



### **Tutorial**

# **Audio Structure Analysis of Music**

### **Meinard Müller**







# Music Structure Analysis

Music Structure Analysis

Example: Weber, Song (No. 4) from "Der Freischütz"

Dialogues

**General goal:** Divide an audio recording into temporal segments corresponding to musical parts and group these segments into musically meaningful categories.

**Challenge:** There are many different principles for creating relationships that form the basis for the musical structure.

Homogeneity: Consistency in tempo, instrumentation, key, ...

Novelty: Sudden changes, surprising elements ...

Repetition: Repeating themes, motives, rhythmic patterns,...

Variation: Modification and transformation

# Music Structure Analysis

**General goal:** Divide an audio recording into temporal segments corresponding to musical parts and group these segments into musically meaningful categories.

### **Examples:**

- Stanzas of a folk song
- Intro, verse, chorus, bridge, outro sections of a pop song
- Exposition, development, recapitulation, coda of a sonata
- Musical form ABACADA ... of a rondo

# Music Structure Analysis

Novelty

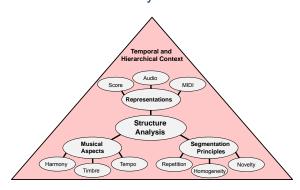








# Music Structure Analysis



# Overview

- Introduction
- Self-Similarity Matrices
- Audio Thumbnailing
- Novelty-based Segmentation
- Converting Path to Block Structures

# Thanks:

- Clausen, Ewert, Kurth, Grohganz, ...
- Dannenberg, Goto
- Grosche, Jiang
- Paulus, Klapuri
- Peeters, Kaiser, ...
- Serra, Gómez, ...
- Smith, Fujinaga, ...
- Wand, Sunkel, Jansen

### Overview

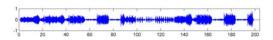
- Introduction
- **Self-Similarity Matrices**
- Audio Thumbnailing
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# **Feature Representation**

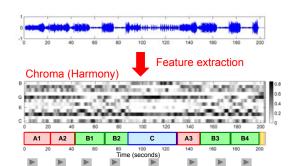
Example: Brahms Hungarian Dance No. 5 (Ormandy)





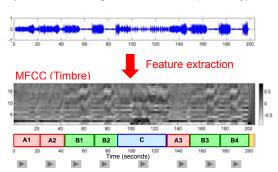
# Feature Representation

Example: Brahms Hungarian Dance No. 5 (Ormandy)



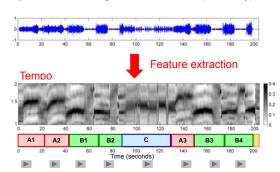
# Feature Representation

**Example:** Brahms Hungarian Dance No. 5 (Ormandy)



# Feature Representation

Example: Brahms Hungarian Dance No. 5 (Ormandy)



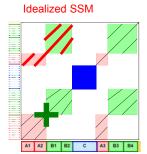
## Self-Similarity Matrix (SSM)

Example: Brahms Hungarian Dance No. 5 (Ormandy)

**Blocks:** Homogeneity

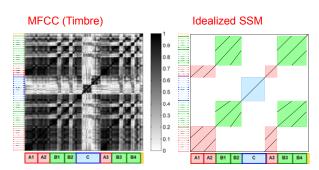
Paths: Repetition

Corners: Novelty



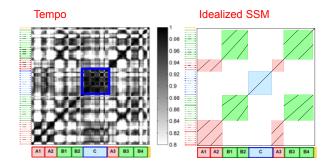
### Self-Similarity Matrix (SSM)

Example: Brahms Hungarian Dance No. 5 (Ormandy)



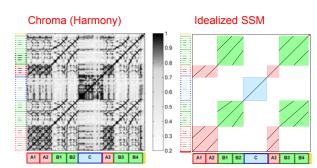
# Self-Similarity Matrix (SSM)

Example: Brahms Hungarian Dance No. 5 (Ormandy)



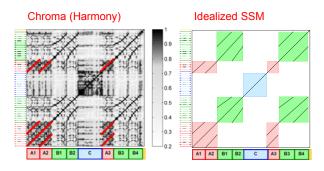
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Example: Brahms Hungarian Dance No. 5 (Ormandy)



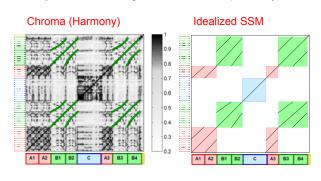
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Example: Brahms Hungarian Dance No. 5 (Ormandy)



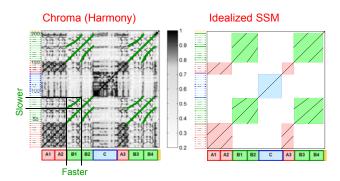
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Example: Brahms Hungarian Dance No. 5 (Ormandy)



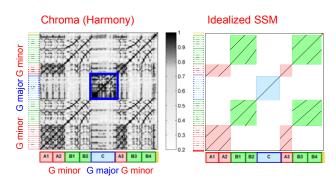
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Example: Brahms Hungarian Dance No. 5 (Ormandy)

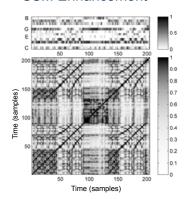


### Self-Similarity Matrix (SSM)

Example: Brahms Hungarian Dance No. 5 (Ormandy)



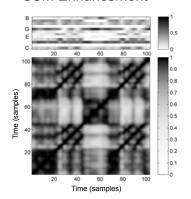
# SSM Enhancement



### **Block Enhancement**

- Feature smoothing
- Coarsening

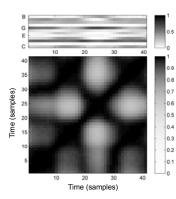
### SSM Enhancement



### **Block Enhancement**

- Feature smoothing
- Coarsening

# SSM Enhancement

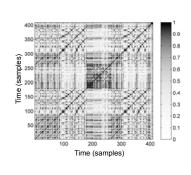


# Block Enhancement

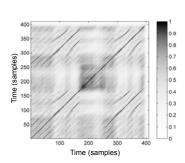
- Feature smoothing
- Coarsening

### SSM Enhancement

# Path Enhancement



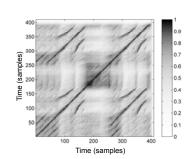
## SSM Enhancement



### Path Enhancement

Diagonal smoothing

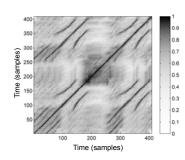
### SSM Enhancement



### Path Enhancement

- Diagonal smoothing
- Multiple filtering

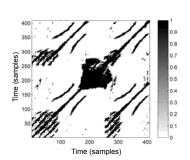
# SSM Enhancement



### Path Enhancement

- Diagonal smoothing
- Multiple filtering
- Forward-backward

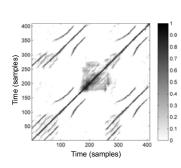
### SSM Enhancement



### Path Enhancement

- Diagonal smoothing
- Multiple filtering
- Forward-backward
- Thresholding (binary)

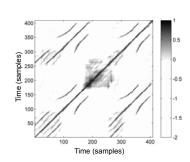
### SSM Enhancement



# Path Enhancement

- Diagonal smoothing
- Multiple filtering
- Forward-backward
- Thresholding (relative)

### SSM Enhancement



# Path Enhancement

- Diagonal smoothing
- Multiple filtering
- Forward-backward
- Thresholding (relative)
- Scaling & penalty

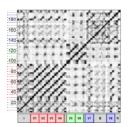
# SSM Enhancement

Example: Zager & Evans "In The Year 2525"



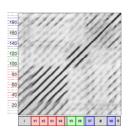
### SSM Enhancement

Example: Zager & Evans "In The Year 2525"



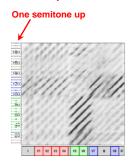
# SSM Enhancement

**Example:** Zager & Evans "In The Year 2525" Missing relations because of transposed sections



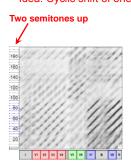
# SSM Enhancement

**Example:** Zager & Evans "In The Year 2525" Idea: Cyclic shift of one of the chroma sequences



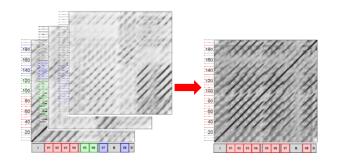
### SSM Enhancement

**Example:** Zager & Evans "In The Year 2525" Idea: Cyclic shift of one of the chroma sequences



### SSM Enhancement

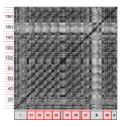
Example: Zager & Evans "In The Year 2525"



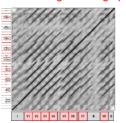
## SSM Enhancement

**Example:** Zager & Evans "In The Year 2525" Note: Order of enhancement steps important!

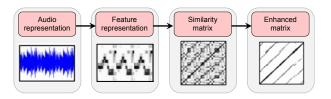
### Adding up



### Smoothing & adding up



### Similarity Matrix Toolbox



Meinard Müller, Nanzhu Jiang, Harald Grohganz SM Toolbox: MATLAB Implementations for Computing and Enhancing Similarity Matrices

http://www.audiolabs-erlangen.de/resources/MIR/SMtoolbox/

### Overview

- Introduction
- Self-Similarity Matrices
- Audio Thumbnailing
- Novelty-based Segmentation
- Converting Path to Block Structures

### Thanks:

- Jiang, Grosche
- Peeters
- Cooper, Foote
- Goto
- Levy, Sandler
- Mauch
- Sapp

# Audio Thumbnailing

**General goal:** Determine the most representative section ("Thumbnail") of a given music recording.

**Example:** Zager & Evans "In The Year 2525"



Example: Brahms Hungarian Dance No. 5 (Ormandy)



Thumbnail is often assumed to be the most repetitive segment

# **Audio Thumbnailing**

### Two steps

### Both steps are problematic!

- 1. Path extraction
- Paths of poor quality (fragmented, gaps)
- Block-like structures
- Curved paths
- 2. Grouping
- Noisy relations
- (missing, distorted, overlapping)
- Transitivity computation difficult

### Main idea: Do both, path extraction and grouping, jointly

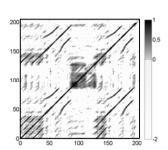
- One optimization scheme for both steps
- Stabilizing effect
- Efficient

# **Audio Thumbnailing**

Main idea: Do both path extraction and grouping jointly

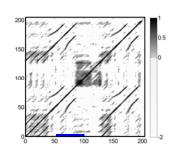
- For each audio segment we define a fitness value
- This fitness value expresses "how well" the segment explains the entire audio recording
- The segment with the highest fitness value is considered to be the thumbnail
- As main technical concept we introduce the notion of a path family

### Fitness Measure



### **Enhanced SSM**

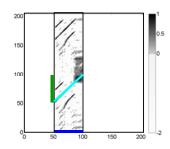
# **Fitness Measure**



### Path over segment

Consider a fixed segment

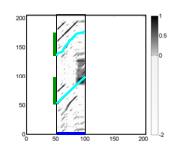
### Fitness Measure



### Path over segment

- Consider a fixed segment
- Path over segment
- Induced segment
- Score is high

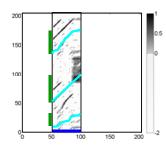
### Fitness Measure



### Path over segment

- Consider a fixed segment
- Path over segment
- Induced segment
- Score is high
- A second path over segment
- Induced segment
- Score is not so high

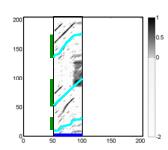
# Fitness Measure



### Path over segment

- Consider a fixed segment
- Path over segment
- Induced segment
- Score is high
- A second path over segment
- Induced segment
- Score is not so high
- A third path over segment
- Induced segment
- Score is very low

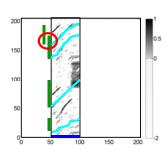
# Fitness Measure



### Path family

- Consider a fixed segment
- A path family over a segment is a family of paths such that the induced segments do not overlap.

### **Fitness Measure**

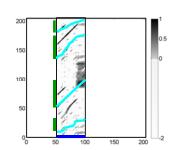


### Path family

- Consider a fixed segment
- A path family over a segment is a family of paths such that the induced segments do not overlap.

This is not a path family!

### **Fitness Measure**

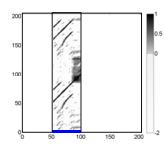


### Path family

- Consider a fixed segment
- A path family over a segment is a family of paths such that the induced segments do not overlap.

This is a path family!
(Even though not a good one)

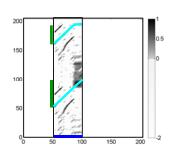
# Fitness Measure



### Optimal path family

Consider a fixed segment

# Fitness Measure



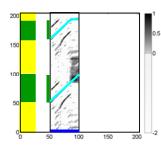
Note: This optimal path family can be computed using dynamic programming.

### Optimal path family

- Consider a fixed segment
- Consider over the segment the optimal path family, i.e., the path family having maximal overall score.
- Call this value:

Score(segment)

### Fitness Measure



### Optimal path family

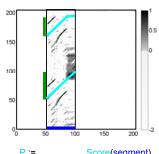
- Consider a fixed segment
- Consider over the segment the optimal path family, i.e., the path family having maximal overall score.
- Call this value:

Score(segment)

- Furthermore consider the amount covered by the induced segments.
- Call this value:

Coverage(segment)

# Fitness Measure



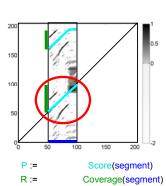
R :=

Score(segment)
Coverage(segment)

# Fitness

Consider a fixed segment

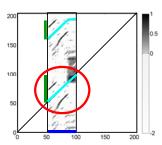
### **Fitness Measure**



### Fitness

- Consider a fixed segment
- Self-explanation are trivial!

### Fitness Measure



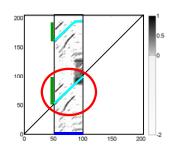
### Fitness

- Consider a fixed segment
- Self-explanation are trivial!
- Subtract length of segment

Fitness Measure

- length(segment) Score(segment)
- R :=
- Coverage(segment) length(segment)

# Fitness Measure



### Fitness

- Consider a fixed segment
- Self-explanation are trivial!
- Subtract length of segment
- Normalization
- P := Normalize( Score(segment) - length(segment) )
- R := Normalize(Coverage(segment) length(segment) )
- $\in$  [0,1]  $\in$  [0,1]

- length(segment) )

**Fitness** 

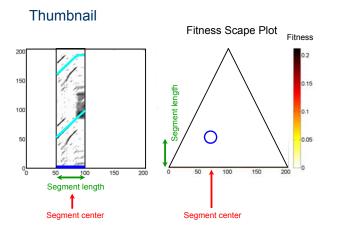
- $\in$  [0,1]

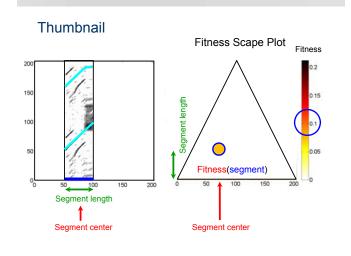
Consider a fixed segment

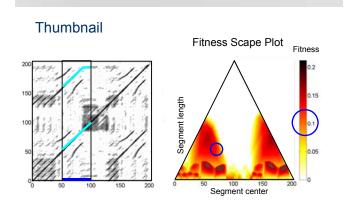
Fitness(segment)

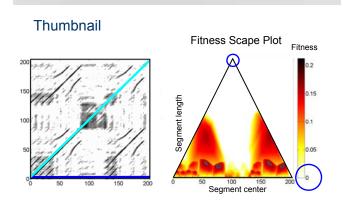
 $F := 2 \cdot P \cdot R / (P + R)$ 

- P := Normalize( Score(segment)
- R := Normalize(Coverage(segment) length(segment) )  $\in$  [0,1]

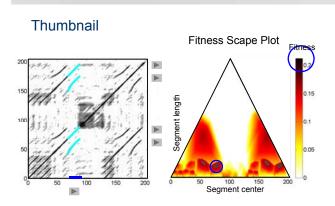




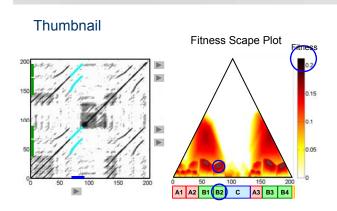




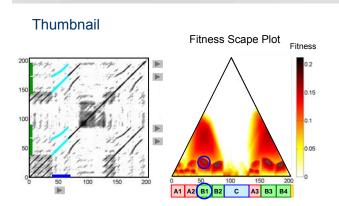
Note: Self-explanations are ignored  $\rightarrow$  fitness is zero



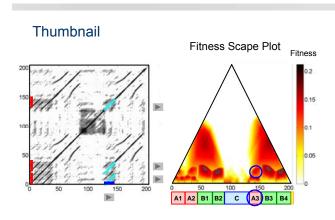
Thumbnail := segment having the highest fitness



Example: Brahms Hungarian Dance No. 5 (Ormandy)



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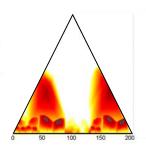


Example: Brahms Hungarian Dance No. 5 (Ormandy)

# Thumbnail Fitness Scape Plot Fitness 200 150 100 150 200 A1 (2 B1 B2 C A3 B3 B4)

Example: Brahms Hungarian Dance No. 5 (Ormandy)

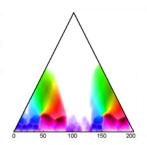
# Scape Plot



Example: Brahms Hungarian Dance No. 5 (Ormandy)

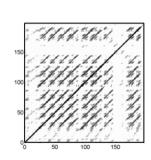
# Scape Plot

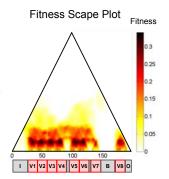
Coloring according to clustering result (grouping)



Example: Brahms Hungarian Dance No. 5 (Ormandy)

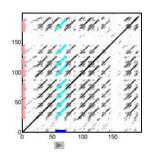
### Thumbnail

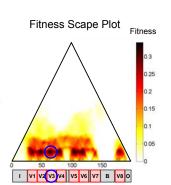




Example: Zager & Evans "In The Year 2525"

# Thumbnail





Example: Zager & Evans "In The Year 2525"

### Overview

- Introduction
- Self-Similarity Matrices
- Audio Thumbnailing

### Thanks:

- Foote
- Serra, Grosche, Arcos
- Goto
- Tzanetakis, Cook
- Novelty-based Segmentation
- Converting Path to Block Structures

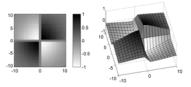
# **Novelty-based Segmentation**

### **General goals:**

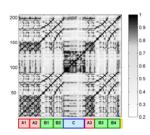
- Find instances where musical changes occur.
- Find transition between subsequent musical parts.

### Idea (Foote):

Use checkerboard-like kernel function to detect corner points on main diagonal of SSM.



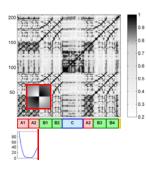
# **Novelty-based Segmentation**



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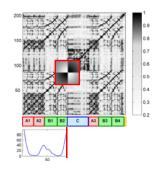
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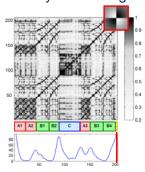
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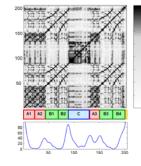
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# Idea (Foote):

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# **Novelty-based Segmentation**



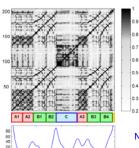
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Novelty function using



# **Novelty-based Segmentation**



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Use checkerboard-like kernel function to detect corner points on main diagonal of SSM.

Novelty function using



Novelty function using



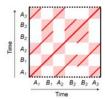
# Novelty-based Segmentation

### Idea:

- Find instances where structural changes occur.
- Combine global and local aspects within a unifying framework

### Structure features

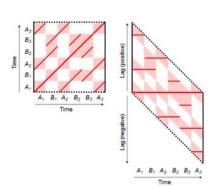
# **Novelty-based Segmentation**



### Structure features

Enhanced SSM

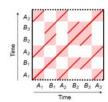
# **Novelty-based Segmentation**

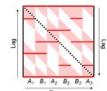


### Structure features

- Enhanced SSM
- Time-lag SSM

# Novelty-based Segmentation

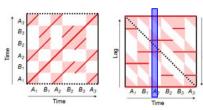




### Structure features

- Enhanced SSM
- Time-lag SSM
- Cyclic time-lag SSM

# **Novelty-based Segmentation**

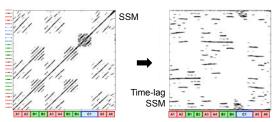


### Structure features

- Enhanced SSM
- Time-lag SSM
- Cyclic time-lag SSM
- Columns as features

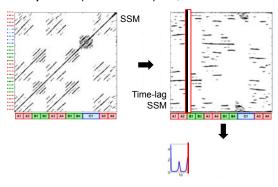
# **Novelty-based Segmentation**

Example: Chopin Mazurka Op. 24, No. 1



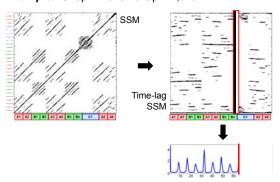
## **Novelty-based Segmentation**

Example: Chopin Mazurka Op. 24, No. 1



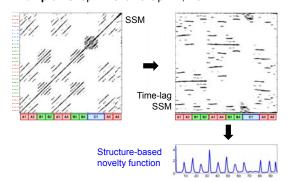
# **Novelty-based Segmentation**

Example: Chopin Mazurka Op. 24, No. 1



# **Novelty-based Segmentation**

Example: Chopin Mazurka Op. 24, No. 1



### Overview

- Introduction
- Self-Similarity Matrices
- Audio Thumbnailing
- Novelty-based Segmentation

Converting Path to Block Structures

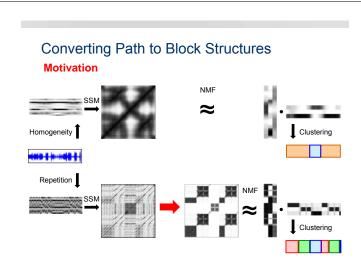
### Thanks:

- Grohganz, Clausen
- Kaiser
- Peeters
- Dubnov, Apel
- Serra, Grosche, Arcos

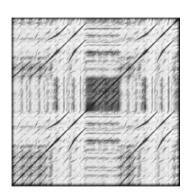
# Converting Path to Block Structures

### Motivation

- Perform joint analysis using repetitive as well as homogeneous aspects
- Make homogeneity-based methods applicable to repetition-based analysis



# Converting Path to Block Structures



### **Procedure**

Enhanced SSM

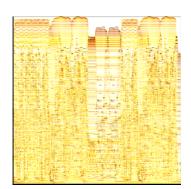
# Converting Path to Block Structures



### Procedure

- Enhanced SSM
- Thresholding & image processing

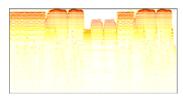
# Converting Path to Block Structures



### **Procedure**

- Enhanced SSM
- Thresholding & image processing
- Eigenvalue decomposition

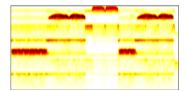
# Converting Path to Block Structures



### **Procedure**

- Enhanced SSM
- Thresholding & image processing
- Eigenvalue decomposition
- Weigthing

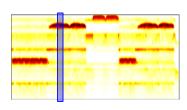
# Converting Path to Block Structures



### **Procedure**

- Enhanced SSM
- Thresholding & image processing
- Eigenvalue decomposition
- Weigthing
- Clustering & smoothing

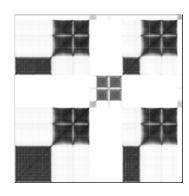
# Converting Path to Block Structures



### **Procedure**

- Enhanced SSM
- Thresholding & image processing
- Eigenvalue decomposition
- Weigthing
- Clustering & smoothing
- Columns as features

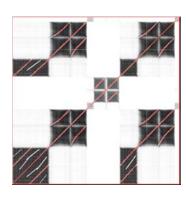
### Converting Path to Block Structures



### **Procedure**

- Enhanced SSM
- Thresholding & image processing
- Eigenvalue decomposition
- Weigthing
- Clustering & smoothing
- Columns as features
- SSM from these features

# Converting Path to Block Structures



### **Procedure**

- Enhanced SSM
- Thresholding & image processing
- Eigenvalue decomposition
- Weigthing
- Clustering & smoothing
- Columns as features
- SSM from these features

Final matrix shows paths as blocks

### Conclusions

- Repetition, Homogeneity, Novelty
- Combined Approaches
- Hierarchical Approaches
- Evaluation
- Explaining Structure



a a a b c b c d d eee a a b c b c

- MIREX
- SALAMI-Project
- Smith, Chew

# PhD Projects (Final Stage)

Nanzhu Jiang

Universtät Erlangen-Nürnberg Supervisor: Meinard Müller

Harald Grohganz

Universität Bonn Supervisors: Michael Clausen, Meinard Müller

Jordan Smith

Queen Mary University of London Supervisor: Elaine Chew

Oriol Nieto

New York University Supervisor: Juan P. Bello

# **Book Project**

### A First Course on Music Processing

Textbook (approx. 500 pages)

- 1. Music Representations
- 2. Fourier Analysis of Signals
- Music Synchronization
- 4. Music Structure Analysis
- 5. Chord Recogntion
- 6. Temo and Beat Tracking
- 7. Content-based Audio Retrieval
- 8. Music Transcription



To appear (plan): End of 2015

### References

- W. CHAI AND B. VERCOE, Music thumbnailing via structural analysis, in Proceedings of the ACM International Conference on Multimedia, Berkeley, CA, USA, 2003, pp. 223–226.
- ACM international conference on multimedia, Berkeley, CA, USA, 2003, pp. 223–226.

  M. COOPER AND J. FOOTE, Automatic music summarization via similarity analysis, in Proceedings of the International Conference on Music Information Retrieval (ISMIR), Paris, France, 2002, pp. 81–85.

  R. B. DANNENBERG AND M. GOTO, Music structure analysis from acoustic signals, in Handbook of Signal Processing in Acoustics, D. Havelock, S.
  J. FOOTE, Visualizing music and audio using self-similarity, in Proceedings of the ACM International Conference on Multimedia, Orlando, FL, USA, 1999, pp. 77–80.

- J. FOOTE, Automatic audio segmentation using a measure of audio novelty, in Proceedings of the IEEE International Conference on Multimedia and Expo (ICME), New York, NY, USA, 2000, pp. 452–455.
- pp. 452–455. M. GOTO, A chorus section detection method for musical audio signals and its application to a music listening station, IEEE Transactions on Audio, Speech and Language Processing, 14 (2006), pp. 1783–1794
  H. GROHGANZ, M. CLAUSEN, N. JIANG, AND M. MÜLLER, Converting path structures into block structures using eigenvalue decompositions of self-similarity matrices, in Proceedings of the 14th International Conference on Music Information Retrieval (ISMIR), Curitiba, Brazil, 2013, pp. 209–214.
- pp. 209–214.
  K. JENSEN, Multiple scale music segmentation using rhythm, timbre, and harmony, EURASIP Journal on Advances in Signal Processing, 2007 (2007).
  F. KAISER AND T. SIKORA, Music structure discovery in popular music using non-negative matrix factorization, in Proceedings of the International Society for Music Information Retrieval Conference (ISMIR), Utrecht, The Netherlands, 2010, pp. 429–434.

### References

- J. PAULUS AND A. P. KLAPURI, Music structure analysis using a probabilistic fitness measure and a greedy search algorithm, IEEE Transactions on Audio, Speech, and Language Processing, 17 (2009), pp. 1159–1170.

  J. PAULUS, M. MÜLLER, AND A. P. KLAPURI, Audio-based music structure analysis, in Proceedings of the 11th International Conference on Music Information Retrieval (ISMIR), Utrecht, The Netherlands, 2010, pp. 625–636.

  G. PEETERS, Deriving musical structure from signal analysis for music audio summary generation: "sequence' and "state" approach, in Computer Music Modeling and Retrieval, vol. 2771 of Lecture Notes in Computer Science, Springer Berlin / Heidelberg, 2004, pp. 143–166.

  G. PEETERS, Sequence representation of music structure using higher-order similarity matrix and maximum-likelihood approach, in Proceedings of the International Conference on Music Information Retrieval (ISMIR), Vienna, Austria, 2007, pp. 35–40.

  C. RHODES AND M. A. CASEY, Algorithms for determining and labelling approximate interactical self-similarity, in Proceedings of the International Conference on Music Information Retrieval (ISMIR), Vienna, Austria, 2007, pp. 41–46.

  J. SERRA, M. MÜLLER, P. GROSCHE, AND J. L. ARCOS, Unsupervised detection of music boundaries by time series structure features, in Proceedings of the AAAI International Conference on Artificial Intelligence, Toronto, Ontario, Canada, 2012, pp. 1613–1619.

  J. B. L. SMITH, J. A. BURGOYNE, I. FUJINAGA, D. D. ROURE, AND J. S. DOWNIE, Design and creation of a large-scale database of structural annotations, in Proceedings of the International Conference in Structural annotations, in Proceedings of Structural annotations, in Proceeding
- J. B. L. SMITH AND E. CHEW, Using quadratic programming to estimate feature relevance in structural analyses of music, in Proceedings of the ACM International Conference on Multimedia, 2013, pp. 113–122.

### References

- M. LEVY, M. SANDLER, AND M. A. CASEY, Extraction of high-level musical structure from audio data and its application to thumbnail generation, in Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Toulouse, France, 2006, pp. 13–16.
- pp. 13–16.

  I. LUKASHEVICH, Towards quantitative measures of evaluating song segmentation, in Proceedings of the International Conference on Music Information Retrieval (ISMIR), Philadelphia, USA, 2008, pp. 375–380.

  M. MÜLLER AND M. CLAUSEN, Transposition-invariant self-similarity matrices, in Proceedings of the 8th International Conference on Music Information Retrieval (ISMIR), Vienna, Austria, 2007, pp. 47–50.

- 2007, pp. 47–50.

  M. MÜLLER AND N. JIANG, A scape plot representation for visualizing repetitive structures of music recordings, in Proceedings of the 13th International Conference on Music Information Retrieval (ISMIR), Porto, Portugal, 2012, pp. 97–102.

  M. MÜLLER, N. JIANG, AND H. GROHGANZ, SM Toolbox: MATLAB implementations for computing and enhancing similiarly matrices, in Proceedings of the 53rd AES Conference on Semantic Audio, London, GB, 2014.

  M. MÜLLER, N. JIANG, AND P. GROSCHE, A robust fitness measure for capturing repetitions in music recordings with applications to audio thumbnailing, IEEE Transactions on Audio, Speech & Language Processing, 21 (2013), pp. 531–543.

  M. MÜLLER AND F. KURTH, Enhancing similarity matrices for music audio analysis, in Proceedings of the International Conference on Acoustics, Speech and Signal Processing (ICASSP), Toulouse, France, 2006, pp. 437–440.

  M. MÜLLER AND F. KURTH, Towards structural analysis of audio recordings in the presence of
- M. MÜLLER AND F. KURTH, Towards structural analysis of audio recordings in the presence of musical variations, EURASIP Journal on Advances in Signal Processing, 2007 (2007).

### References

- M. SUNKEL, S. JANSEN, M. WAND, E. EISEMANN, H.-P. SEIDEL, Learning Line Features in 3D Geometry, in Computer Graphics Forum (Proc. Eurographics), 2011.

  D. TURNBULL, G. LANCKRIET, E. PAMPALK, AND M. GOTO, A supervised approach for detecting boundaries in music using difference features and boosting, in Proceedings of the International Conference on Music Information Retrieval (ISMIR), Vienna, Austria, 2007, pp. 51–54.
- 54. C. TZANETAKIS AND P. COOK, Multifeature audio segmentation for browsing and annotation, in Proceedings of the IEEEWorkshop on Applications of Signal Processing to Audio and Acoustics (WASPAA). New Platz, NY, USA, 1999, pp. 103–106.