (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$   
  $\rightarrow$  Complexity = 0

- Flat chroma vector:  $\mathbf{c}^{\text{flat}} := \frac{1}{12} \cdot (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)^T$   
  $\rightarrow$  Complexity = 1

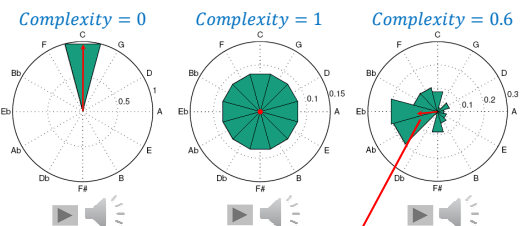
- All other chroma vectors:  $\rightarrow 0 < \text{Complexity} < 1$   


## Tonal Complexity

### Fifth-width measure

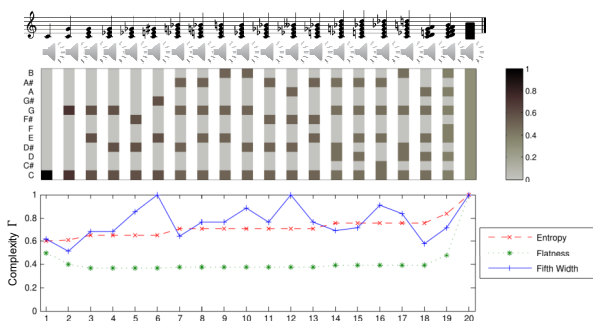
- Compute **Width of Distribution over Circle of Fifths (angular deviation)**:

$$\Gamma_{\text{Fifth}}(\mathbf{c}) := \sqrt{1 - r_{\text{fifth}}(\mathbf{c})} \quad \text{with} \quad r_{\text{fifth}}(\mathbf{c}) = \left| \sum_{q=0}^{11} c_q^{\text{fifth}} \exp\left(\frac{2\pi i q}{12}\right) \right|$$



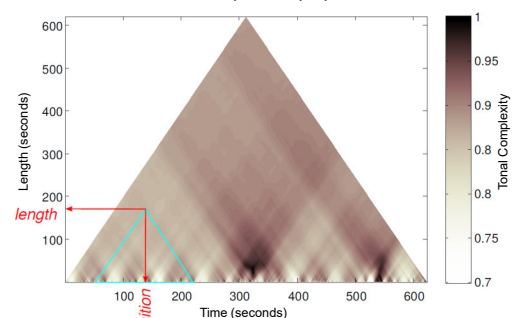
$$\text{Complexity} = 1 - r_{\text{fifth}}(\mathbf{c})$$

## Tonal Complexity – Chords



## Tonal Complexity – Scape Plot Representations

- Complexity relates to multiple time scales
- Multi-scale visualization technique: Scape plot



## Tonal Complexity – Beethoven's Sonatas

