



Nonnegative Autoencoders with Applications to Music Audio Decomposing

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Guest Lecture Sound and Music Computing (CS4347/CS5647) National University of Singapore 16.10.2023





Meinard Müller

- Mathematics (Diplom/Master, 1997) Computer Science (PhD, 2001) Information Retrieval (Habilitation, 2007)
- Senior Researcher (2007-2012)
- Professor Semantic Audio Processing (since 2012)
- Former President of the International Society for Music Information Retrieval (MIR)
- IEEE Fellow for contributions to Music Signal Processing











PIEEE

Nonnegative Autoencoders with Applications to Music Audio Decomposing



Meinard Müller: Research Group Semantic Audio Processing

- Michael Krause
- Yigitcan Özer
- Simon Schwär
- Johannes Zeitler
- Peter Meier (external)
- Christof Weiß
- Sebastian Rosenzweig
- Frank Zalkow
- Christian Dittmar
- Stefan Balke
- Jonathan Driedger Thomas Prätzlich



















International Audio Laboratories Erlangen



- Fraunhofer Institute for Integrated Circuits IIS
- Largest Fraunhofer institute with ≈ 1000 members
- Applied research for sensor, audio, and media technology







- Friedrich-Alexander Universität Erlangen-Nürnberg (FAU)
- One of Germany's largest universities with ≈ 40,000 students
- Strong Technical Faculty



3D Audio

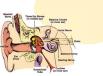
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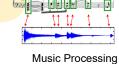
Audio Coding













Internet of Things

Psychoacoustics



AudioLabs – FAU

- Prof. Dr. Jürgen Herre Audio Coding
- Prof. Dr. Bernd Edler
- Prof. Dr. Meinard Müller Semantic Audio Processing
- Prof. Dr. Emanuël Habets Spatial Audio Signal Processing
- Prof. Dr. Nils Peters
- Dr. Stefan Turowski Coordinator AudioLabs-FAU









negative Autoencoders with ications to Music Audio Decomposing

LABS

Source Separation

- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- "Cocktail party effect"





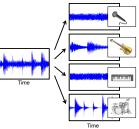
Source Separation

- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- "Cocktail party effect"
- Several input signals
- Sources are assumed to be statistically independent



Source Separation (Music)

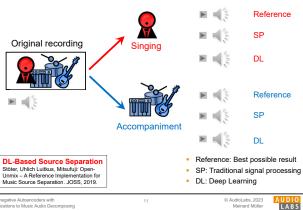
- Main melody, accompaniment, drum track
- Instrumental voices
- Individual note events
- Only mono or stereo
- Sources are often highly dependent



Prior Knowledge
Ewert, Pardo, Müller, Plumbley:
Score-Informed Source Separatio
for Musical Audio Recordings.
IEEE SPM 31(3), 2014.



Source Separation (Singing Voice)



Score-Informed Source Separation

Exploit musical score to support decomposition process

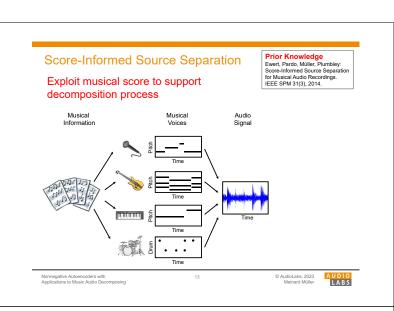
Musical

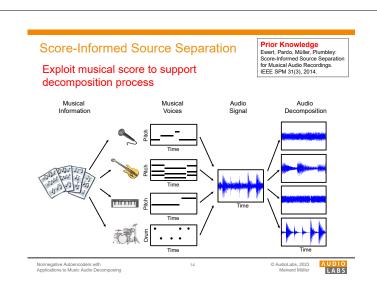
Audio Signal











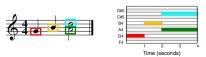
Score-Informed Audio Decomposition







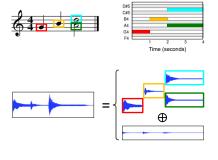
Score-Informed Audio Decomposition







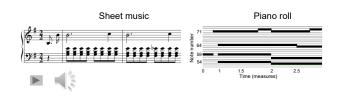
Score-Informed Audio Decomposition



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Score-Informed Audio Decomposition

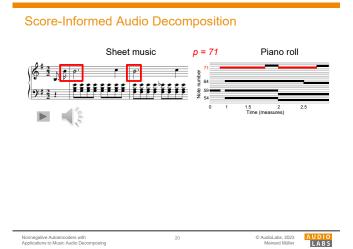


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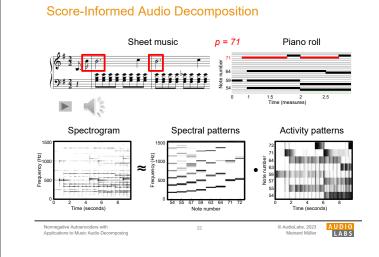


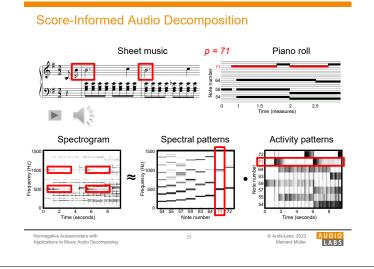
Sheet music p = 59 Piano roll

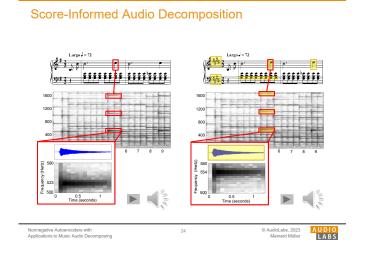
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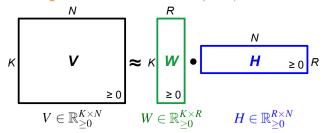
Score-Informed Audio Decomposition Sheet music p = 71 Piano roll Piano rol





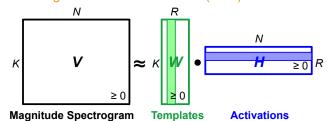


Nonnegative Matrix Factorization (NMF)





Nonnegative Matrix Factorization (NMF)



Templates:

Pitch + Timbre

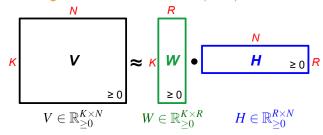
"How does it sound"

Activations: Onset time + Duration "When does it sound"



AUDIO

Nonnegative Matrix Factorization (NMF)

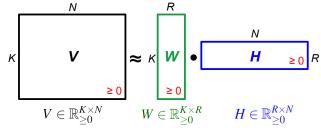


Dimensionality reduction

- K, N typically much larger than R (maximal rank)
- Example: N = 1000, K = 500, R = 20 $K \times N = 500,000, K \times R = 10,000,$ $R \times N = 20,000$



Nonnegative Matrix Factorization (NMF)



Nonnegativity:

- Prevents mutual cancellation of template vectors
- Encourages semantically meaningful decomposition



NMF Optimization

Optimization problem:

Given $V \in \mathbb{R}_{\geq 0}^{K imes N}$ and rank parameter \emph{R} minimize

$$||V - WH||^2$$

with respect to $\ W \in \mathbb{R}_{\geq 0}^{K imes R} \ \ \ \ \ M \in \mathbb{R}_{\geq 0}^{R imes N} \,.$

Optimization not easy:

- Nonnegativity constraints
- Nonconvexity when jointly optimizing W and H

Strategy: Iteratively optimize W and H via gradient descent

NMF Optimization

Computation of gradient with respect to H (fixed W)

D := RN

 $oldsymbol{arphi}^W:\mathbb{R}^D o\mathbb{R}$

 $\varphi^W(H) := \|V - WH\|^2$

Variables

 $H \in \mathbb{R}^{R \times N}$

 $\rho \in [1:R]$

 $v \in [1:N]$



NMF Optimization

Computation of gradient with respect to *H* (fixed *W*)

$$\begin{split} & D := RN \\ & \phi^W : \mathbb{R}^D \to \mathbb{R} \\ & \phi^W(H) := \|V - WH\|^2 \end{split} \qquad \frac{\partial \phi^W}{\partial H_{\rho \nu}} = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{rn} \right)^2 \right)}{\partial H_{\rho \nu}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$
 $H_{\rho v}$
 $\rho \in [1:R]$

 ν ∈ [1 : *N*]

Nonnegative Autoencoders with Applications to Music Audio Decomposing



NMF Optimization

Computation of gradient with respect to *H* (fixed *W*)

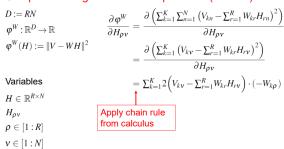
$$\begin{array}{ll} D := RN & \frac{\partial \varphi^W}{\varphi^W : \mathbb{R}^D \to \mathbb{R}} & \frac{\partial \varphi^W}{\partial H_{\rho \nu}} = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{rn}\right)^2\right)}{\partial H_{\rho \nu}} \\ \varphi^W(H) := \|V - WH\|^2 & = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Variables} & = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Summand that does not depend on } H_{\rho \nu} \\ H_{\rho \nu} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{n=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{n=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{n=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{n=1}^K W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^K W_{kr} H_{r\nu}\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^K W_{kr} H_{r\nu}\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^K W_{kr} H_{r\nu}\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} \\ \text{Must be zero} & = \frac{\partial \left(\sum_{k=1}^K W_{kr} H_{r\nu}\right)}{\partial H_{\rho \nu}} \\ \text{Must be zero} \\ \text{Must be$$

 ν ∈ [1 : *N*]



NMF Optimization

Computation of gradient with respect to *H* (fixed *W*)





NMF Optimization

Computation of gradient with respect to *H* (fixed *W*)

$$\begin{array}{ll} D := RN \\ \phi^W : \mathbb{R}^D \to \mathbb{R} \\ \phi^W(H) := \|V - WH\|^2 \\ \end{array} \qquad \begin{array}{ll} \frac{\partial \phi^W}{\partial H_{\rho \nu}} = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{rn}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ \end{array} \\ \text{Variables} \\ H \in \mathbb{R}^{R \times N} \\ H_{\rho \nu} \\ \rho \in [1:R] \\ \nu \in [1:N] \end{array} \qquad \begin{array}{ll} \frac{\partial \phi^W}{\partial H_{\rho \nu}} = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{\nu\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu} H_{r\nu}\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{k\nu}\right)}{\partial H_{\rho \nu}} \\ = \frac{\partial \left(\sum_{k=$$



NMF Optimization

Computation of gradient with respect to *H* (fixed *W*)

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \to \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$= \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{rn}\right)^2\right)}{\partial H_{\rho \nu}}$$

$$= \frac{\partial \left(\sum_{k=1}^K \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right)^2\right)}{\partial H_{\rho \nu}}$$

$$= \sum_{k=1}^K 2 \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right) \cdot \left(-W_{k\rho}\right)$$

$$= \sum_{k=1}^K 2 \left(V_{k\nu} - \sum_{r=1}^R W_{kr} H_{r\nu}\right) \cdot \left(-W_{k\rho}\right)$$

$$= 2 \left(\sum_{r=1}^R \sum_{k=1}^K W_{k\rho} W_{kr} H_{r\nu} - \sum_{k=1}^K W_{k\rho} V_{k\nu}\right)$$

$$= 2 \left(\sum_{r=1}^R \left(\sum_{k=1}^K W_{k\rho} W_{kr}\right) H_{r\nu} - \sum_{k=1}^K W_{\rho k} V_{k\nu}\right)$$
Introduce transposed W^\top

NMF Optimization

Computation of gradient with respect to H (fixed W)

$$\begin{split} D &:= RN \\ \phi^W : \mathbb{R}^D \to \mathbb{R} \\ \phi^W(H) &:= \|V - WH\|^2 \\ \text{Variables} \\ H &\in \mathbb{R}^{R \times N} \\ H_{\rho v} \\ \rho &\in [1:R] \\ v &\in [1:N] \end{split} \qquad \begin{aligned} & \frac{\partial \phi^W}{\partial H_{\rho v}} &= \frac{\partial \left(\sum_{k=1}^K \sum_{n=1}^N \left(V_{kn} - \sum_{r=1}^R W_{kr} H_{rn}\right)^2\right)}{\partial H_{\rho v}} \\ &= \frac{\partial \left(\sum_{k=1}^K \left(V_{kv} - \sum_{r=1}^R W_{kr} H_{rv}\right)^2\right)}{\partial H_{\rho v}} \\ &= \sum_{k=1}^K 2 \left(V_{kv} - \sum_{r=1}^R W_{kr} H_{rv}\right) \cdot \left(-W_{k\rho}\right) \\ &= 2 \left(\sum_{r=1}^R \sum_{k=1}^K W_{k\rho} W_{kr} H_{rv} - \sum_{k=1}^K W_{k\rho} V_{kv}\right) \\ &= 2 \left(\sum_{r=1}^R \left(\sum_{k=1}^K W_{\rho k} W_{kr}\right) H_{rv} - \sum_{k=1}^K W_{\rho k} V_{kv}\right) \\ &= 2 \left(\left(W^\top W H\right)_{\rho v} - \left(W^\top V\right)_{\rho v}\right). \end{aligned}$$

Nonnegative Autoencoders with Applications to Music Audio Decomposing



NMF Optimization

Gradient descent

Initialization $H^{(0)} \in \mathbb{R}^{R \times N}$ Iteration for $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left(\left(W^ op W H^{(\ell)}
ight)_{rn} - \left(W^ op V
ight)_{rn}
ight)$$

with suitable learning rate $\gamma_{rn}^{(\ell)} \geq 0$



NMF Optimization

Gradient descent

Initialization $H^{(0)} \in \mathbb{R}^{R \times N}$ Iteration for $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left(\left(W^\top W H^{(\ell)} \right)_{rn} - \left(W^\top V \right)_{rn} \right)$$

with suitable learning rate $\gamma_m^{(\ell)} \geq 0$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

Nonnegative Autoencoders with Applications to Music Audio Decomposing



NMF Optimization Gradient descent

Initialization $H^{(0)} \in \mathbb{R}^{R \times N}$ Iteration for $\ell = 0, 1, 2, \dots$ Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := rac{H_{rn}^{(\ell)}}{\left(W^ op W H^{(\ell)}
ight)_{rn}}$$

$$\begin{split} H_{rn}^{(\ell+1)} &= H_{rn}^{(\ell)} - \underbrace{\begin{pmatrix} \chi^{(\ell)}_{rn} \end{pmatrix} \left(\begin{pmatrix} W^\top W H^{(\ell)} \end{pmatrix}_{rn} - \begin{pmatrix} W^\top V \end{pmatrix}_{rn} \right)}_{rn} - \begin{pmatrix} W^\top V \end{pmatrix}_{rn} \\ &= H_{rn}^{(\ell)} \cdot \frac{\begin{pmatrix} W^\top V \end{pmatrix}_{rn}}{\begin{pmatrix} W^\top W H^{(\ell)} \end{pmatrix}_{rn}} \end{split}$$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?



NMF Optimization Gradient descent

Initialization $H^{(0)} \in \mathbb{R}^{R \times N}$ Iteration for $\ell = 0, 1, 2, \dots$ $\gamma_{rn}^{(\ell)} := rac{1}{\left(W^ op W H^{(\ell)}
ight)_{rn}}$

Choose adaptive

learning rate:

$$\begin{aligned} H_{rn}^{(\ell+1)} &= H_{rn}^{(\ell)} - \overbrace{\mathbf{v}_{rn}^{(\ell)}} \left(\left(W^{\top} W H^{(\ell)} \right)_{rn} - \left(W^{\top} V \right)_{rn} \right) \\ &= H_{rn}^{(\ell)} \cdot \frac{\left(W^{\top} V \right)_{rn}}{\left(W^{\top} W H^{(\ell)} \right)} \end{aligned}$$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?
- Update rule become multiplicative
- Nonnegative values stav nonnegative

LABS

NMF Optimization

NMF Algorithm

Algorithm: NMF $(V \approx WH)$

Nonnegative matrix V of size $K \times N$ Input:

Threshold ε used as stop criterion

Output: Nonnegative template matrix W of size $K \times R$ Nonnegative activation matrix H of size $R \times N$

Procedure: Define nonnegative matrices $W^{(0)}$ and $H^{(0)}$ by some random or informed initialization. Furthermore set $\ell=0$. Apply the following update rules (written in matrix notation):

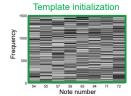
$$(1) \quad H^{(\ell+1)} = H^{(\ell)} \odot \left(((W^{(\ell)})^\top V) \oslash ((W^{(\ell)})^\top W^{(\ell)} H^{(\ell)}) \right)$$

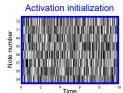
$$(2) \quad W^{(\ell+1)} = W^{(\ell)} \odot \left((V(H^{(\ell+1)})^\top) \oslash (W^{(\ell)}H^{(\ell+1)}(H^{(\ell+1)})^\top) \right)$$

(3) Increase ℓ by one.

Repeat the steps (1) to (3) until $\|H^{(\ell)}-H^{(\ell-1)}\| \le \varepsilon$ and $\|W^{(\ell)}-W^{(\ell-1)}\| \le \varepsilon$ (or until some other stop criterion is fulfilled). Finally, set $H=H^{(\ell)}$ and $W=W^{(\ell)}$.

NMF-based Spectrogram Decomposition

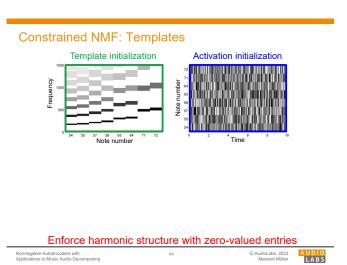


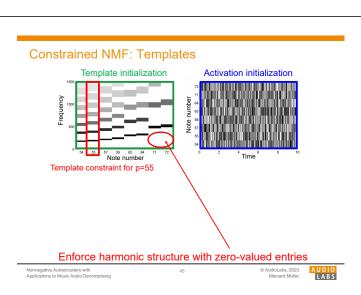


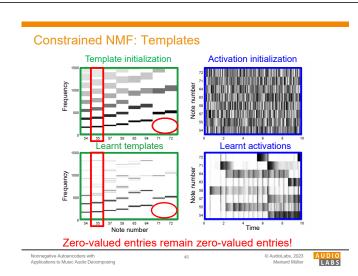
Random initialization

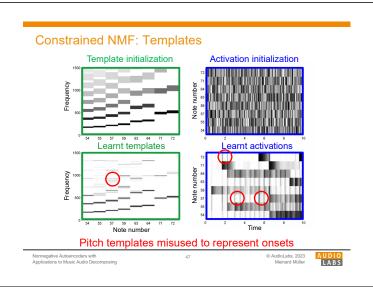


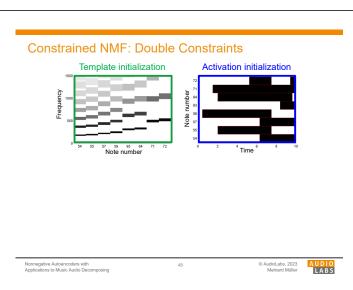
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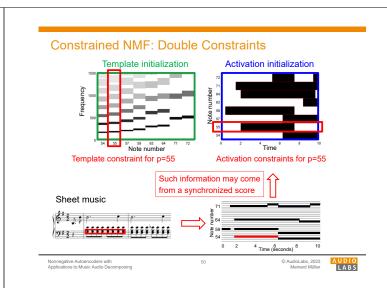


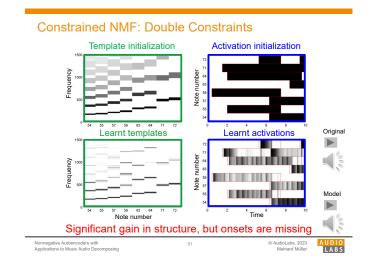


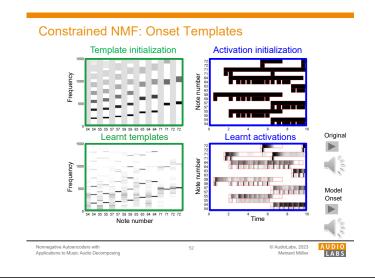


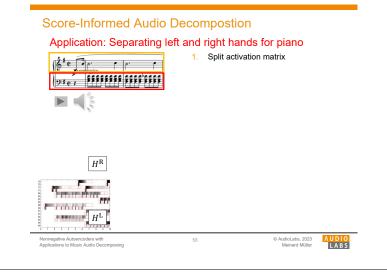


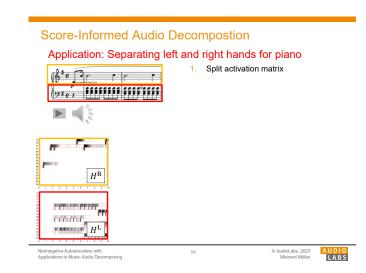
Constrained NMF: Double Constraints Activation initialization Activation initialization Activation constraints for p=55 Activation constraints for p=55







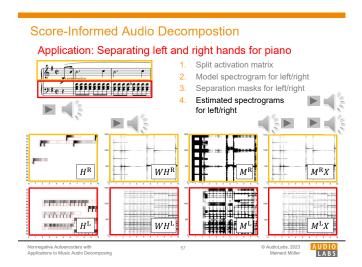


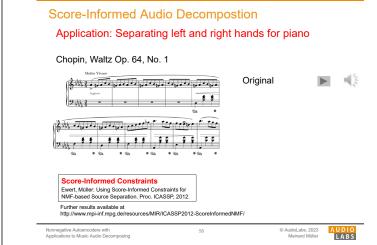


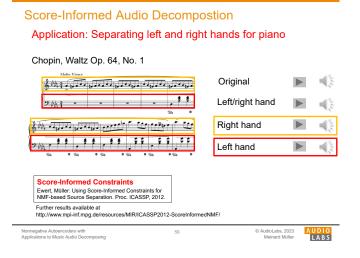
Score-Informed Audio Decompostion Application: Separating left and right hands for piano 1. Split activation matrix 2. Model spectrogram for left/right

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Score-Informed Audio Decompostion Application: Separating left and right hands for piano 1. Split activation matrix 2. Model spectrogram for left/right 3. Separation masks for left/right WHR WHR Applications to Music Audio Decomposition







Conclusions (NMF)

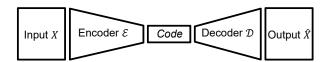
- NMF used for spectrogram decomposition
- Multiplicative update rules make it easy to constrain NMF model via zero initialization
- Exploiting score information to guide separation process (requires score—audio synchronization)
- Application: Separation of arbitrary note groups from given audio recording

Nonnegative Autoencoders with Applications to Music Audio Decomposing

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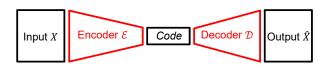
Autoencoder



- Specific type of neural network
- Encoder: Compress input X into a low-dimensional code
- Decoder: Reconstruct output \hat{X} from code

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Autoencoder



- Specific type of neural network
- Encoder: Compress input X into a low-dimensional code
- Decoder: Reconstruct output \widehat{X} from code
- Goal: Learn parameters for encoder and decoder such that output is close to input with respect to some loss function:

$$\mathcal{L}\big(X,\widehat{X}\big)\approx 0$$

NMF

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NMF and Autoencoder (AE)

Nonnegative Autoencoder Smaragdis, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017. NMF

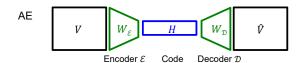
 $V \approx WH$ implies $W^+V \approx H$ with pseudoinverse W^+



NMF and Autoencoder (AE)

Nonnegative Autoencoder Smaragdis, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.

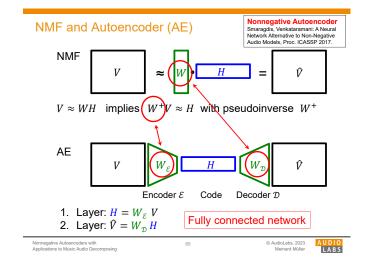
 $V \approx WH$ implies $W^+V \approx H$ with pseudoinverse W^+



1. Layer: $H = W_{\varepsilon} V$

2. Layer: $\hat{V} = W_{\mathcal{D}} H$

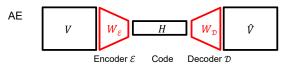






Nonnegative Autoencoder Smaragdis, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017. Ŷ

 $V \approx WH$ implies $W^+V \approx H$ with pseudoinverse W^+



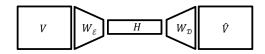
1. Layer: $H = W_{\varepsilon} V$ 2. Layer: $\hat{V} = W_{\mathcal{D}} H$

NMF: Learn H and W AE: Learn W_{ε} and $W_{\mathcal{D}}$

NMF



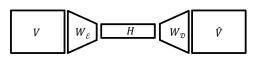
Nonnegative Autoencoder (NAE)



- 1. Layer: $H = W_{\varepsilon} V$ 2. Layer: $\hat{V} = W_{\mathcal{D}} H$
- How can one adjust the AE to simulate NMF?
- How can one achieve nonnegativity?
- How can one incorporate musical knowledge?



Nonnegative Autoencoder (NAE)



1. Layer: $H = W_{\varepsilon} V$ 2. Layer: $\hat{V} = W_{\mathcal{D}} H$

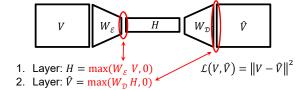
 $\mathcal{L}(V, \widehat{V}) = \|V - \widehat{V}\|^2$

Loss function: same as in NMF

Nonnegative Autoencoders with Applications to Music Audio Decomposing



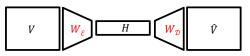
Nonnegative Autoencoder (NAE)



- Loss function: same as in NMF
- Activation function (ReLU) makes H and \hat{V} nonnegative



Nonnegative Autoencoder (NAE)



1. Layer: $H = \max(W_{\varepsilon} V, 0)$

 $\mathcal{L}(V, \widehat{V}) = \|V - \widehat{V}\|^2$

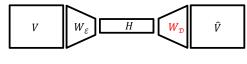
2. Layer: $\hat{V} = \max(W_D H, 0)$

 $W_{\mathcal{D}} \leftarrow \max \left(W_{\mathcal{D}} - \gamma \frac{\partial \mathcal{L}}{\partial W_{\mathcal{D}}}, 0 \right)$

- Loss function: same as in NMF
- Activation function (ReLU) makes H and \hat{V} nonnegative
- Projected gradient descent can be used to keep $W_{\mathcal{D}}$ (and $W_{\mathcal{E}}$) nonnegative



Musical Constraints

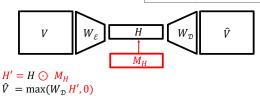


 $H = \max(W_{\varepsilon} V, 0)$ $\widehat{V} = \max(W_{\mathcal{D}} H, 0)$

• Template constraints: Project certain entries in $W_{\mathcal{D}}$ to zero values (using projected gradient decent)

Musical Constraints

Ewert, Sandler: Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation. Proc. ICASSP, 2017.



- Template constraints: Project certain entries in $W_{\mathcal{D}}$ to zero values (using projected gradient decent)
- Activation constraints: Use structured dropout by applying pointwise multiplication with binary mask M_H

NAE with Multiplicative Update Rules

- Multiplicative update rules in NMF:
 - Preserve nonnegativity
 - Lead to fast convergence
- Question: Can one introduce multiplicative update rules to train network weights for NAE?
- Use in additive gradient descent

$$W^{(\ell+1)} = W^{(\ell)} - \gamma \cdot \frac{\partial \mathcal{L}}{\partial W}$$

a suitable (adaptive) learning rate $\ \gamma$.

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NAE with Multiplicative Update Rules

Encoder:

$$H = W_{\mathcal{E}}V$$

Structured Dropout:

$$H' = H \odot M_H$$

Decoder:

$$\hat{V} = W_{\mathcal{D}}H'$$

NMF vs. NAE

Ozer, Hansen, Zunner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.



NAE with Multiplicative Update Rules

Encoder:

$$H = W_{\mathcal{E}}V$$

$$W_{\mathcal{E},rk}^{(\ell+1)} = W_{\mathcal{E},rk}^{(\ell)} \cdot \frac{\left(\left(\left(W_{\mathcal{D}}^{\top}V\right) \odot M_{H}\right)V^{\top}\right)_{rk}}{\left(\left(\left(W_{\mathcal{D}}^{\top}W_{\mathcal{D}}H^{\prime(\ell)}\right) \odot M_{H}\right)V^{\top}\right)_{rk}}$$

Structured Dropout:

$$H' = H \odot M_H$$

Decoder:

$$\hat{V} = W_{\mathcal{D}}H'$$

$$W_{\mathcal{D},kr}^{(\ell+1)} = W_{\mathcal{D},kr}^{(\ell)} \cdot \frac{\left(VH'^{\intercal}\right)_{kr}}{\left(W_{\mathcal{D}}^{(\ell)}H'H'^{\intercal}\right)_{kr}}$$

Similar idea and

computation as for NMF

NMF vs. NAE

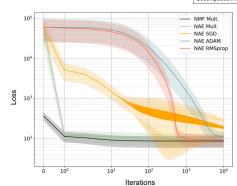
Ozer, Hansen, Zunner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.



Approximation Loss

NMF vs. NAE

Özer, Hansen, Zunner, Müller: Investigating Nonnegative Autoencoders for Efficient Audi Decomposition. Proc. EUSIPCO, 2022.

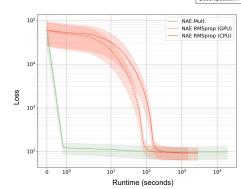


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Approximation Loss

NMF vs. NAE

Özer, Hansen, Zunner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.



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Conclusions (NAE)

- Simulation of NMF:
 - Decoder corresponds to NMF templates
 - Encoder learns a kind of pseudo-inverse
 - Code corresponds to NMF activations
- Nonnegativity can be achieved via
 - activation function (ReLU)
 - projected gradient descent
 - multiplicative update rules
- Musical knowledge can be integrated via
 - removing network weights (template constraints)
 - structured dropout (activation constraints)

Nonnegative Autoencoders with Applications to Music Audio Decomposing

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Outlook

- More complex networks
 - Deeper networks (more layers)
 - Different layer types (CNN, RNN, ...) and activation functions
 - Modification of loss function and regularization terms
- Understanding encoder decoder relationship
 - Nonnegativity
 - Pseudo-inverse
- Update rules
 - Constraints and convergence issues
 - Adaptive learning rates and projected gradient descent



Score-Informed Audio Decomposition

Audio mosaicing (style transfer)

Target signal: Beatles-Let it be

Source signal: Bees

એઇ એઇ એઇ એઇ Mosaic signal: Let it Bee

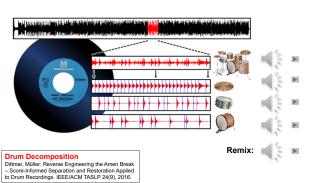
Audio Mosaicing Driedger, Prätzlich, Müller: Let It Bee – Towards NMF-Inspired Audio Mosaicing. ISMIR, 2015.

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Score-Informed Audio Decomposition

Informed Drum-Sound Decomposition





Score-Informed Audio Decomposition

Major challenge: Reconstructed sound events often have artifacts

Approaches:

- Resynthesize certain sound components
- Differentiable Digital Signal Processing (DDSP) combines classical DSP and deep learning
- Generative adversarial networks may help to reduce the artifacts

DDSP Engel et al.: DDSP: Differentiable Digital Signal Processing. ICLR, 2020.



Source Separation (Piano Concerto)

- Yigitcan Özer
- PhD student in engineering
- Pianist





Source Separation (Piano Concerto)

- Yigitcan Özer
- PhD student in engineering
- Pianist



Only Piano!



Where is the orchestra?



Source Separation (Piano Concerto)





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Source Separation (Piano Concerto)



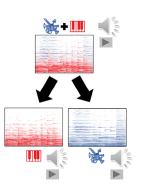


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Source Separation (Piano Concerto)





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Source Separation (Piano Concerto)



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Fundamentals of Music Processing (FMP)



Meinard Müller Fundamentals of Music Processing Audio, Analysis, Algorithms, Applications Springer, 2015

Accompanying website: www.music-processing.de

Fundamentals of Music Processing (FMP)



Meinard Müller Fundamentals of Music Processing Audio, Analysis, Algorithms, Applications Springer, 2015

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2nd edition Meinard Müller Fundamentals of Music Processing Using Python and Jupyter Notebooks Springer, 2021



Fundamentals of Music Processing (FMP)



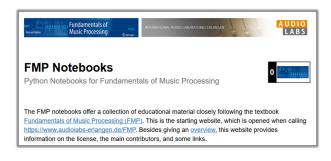
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2nd edition Meinard Müller Fundamentals of Music Processing Using Python and Jupyter Notebooks Springer, 2021



FMP Notebooks: Education & Research



https://www.audiolabs-erlangen.de/FMP

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librosa

ESSENTIA

Resources (Group Meinard Müller)

FMP Notebooks:

https://www.audiolabs-erlangen.de/FMP

libfmp:

https://github.com/meinardmueller/libfmp

synctoolbox:

https://github.com/meinardmueller/synctoolbox

https://github.com/meinardmueller/libtsm

Preparation Course Python (PCP) Notebooks:

https://www.audiolabs-erlangen.de/resources/MIR/PCP/PCP.html https://github.com/meinardmueller/PCP



Resources

librosa:

https://librosa.org/

madmom:

https://github.com/CPJKU/madmom

Essentia Python tutorial:

https://essentia.upf.edu/essentia_python_tutorial.html

https://github.com/mir-dataset-loaders/mirdata

open-unmix:

https://github.com/sigsep/open-unmix-pytorch

Open Source Tools & Data for Music Source Separation:

https://source-separation.github.io/tutorial/landing.html





References (FMP Textbook & Notebooks)

- Meinard Müller: Fundamentals of Music Processing Using Python and Jupyter Notebooks. 2nd Edition, Springer, 2021.
- Meinard Müller and Frank Zalkow: libfmp: A Python Package for Fundamentals of Music Processing. Journal of Open Source Software (JOSS), 6(63): 1–5, 2021. https://joss.theoj.org/papers/10.21105/joss.03326
- Meinard Müller: An Educational Guide Through the FMP Notebooks for Teaching and Learning Fundamentals of Music Processing. Signals, 2(2): 245–285, 2021.
- Meinard Müller and Frank Zalkow: FMP Notebooks: Educational Material for Teaching and Learning Fundamentals of Music Processing. Proc. International Society for Music Information Retrieval Conference (ISMIR): 573-580, 2019.
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- Sebastian Ewert and Meinard Müller: Using Score-Informed Constraints for NMF-Based Source Separation, Proc. ICASSP, 2012.
- Paris Smaragdis and Shrikant Venkataramani: A Neural Network Alternative to Non-Negative Audio Models. Proc. ICASSP, 2017.
- Sebastian Ewert and Mark B. Sandler: Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation. Proc. ICASSP, 2017
- Yigitcan Özer, Jonathan Hansen, Tim Zunner, and Meinard Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.

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