INTERNATIONAL AUDIO LABORATORIES ERLANGEN

AUDIO LABS

Hochschule für Musik Karlsruhe

Blockvorlesung

Advanced Audio-Based Music Processing

3. Audio Processing Basics

Christof Weiß, Frank Zalkow, Meinard Müller
International Audio Laboratories Erlangen

christof.weiss@audiolabs-erlangen.de
frank.zalkow@audiolabs-erlangen.de
meinard.mueller@audiolabs-erlangen.de

FAU FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

Hochschule für Musik Karlsruhe

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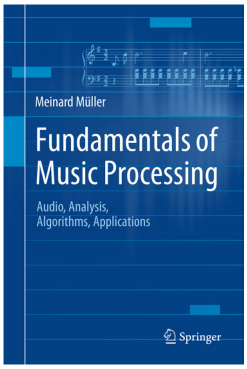
Book: Fundamentals of Music Processing



Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
483 p., 249 illus., hardcover
ISBN: 978-3-319-21944-8
Springer, 2015

Accompanying website:
www.music-processing.de

Book: Fundamentals of Music Processing

Chapter	Music Processing Scenario
1	Music Representations
2	Fourier Analysis of Signals
3	Music Synchronization
4	Music Structure Analysis
5	Chord Recognition
6	Tempo and Beat Tracking
7	Content-Based Audio Retrieval
8	Musically Informed Audio Decomposition

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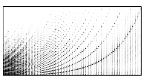
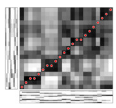
Chapter 2: Fourier Analysis of Signals



Important technical terminology is covered in Chapter 2. In particular, we approach the Fourier transform—which is perhaps the most fundamental tool in signal processing—from various perspectives. For the reader who is more interested in the musical aspects of the book, Section 2.1 provides a summary of the most important facts on the Fourier transform. In particular, the notion of a spectrogram, which yields a time–frequency representation of an audio signal, is introduced. The remainder of the chapter treats the Fourier transform in greater mathematical depth and also includes the fast Fourier transform (FFT)—an algorithm of great beauty and high practical relevance.

Chapter 3: Music Synchronization



As a first music processing task, we study in Chapter 3 the problem of music synchronization. The objective is to temporally align compatible representations of the same piece of music. Considering this scenario, we explain the need for musically informed audio features. In particular, we introduce the concept of chroma-based music features, which capture properties that are related to harmony and melody. Furthermore, we study an alignment technique known as dynamic time warping (DTW), a concept that is applicable for the analysis of general time series. For its efficient computation, we discuss an algorithm based on dynamic programming—a widely used method for solving a complex problem by breaking it down into a collection of simpler subproblems.

Audio Processing Basics
Overview

- Fourier Transform: Motivation & Definition
- Short-Time Fourier Transform and Spectrograms
- Audio Features and Chromagrams

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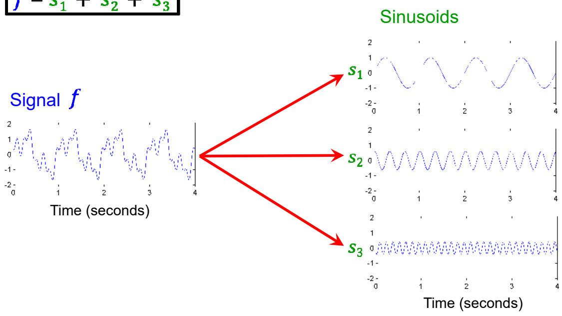
Fourier Transform

Idea: Decompose a given signal into a superposition of sinusoids (elementary signals).

$\boxed{f = s_1 + s_2 + s_3}$



Fourier Transform

Each sinusoid has a physical meaning and can be described by three parameters:

$$\boxed{s_{(A,\omega,\varphi)}(t) = A \cdot \sin(2\pi(\omega t - \varphi))}$$

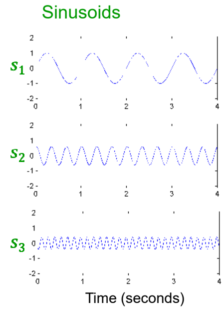
$\omega =$ frequency
$A =$ amplitude
$\varphi =$ phase

Interpretation:
The amplitude A reflects the intensity at which the sinusoidal of frequency ω appears in f.
The phase φ reflects how the sinusoidal has to be shifted to best correlate with f.

$A_1 = 1$, $\omega_1 = 1$, $\varphi_1 = 0$

$A_2 = 0.6$, $\omega_2 = 3$, $\varphi_2 = -0.2$

$A_3 = 0.4$, $\omega_3 = 7$, $\varphi_3 = 0.4$



Fourier Transform

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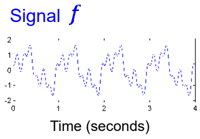
$\boxed{f = s_1 + s_2 + s_3}$



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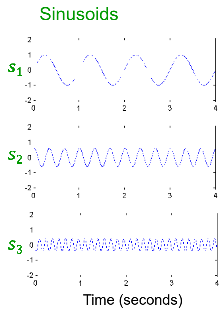

Fourier Transform

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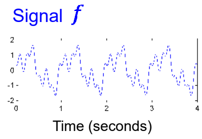

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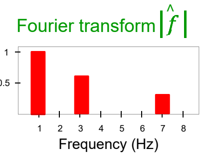
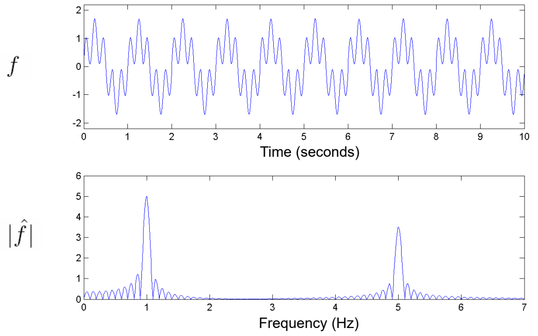


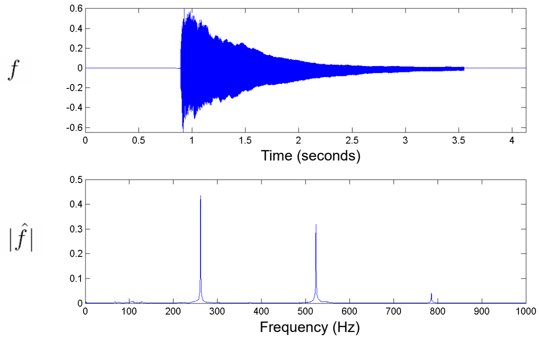
Fourier Transform

Example: Superposition of two sinusoids



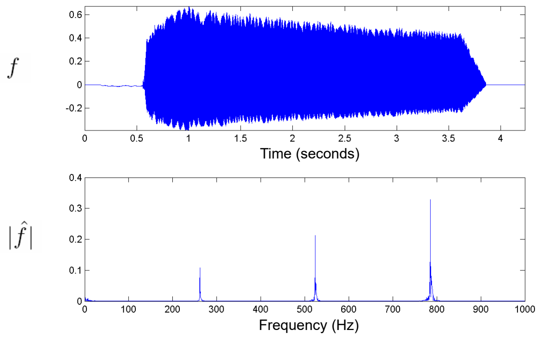
Fourier Transform

Example: C4 played by piano



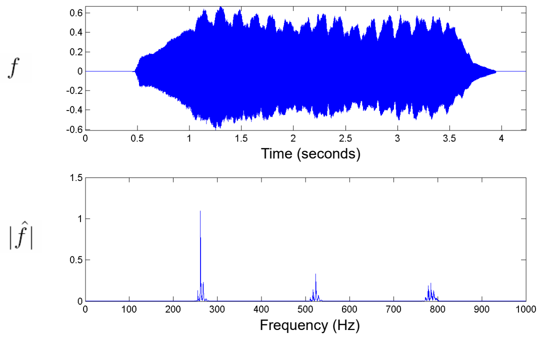
Fourier Transform

Example: C4 played by trumpet



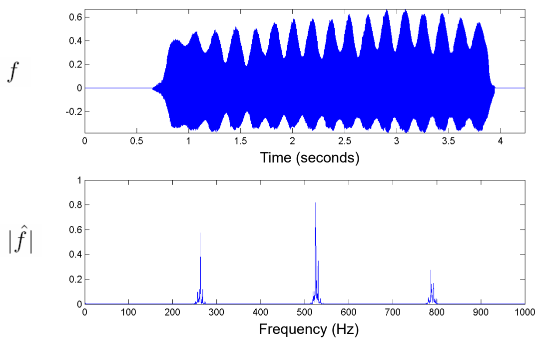
Fourier Transform

Example: C4 played by violin



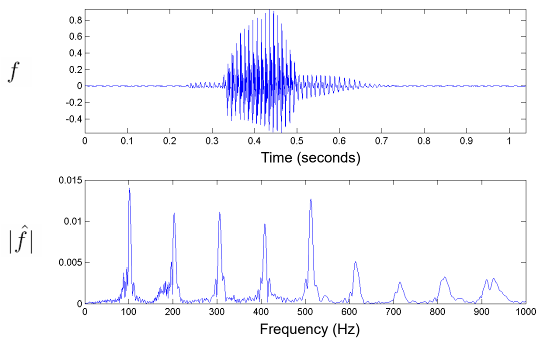
Fourier Transform

Example: C4 played by flute



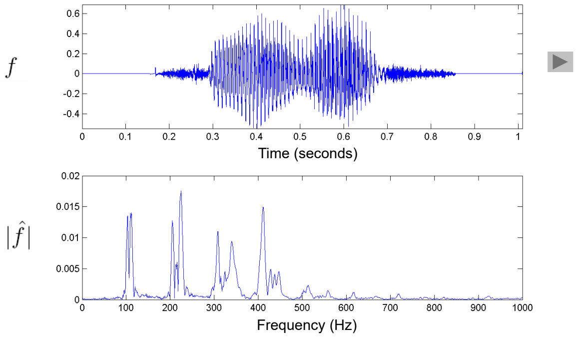
Fourier Transform

Example: Speech “Bonn”



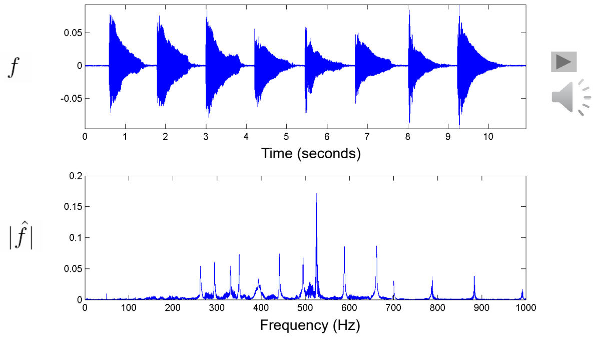
Fourier Transform

Example: Speech “Zürich”



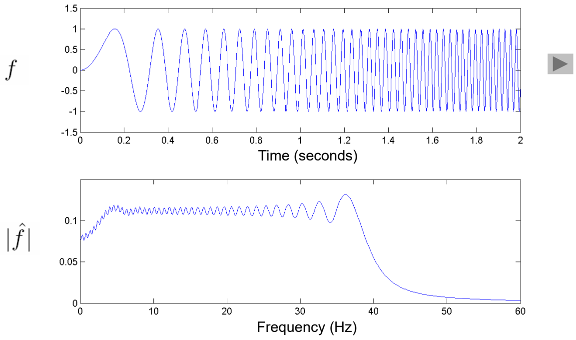
Fourier Transform

Example: C-major scale (piano)



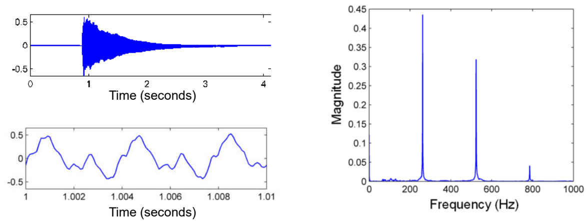
Fourier Transform

Example: Chirp signal



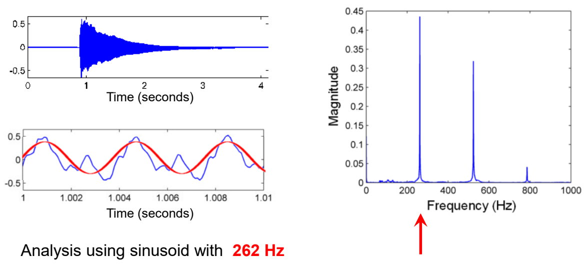
Fourier Transform

Example: Piano tone (C4, 261.6 Hz)



Fourier Transform

Example: Piano tone (C4, 261.6 Hz)



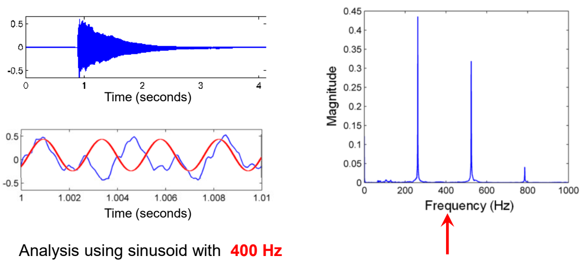
Analysis using sinusoid with **262 Hz**

→ high correlation

→ large Fourier coefficient

Fourier Transform

Example: Piano tone (C4, 261.6 Hz)



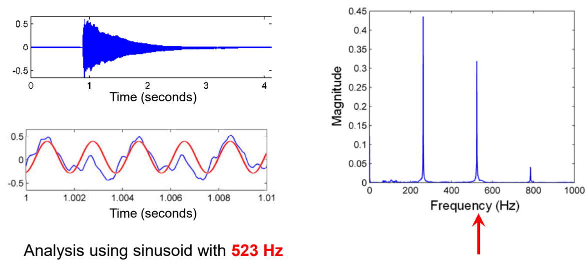
Analysis using sinusoid with **400 Hz**

→ low correlation

→ small Fourier coefficient

Fourier Transform

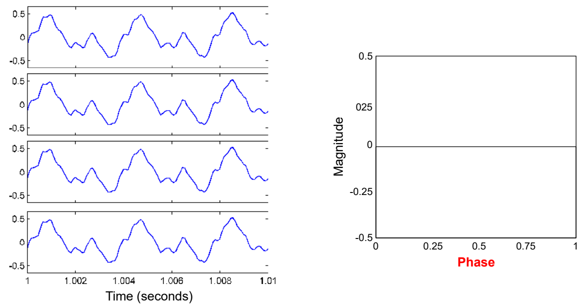
Example: Piano tone (C4, 261.6 Hz)



Analysis using sinusoid with **523 Hz**
→ high correlation
→ large Fourier coefficient

Fourier Transform

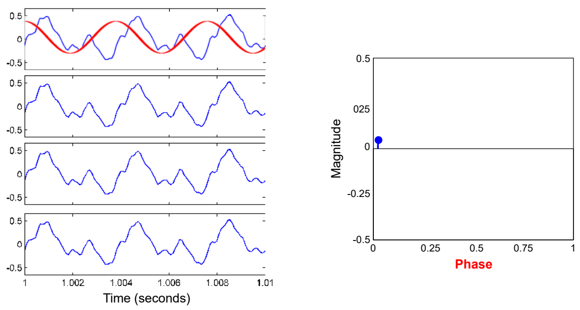
Role of phase



Fourier Transform

Role of phase

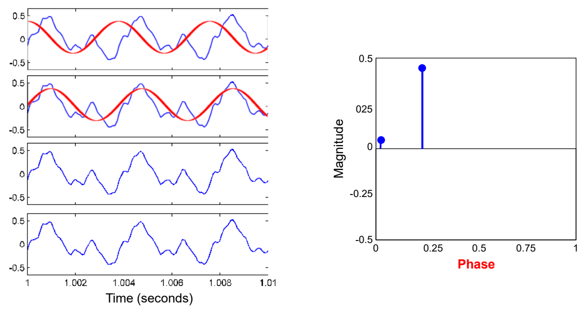
Analysis with sinusoid having frequency 262 Hz and phase **φ = 0.05**



Fourier Transform

Role of phase

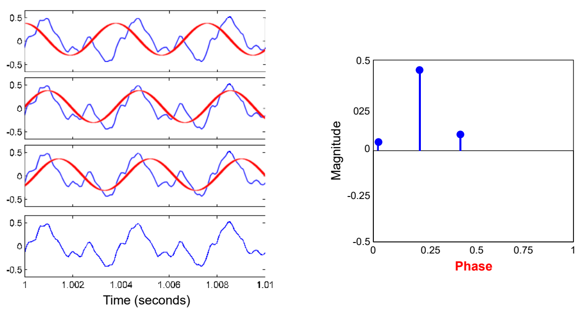
Analysis with sinusoid having frequency 262 Hz and phase **φ = 0.24**



Fourier Transform

Role of phase

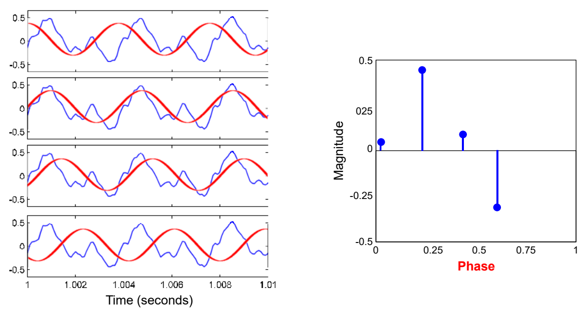
Analysis with sinusoid having frequency 262 Hz and phase **φ = 0.45**



Fourier Transform

Role of phase

Analysis with sinusoid having frequency 262 Hz and phase **φ = 0.6**



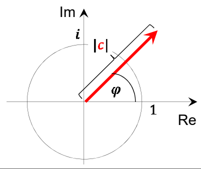
Fourier Transform

Each sinusoid has a physical meaning and can be described by three parameters:

$$\boxed{s_{(A,\omega,\varphi)}(t) = A \cdot \sin(2\pi(\omega t - \varphi))}$$

ω = frequency
A = amplitude
φ = phase

Polar coordinates:
$c = |c| \cdot \exp(2\pi i \varphi)$



Complex formulation of sinusoids:

$$\boxed{e_{(c,\omega)}(t) = c \cdot \exp(2\pi i \omega t) = c \cdot (\cos(2\pi\omega t) + i \cdot \sin(2\pi\omega t))}$$

ω = frequency
A = amplitude = $|c|$
φ = phase = $\arg(c)$

Fourier Transform

Signal $\quad f : \mathbb{R} \to \mathbb{R}$

Fourier representation $\quad f(t) = \int_{\omega \in \mathbb{R}} c_\omega \exp(2\pi i \omega t) d\omega$

Fourier transform $\quad c_\omega = \hat{f}(\omega) = \int_{t \in \mathbb{R}} f(t) \exp(-2\pi i \omega t) dt$

Fourier Transform

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Fourier representation $\quad f(t) = \int_{\omega \in \mathbb{R}} c_\omega \exp(2\pi i \omega t) d\omega$

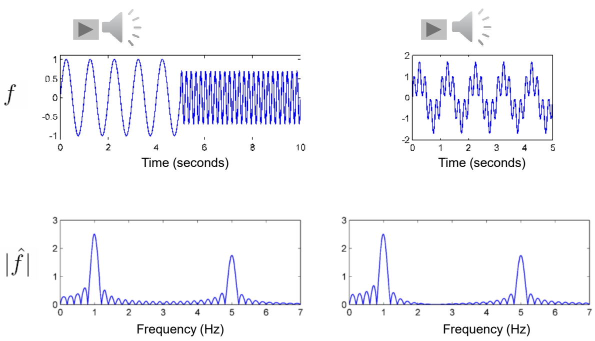
Fourier transform $\quad c_\omega = \hat{f}(\omega) = \int_{t \in \mathbb{R}} f(t) \exp(-2\pi i \omega t) dt$

- Tells which frequencies occur, but does not tell when the frequencies occur.
- Frequency information is averaged over the entire time interval.
- Time information is hidden in the phase

Audio Processing Basics
Overview

- Fourier Transform: Motivation & Definition
- Short-Time Fourier Transform and Spectrograms
- Audio Features and Chromagrams

Fourier Transform



Short-Time Fourier Transform

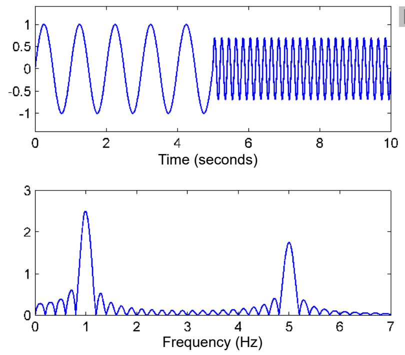
Idea (Dennis Gabor, 1946):

- Consider only a small section of the signal for the spectral analysis

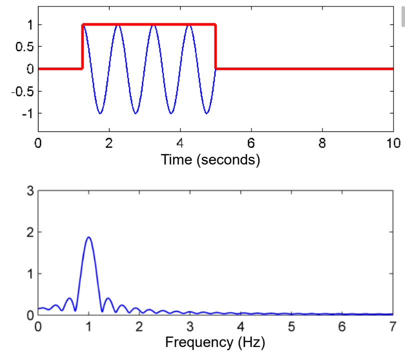
  → recovery of time information

- Short-Time Fourier Transform (STFT)
- Section is determined by pointwise multiplication of the signal with a localizing window function

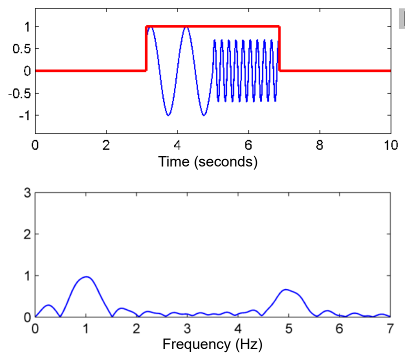
Short-Time Fourier Transform



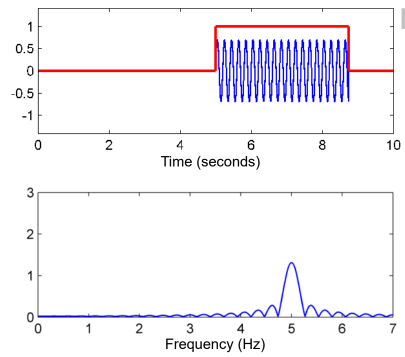
Short-Time Fourier Transform



Short-Time Fourier Transform



Short-Time Fourier Transform



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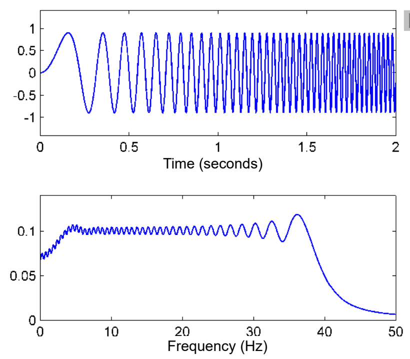
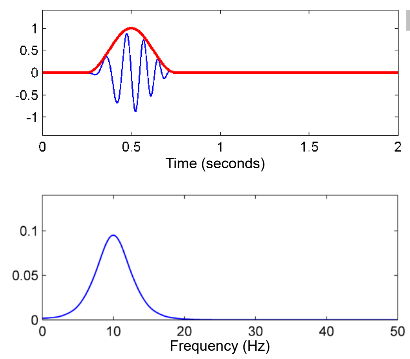
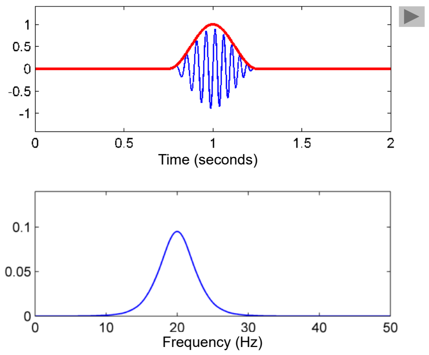


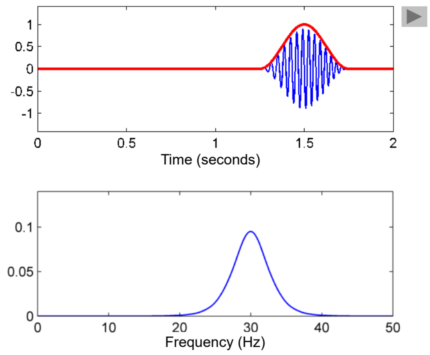
Short-Time Fourier Transform



Short-Time Fourier Transform



Short-Time Fourier Transform



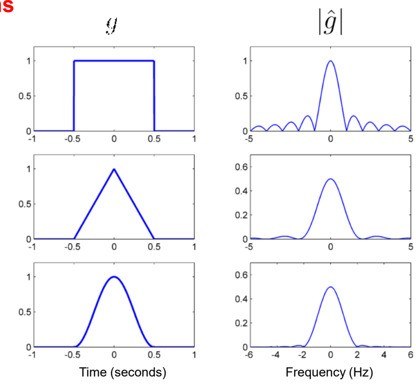
Short-Time Fourier Transform

Window functions

Rectangular window

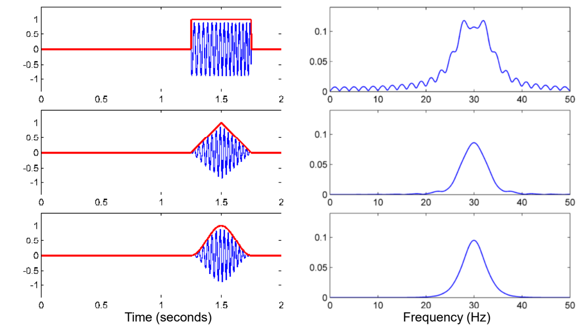
Triangular window

Hann window



Short-Time Fourier Transform

Window functions



→ Trade off between smoothing and “ringing”

Short-Time Fourier Transform

Definition

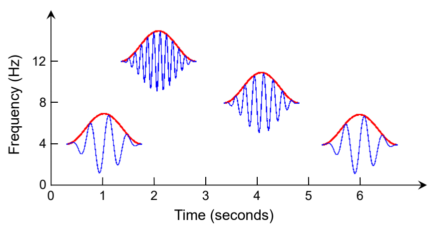
- Signal $f : \mathbb{R} \to \mathbb{R}$
- Window function $g : \mathbb{R} \to \mathbb{R}$ $\quad (g \in L^2(\mathbb{R}), \|g\|_2 \neq 0)$
- STFT $\tilde{f}_g(t,\omega) = \int_{u\in\mathbb{R}} f(u)\overline{g}(u-t)\exp(-2\pi i\omega u)du = \langle f | g_{t,\omega} \rangle$

  with $g_{t,\omega}(u) = \exp(2\pi i\omega(u-t))g(u-t)$ for $u \in \mathbb{R}$

Short-Time Fourier Transform

Intuition:

- $g_{t,\omega}$ is “musical note” of frequency ω centered at time t
- Inner product $\langle f | g_{t,\omega} \rangle$ measures the correlation between the musical note $g_{t,\omega}$ and the signal f



Short-Time Fourier Transform

Discrete STFT

$$\mathcal{X}(m,k) := \sum_{n=0}^{N-1} x(n+mH)w(n)\exp(-2\pi ikn/N)$$

$x:\mathbb{Z}\to\mathbb{R}$	DT-signal
$w:[0:N-1]\to\mathbb{R}$	Window function of length $N\in\mathbb{N}$
$H\in\mathbb{N}$	Hop size
$K=N/2$	Index corresponding to Nyquist frequency
$\mathcal{X}(m,k)$	Fourier coefficient for frequency index $k\in[0:K]$ and time frame $m\in\mathbb{Z}$

Short-Time Fourier Transform

Discrete STFT

$$\mathcal{X}(m,k) := \sum_{n=0}^{N-1} x(n+mH)w(n)\exp(-2\pi ikn/N)$$

Physical time position associated with $\mathcal{X}(m,k)$:

$$T_{\mathrm{coef}}(m) := \frac{m\cdot H}{F_s} \quad \text{(seconds)}$$

H = Hop size

F_s = Sampling rate

Physical frequency associated with $\mathcal{X}(m,k)$:

$$F_{\mathrm{coef}}(k) := \frac{k\cdot F_s}{N} \quad \text{(Hertz)}$$

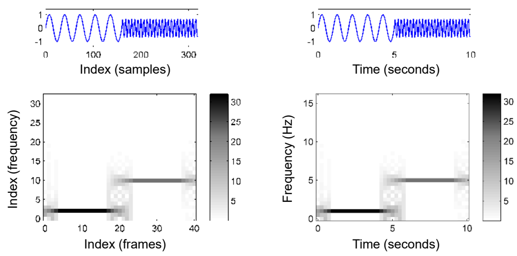
Short-Time Fourier Transform

Discrete STFT

Parameters
N = 64
H = 8
F_s = 32 Hz

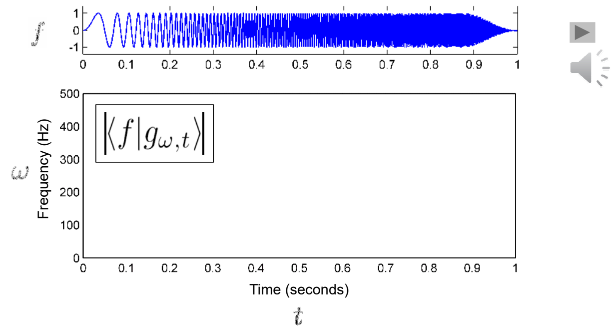
Computational world

Physical world



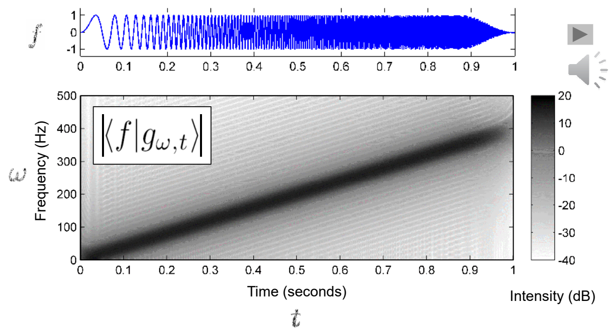
Time–Frequency Representation

Spectrogram



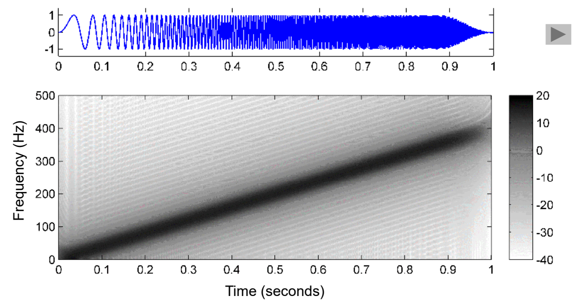
Time–Frequency Representation

Spectrogram



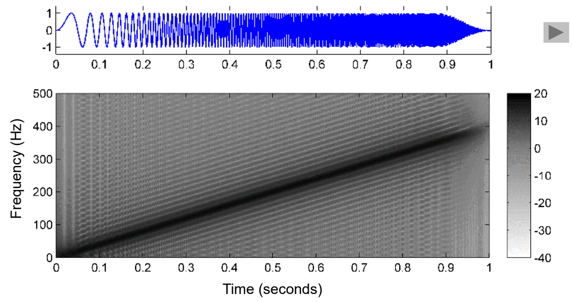
Time–Frequency Representation

Chirp signal and STFT with Hann window of length 50 ms



Time–Frequency Representation

Chirp signal and STFT with box window of length 50 ms



Time–Frequency Representation

Time–Frequency Localization

- Size of window constitutes a trade-off between time resolution and frequency resolution:

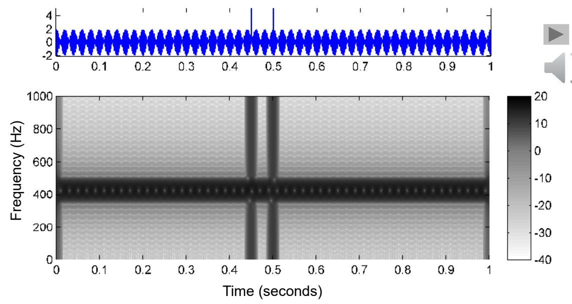
  Large window : poor time resolution
  good frequency resolution

  Small window : good time resolution
  poor frequency resolution

- Heisenberg Uncertainty Principle: there is no window function that localizes in time and frequency with arbitrary precision.

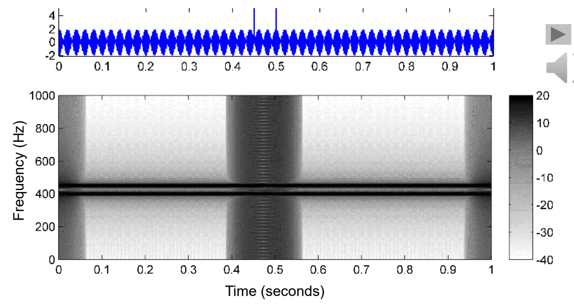
Time–Frequency Representation

Signal and STFT with Hann window of length 20 ms



Time–Frequency Representation

Signal and STFT with Hann window of length 100 ms



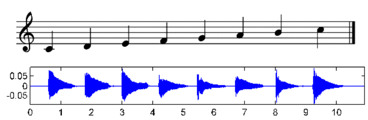
Audio Processing Basics

Overview

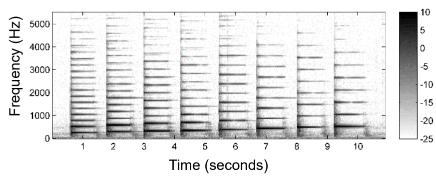
- Fourier Transform: Motivation & Definition
- Short-Time Fourier Transform and Spectrograms
- Audio Features and Chromagrams

Audio Features

Example: C-major scale (piano)

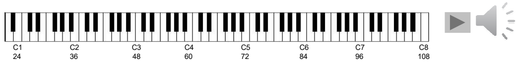


Spectrogram

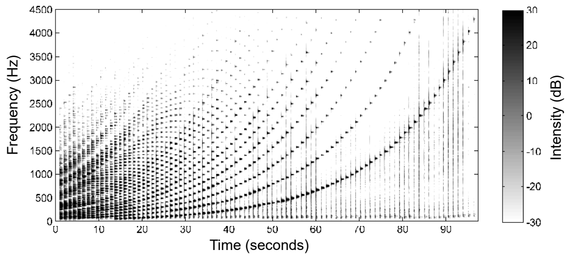


Audio Features

Example: Chromatic scale

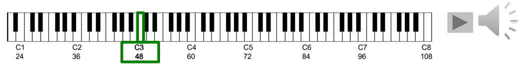


Spectrogram

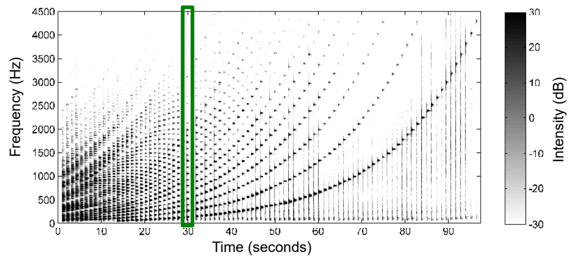


Audio Features

Example: Chromatic scale



Spectrogram



Audio Features

Model assumption: Equal-tempered scale

- MIDI pitches: $p \in [1:128]$
- Piano notes: p = 21 (A0) to p = 108 (C8)
- Concert pitch: p = 69 (A4) ≙ 440 Hz
- Center frequency: $F_{\mathrm{pitch}}(p) = 2^{(p-69)/12} \cdot 440$ Hz

→ Logarithmic frequency distribution
Octave: doubling of frequency

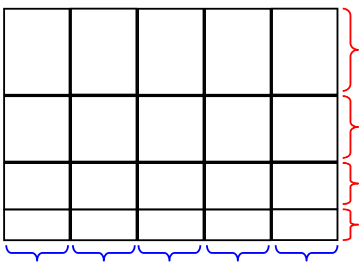
Audio Features

Idea: Binning of Fourier coefficients

Divide up the frequency axis into logarithmically spaced "pitch regions" and combine spectral coefficients of each region to a single pitch coefficient.

Audio Features

Time-frequency representation



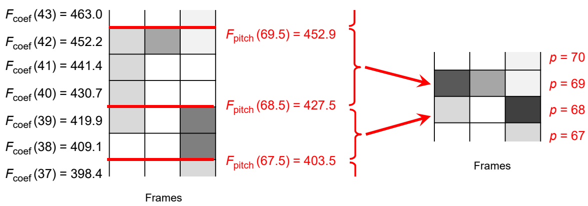
Windowing in the frequency domain

Windowing in the time domain

Log-Frequency Spectrogram

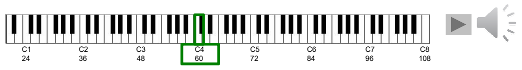
Pooling procedure for discrete STFT

Parameters
N = 4096
H = 2048
F_s = 44100 Hz

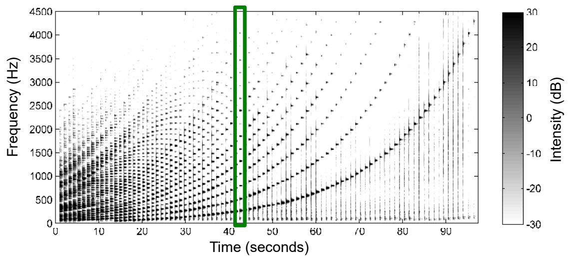


Audio Features

Example: Chromatic scale



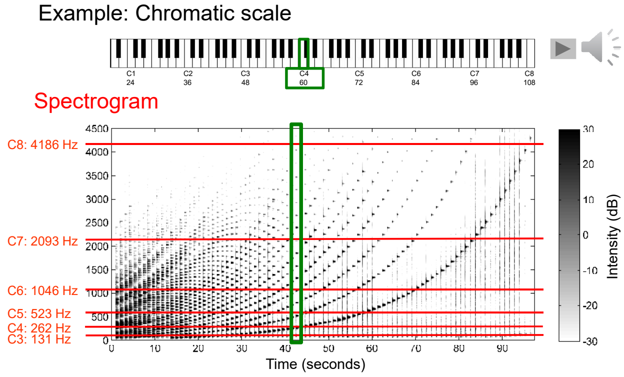
Spectrogram



Audio Features

Example: Chromatic scale

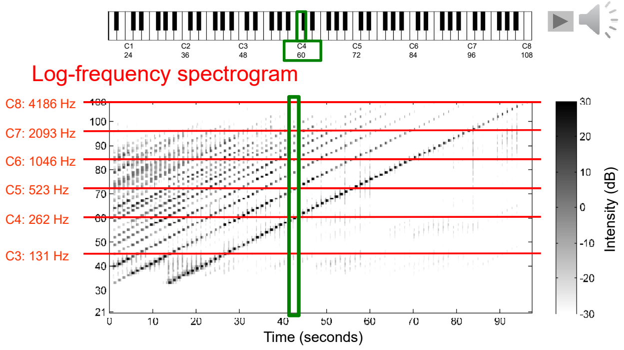
Spectrogram



Audio Features

Example: Chromatic scale

Log-frequency spectrogram



Audio Features

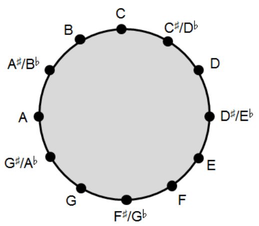
Frequency ranges for pitch-based log-frequency spectrogram

Note	MIDI pitch p	Center [Hz] frequency $F_{\mathrm{pitch}}(p)$	Left [Hz] boundary $F_{\mathrm{pitch}}(p-0.5)$	Right [Hz] boundary $F_{\mathrm{pitch}}(p+0.5)$	Width [Hz]
A3	57	220.0	213.7	226.4	12.7
A#3	58	233.1	226.4	239.9	13.5
B3	59	246.9	239.9	254.2	14.3
C4	60	261.6	254.2	269.3	15.1
C#4	61	277.2	269.3	285.3	16.0
D4	62	293.7	285.3	302.3	17.0
D#4	63	311.1	302.3	320.2	18.0
E4	64	329.6	320.2	339.3	19.0
F4	65	349.2	339.3	359.5	20.2
F#4	66	370.0	359.5	380.8	21.4
G4	67	392.0	380.8	403.5	22.6
G#4	68	415.3	403.5	427.5	24.0
A4	69	440.0	427.5	452.9	25.4

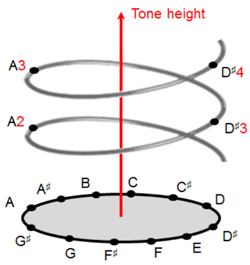
Audio Features

Chroma features

Chromatic circle



Shepard's helix of pitch



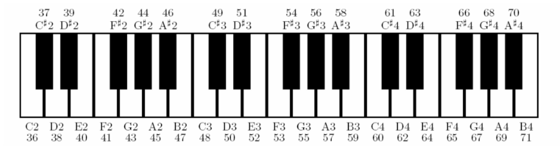
Audio Features

Chroma features

- Human perception of pitch is periodic in the sense that two pitches are perceived as similar in color if they differ by an octave (same pitch class).
- Separation of pitch into two components: tone height (octave number) and chroma / pitch class.
- Chroma : 12 pitch classes of the equal-tempered scale. For example:
  Chroma C $\widehat{=} \{\ldots, \mathrm{C0}, \mathrm{C1}, \mathrm{C2}, \mathrm{C3}, \ldots\}$
- Computation: pitch features → chroma features
  Add up all pitches belonging to the same pitch class
- Result: 12-dimensional chroma vector.

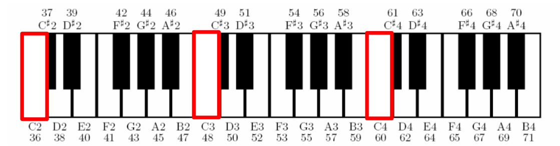
Audio Features

Chroma features



Audio Features

Chroma features

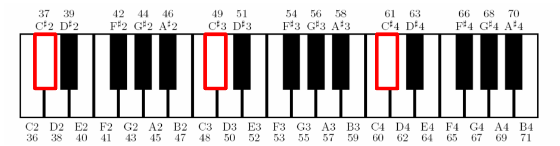


C2 C3 C4

Chroma C

Audio Features

Chroma features

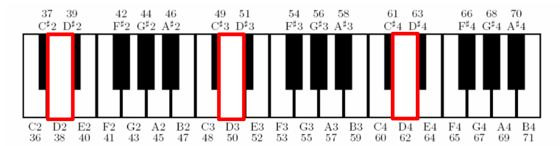


C#2 C#3 C#4

Chroma C#

Audio Features

Chroma features

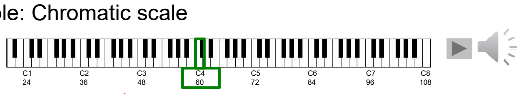


D2 D3 D4

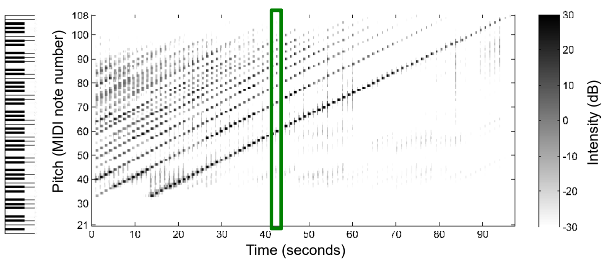
Chroma D

Audio Features

Example: Chromatic scale

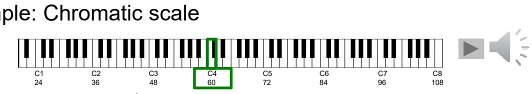


Log-frequency spectrogram

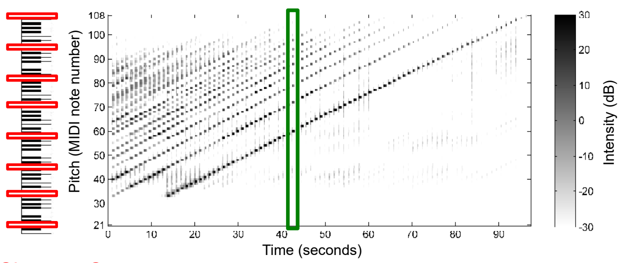


Audio Features

Example: Chromatic scale



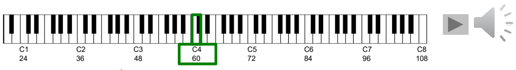
Log-frequency spectrogram



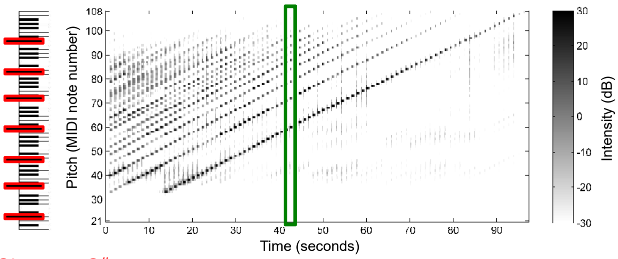
Chroma C

Audio Features

Example: Chromatic scale



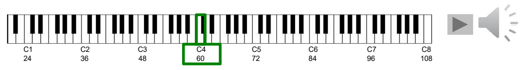
Log-frequency spectrogram



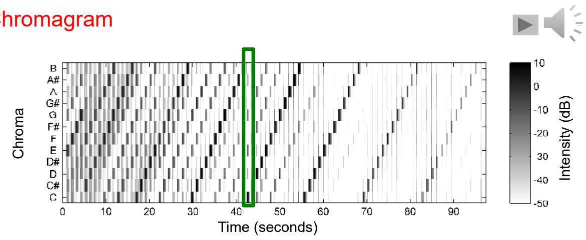
Chroma $C^{\#}$

Audio Features

Example: Chromatic scale

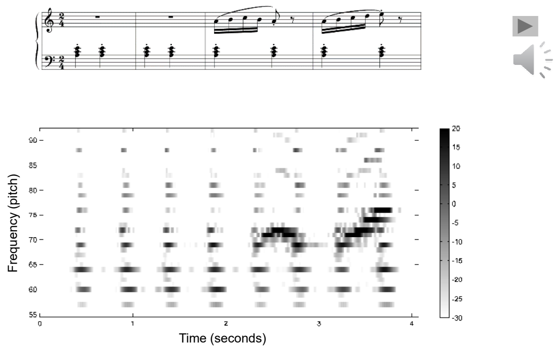


Chromagram



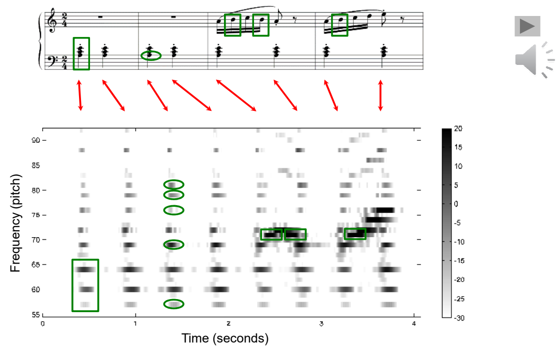
Audio Features

Chroma features



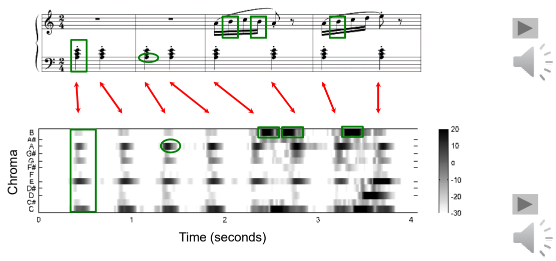
Audio Features

Chroma features



Audio Features

Chroma features



Audio Features

Chroma features

- Sequence of chroma vectors correlates to the harmonic progression
- Normalization $x \rightarrow x/\|x\|$ makes features invariant to changes in dynamics
- Further denoising and smoothing
- Taking logarithm before adding up pitch coefficients accounts for logarithmic sensation of intensity

Audio Features

Logarithmic compression

For a positive constant $\gamma \in \mathbb{R}_{>0}$
the **logarithmic compression**

$$\Gamma_\gamma : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$$

is defined by

$$\Gamma_\gamma(v) := \log(1 + \gamma \cdot v)$$

A value $v \in \mathbb{R}_{>0}$ is replaced
by a compressed value $\Gamma_\gamma(v)$

Audio Features

Logarithmic compression

For a positive constant $\gamma \in \mathbb{R}_{>0}$
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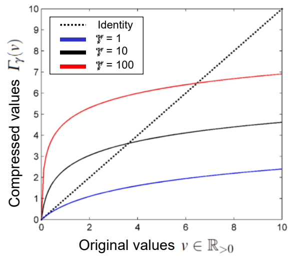
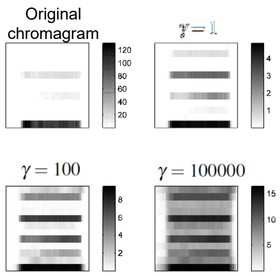


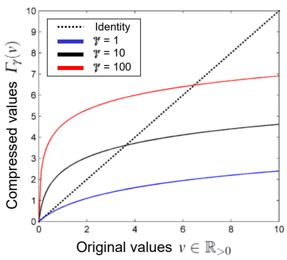
A value $v \in \mathbb{R}_{>0}$ is replaced
by a compressed value $\Gamma_\gamma(v)$

The higher $\gamma \in \mathbb{R}_{>0}$
the stronger the compression

Audio Features

Logarithmic compression





A value $v \in \mathbb{R}_{>0}$ is replaced
by a compressed value $\Gamma_\gamma(v)$

The higher $\gamma \in \mathbb{R}_{>0}$
the stronger the compression

Audio Features

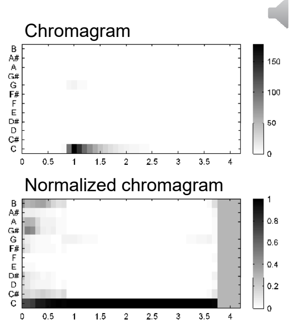
Normalization

Example: C4 played by piano

Replace a vector
by the normalized vector

$$x / \|x\|$$

using a suitable norm $\|\cdot\|$



Example:
Chroma vector $x \in \mathbb{R}^{12}$
Euclidean norm

$$\|x\| := \left(\sum_{i=0}^{11} |x(i)|^2 \right)^{1/2}$$

Audio Features

Normalization

Example: C4 played by piano

Replace a vector
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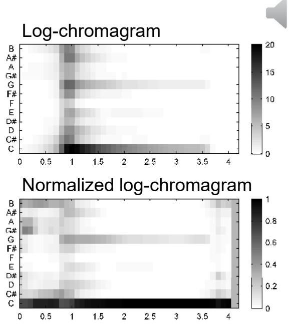
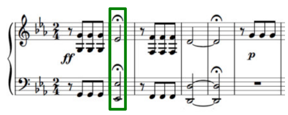


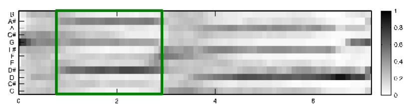
Example:
Chroma vector $x \in \mathbb{R}^{12}$
Euclidean norm

$$\|x\| := \left(\sum_{i=0}^{11} |x(i)|^2 \right)^{1/2}$$

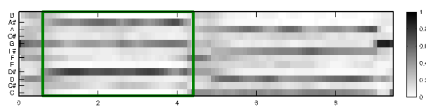
Audio Features

Chroma features (normalized)





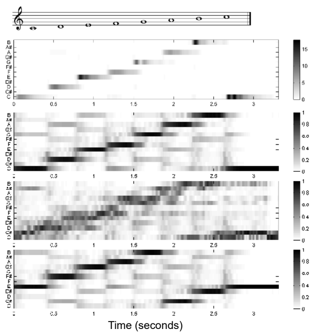
Karajan



Scherbakov

Audio Features

Chroma features



Chromagram

Chromagram after logarithmic compression and normalization

Chromagram based on a piano tuned 40 cents upwards

Chromagram after applying a cyclic shift of four semitones upwards