



# Fundamentals of Music Processing

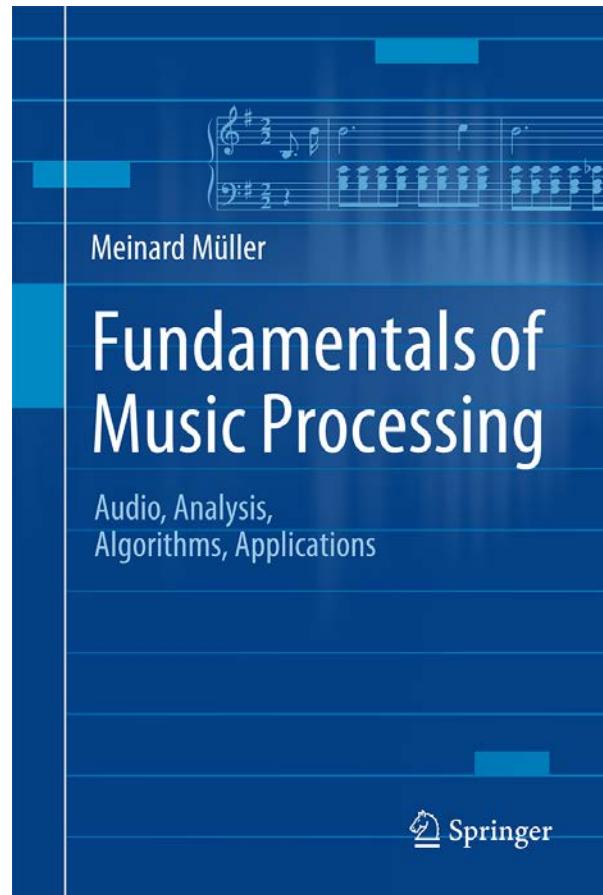
## Chapter 6: Tempo and Beat Tracking

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[www.music-processing.de](http://www.music-processing.de)

# Book: Fundamentals of Music Processing

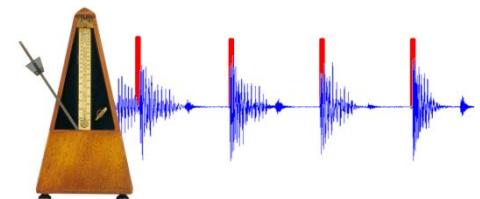


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# Chapter 6: Tempo and Beat Tracking

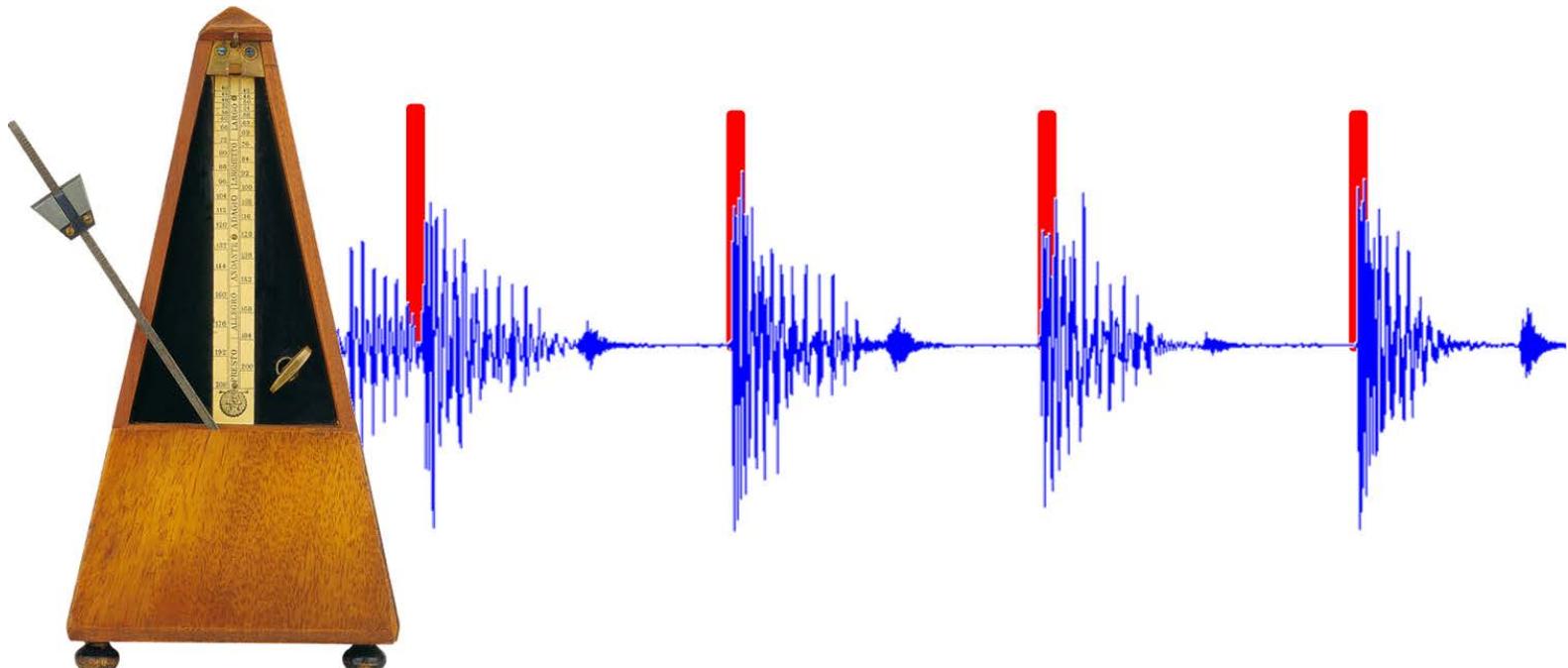
- 6.1 Onset Detection
- 6.2 Tempo Analysis
- 6.3 Beat and Pulse Tracking
- 6.4 Further Notes



Tempo and beat are further fundamental properties of music. In Chapter 6, we introduce the basic ideas on how to extract tempo-related information from audio recordings. In this scenario, a first challenge is to locate note onset information—a task that requires methods for detecting changes in energy and spectral content. To derive tempo and beat information, note onset candidates are then analyzed with regard to quasiperiodic patterns. This leads us to the study of general methods for local periodicity analysis of time series.

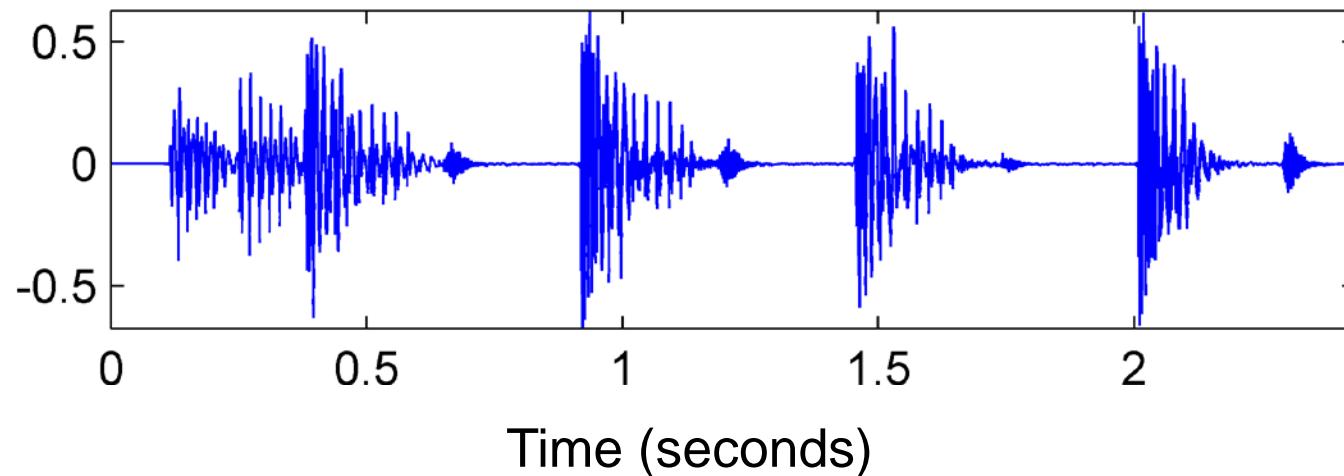
# 6 Tempo and Beat Tracking

Teaser



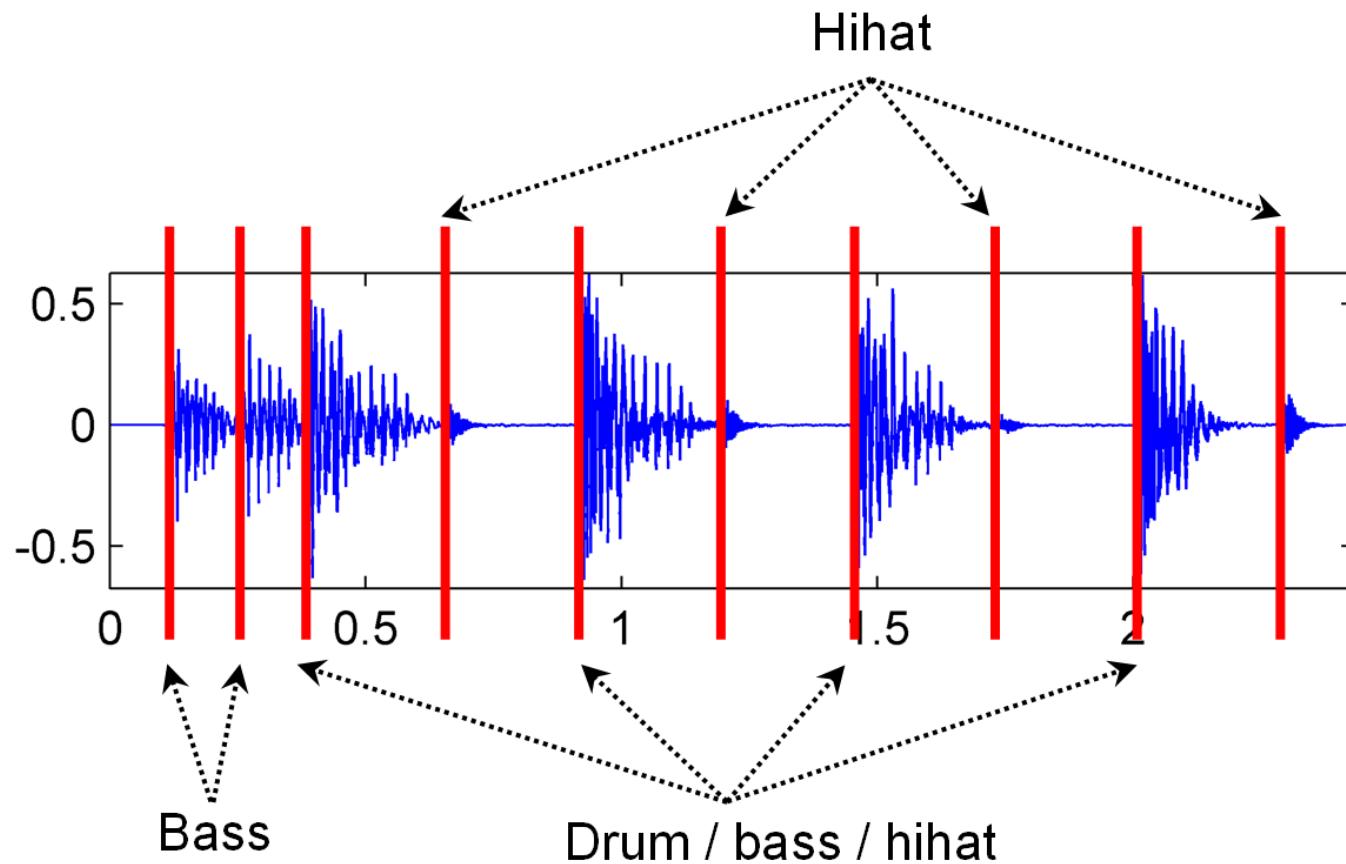
# 6 Tempo and Beat Tracking

Fig. 6.1



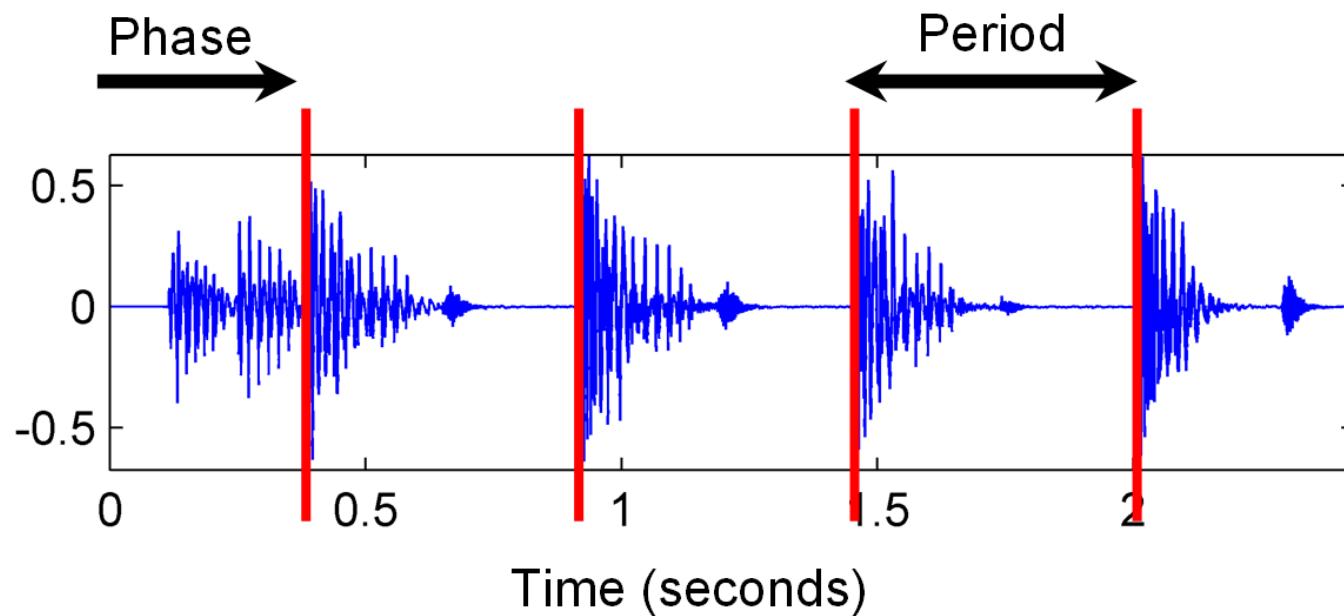
# 6 Tempo and Beat Tracking

Fig. 6.1



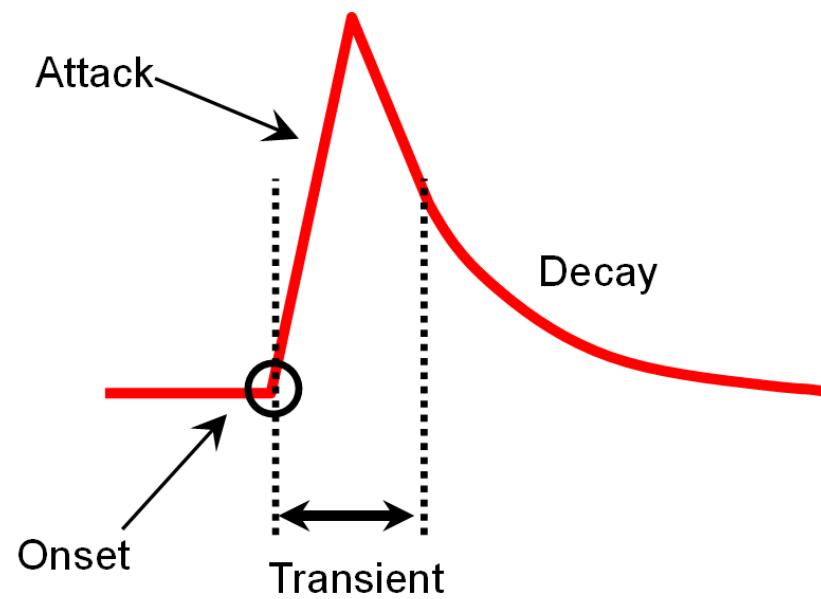
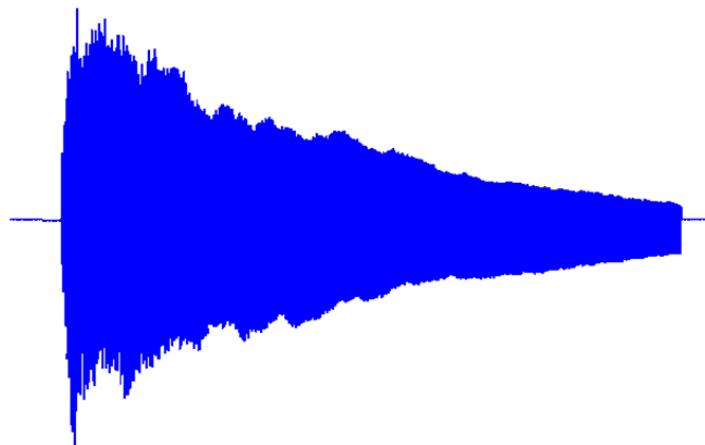
# 6 Tempo and Beat Tracking

Fig. 6.1



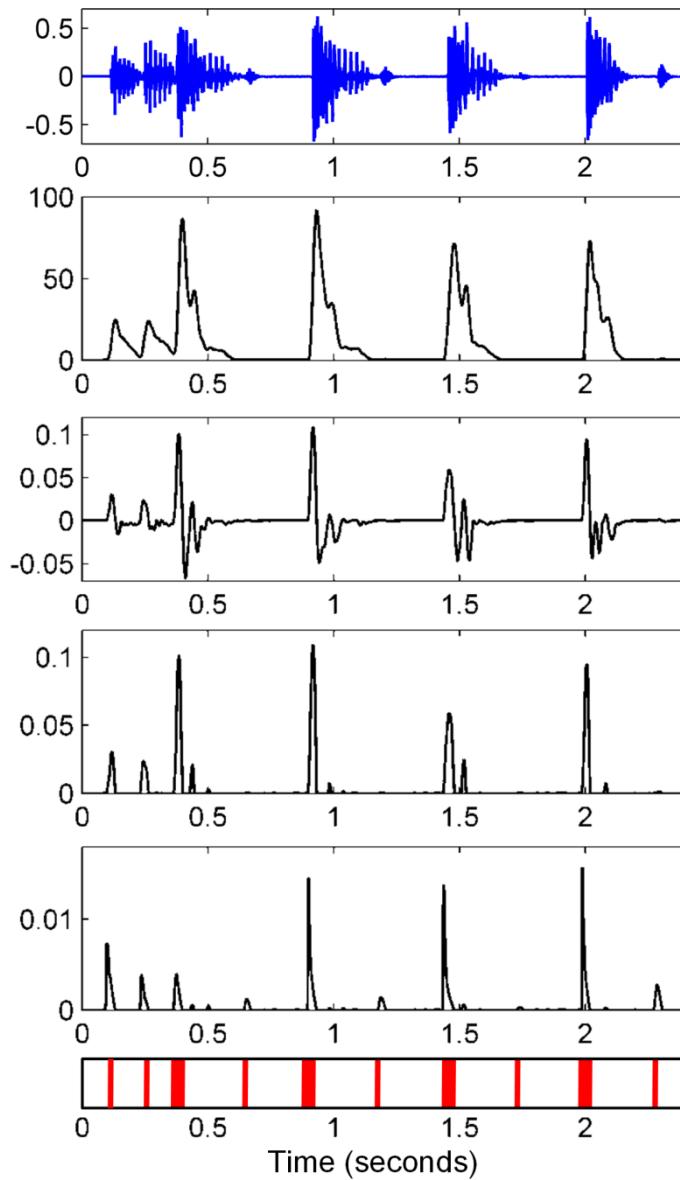
## 6.1 Onset Detection

Fig. 6.2



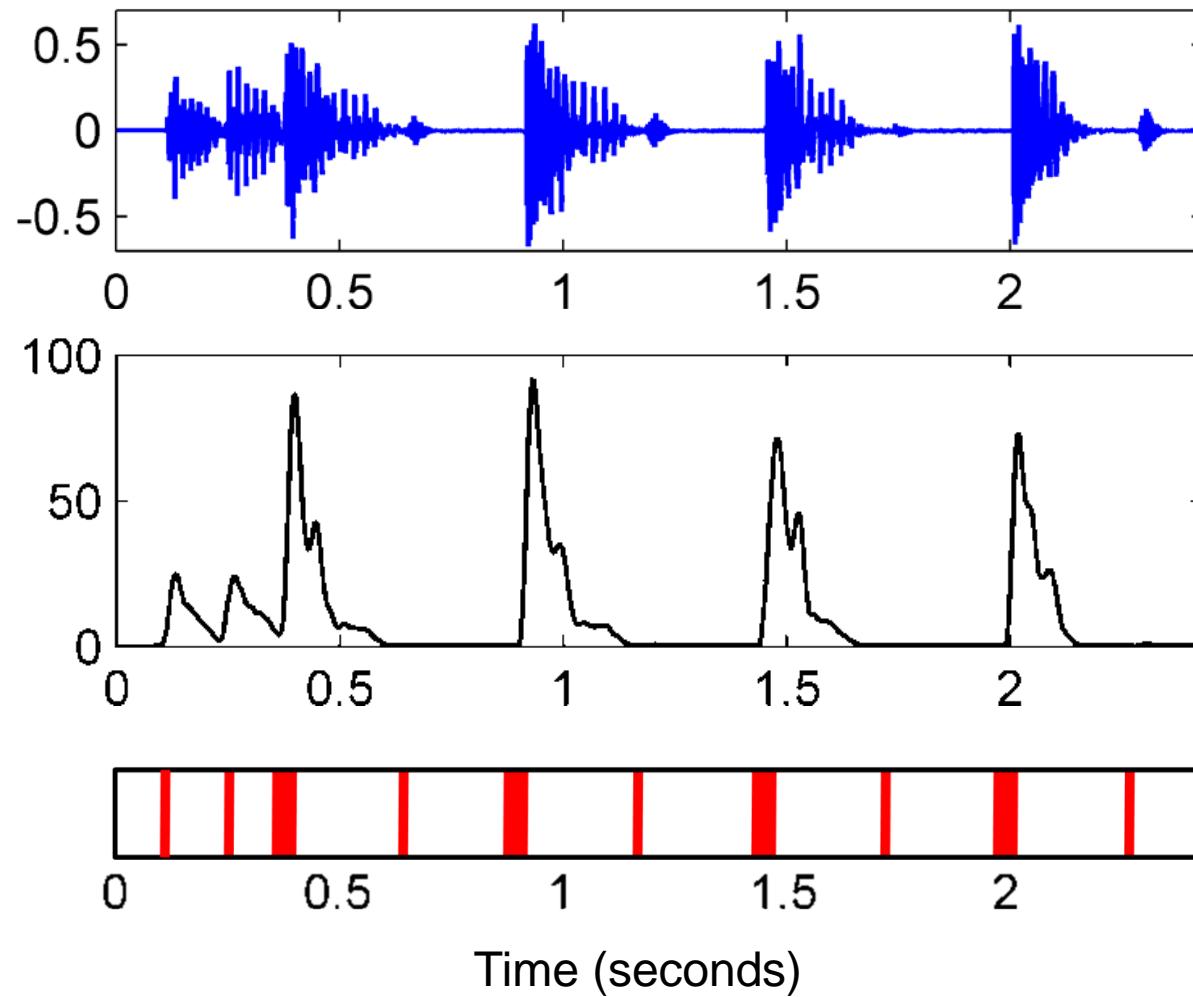
# 6.1 Onset Detection

Fig. 6.3



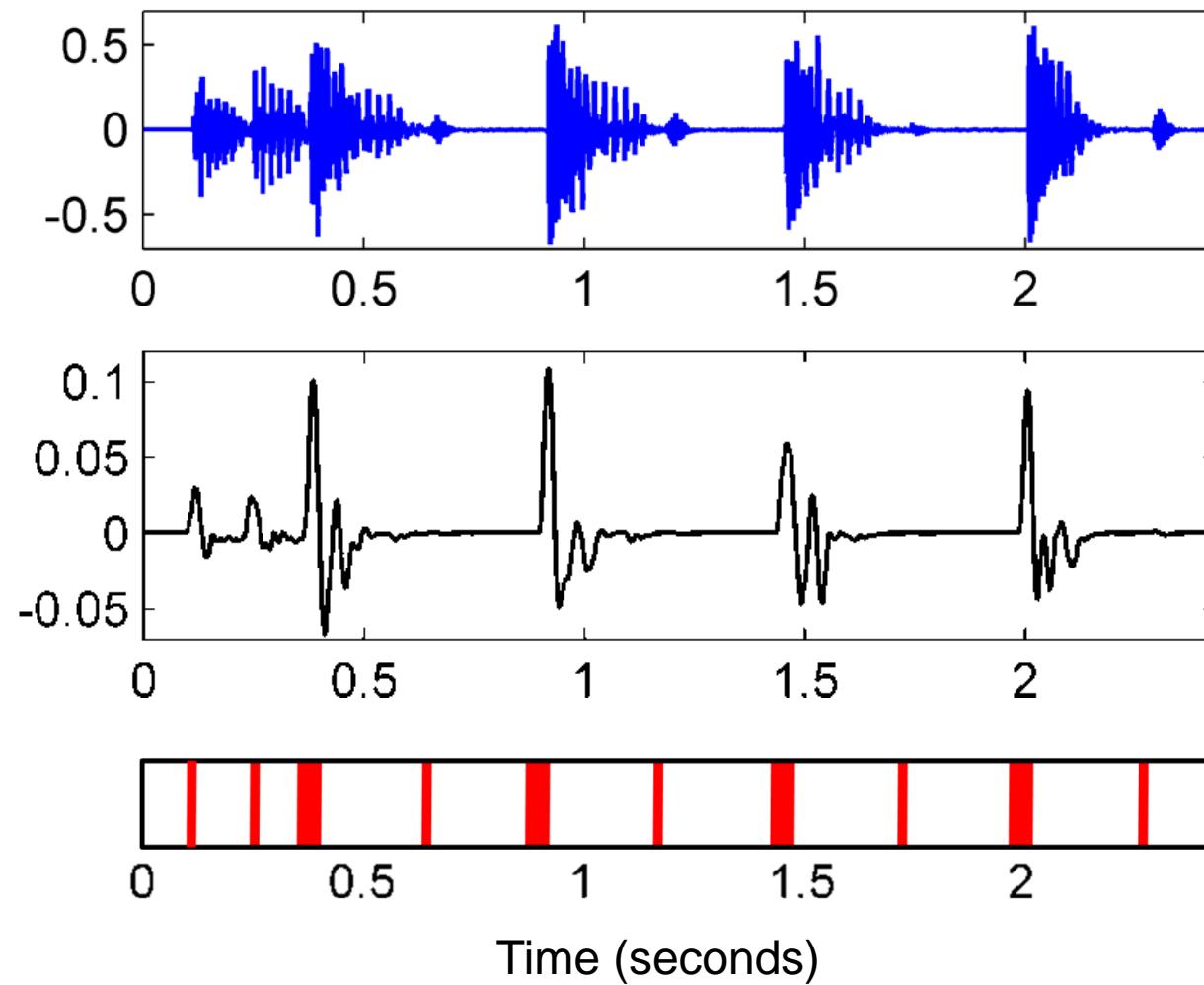
## 6.1 Onset Detection

Fig. 6.3



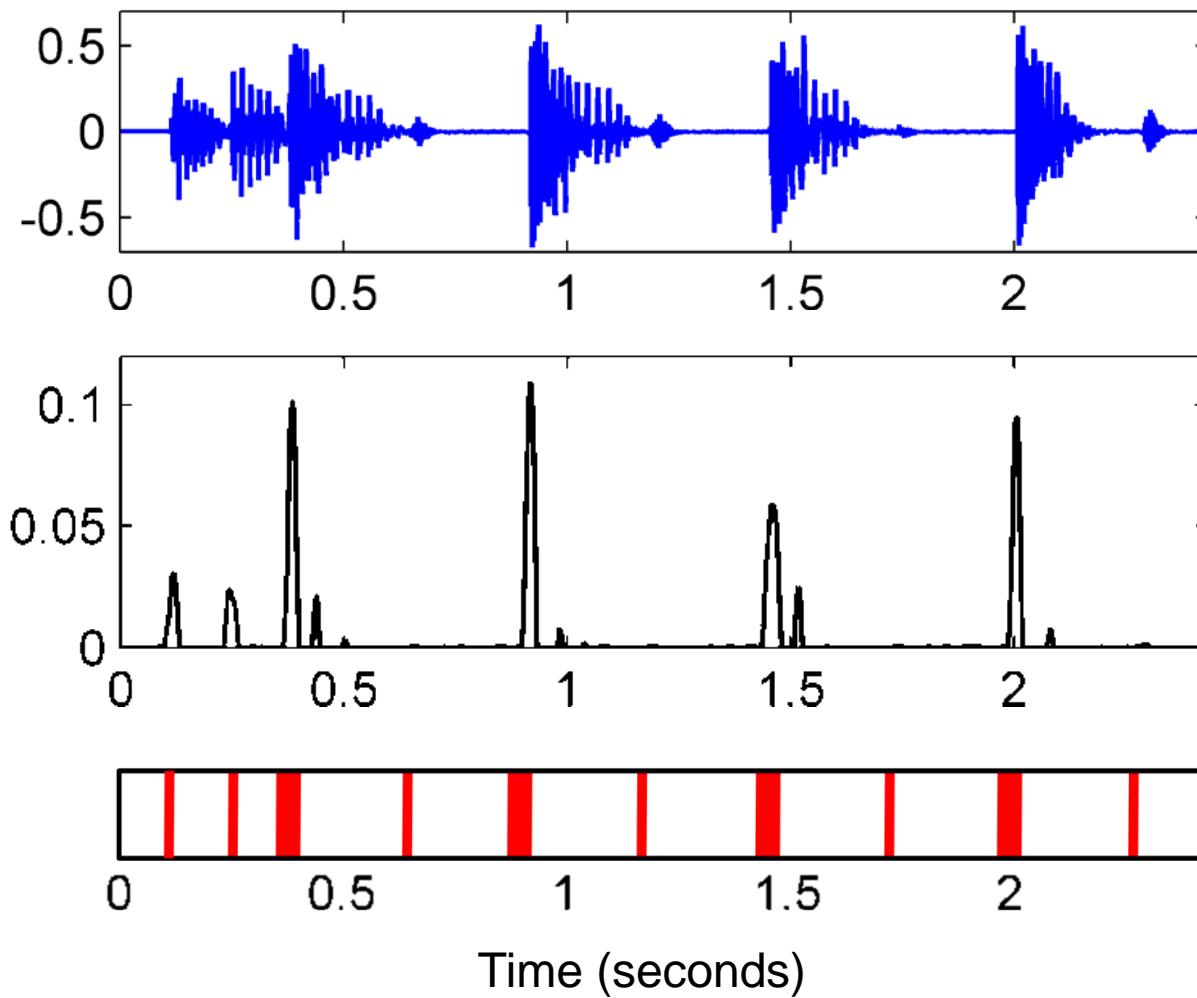
## 6.1 Onset Detection

Fig. 6.3



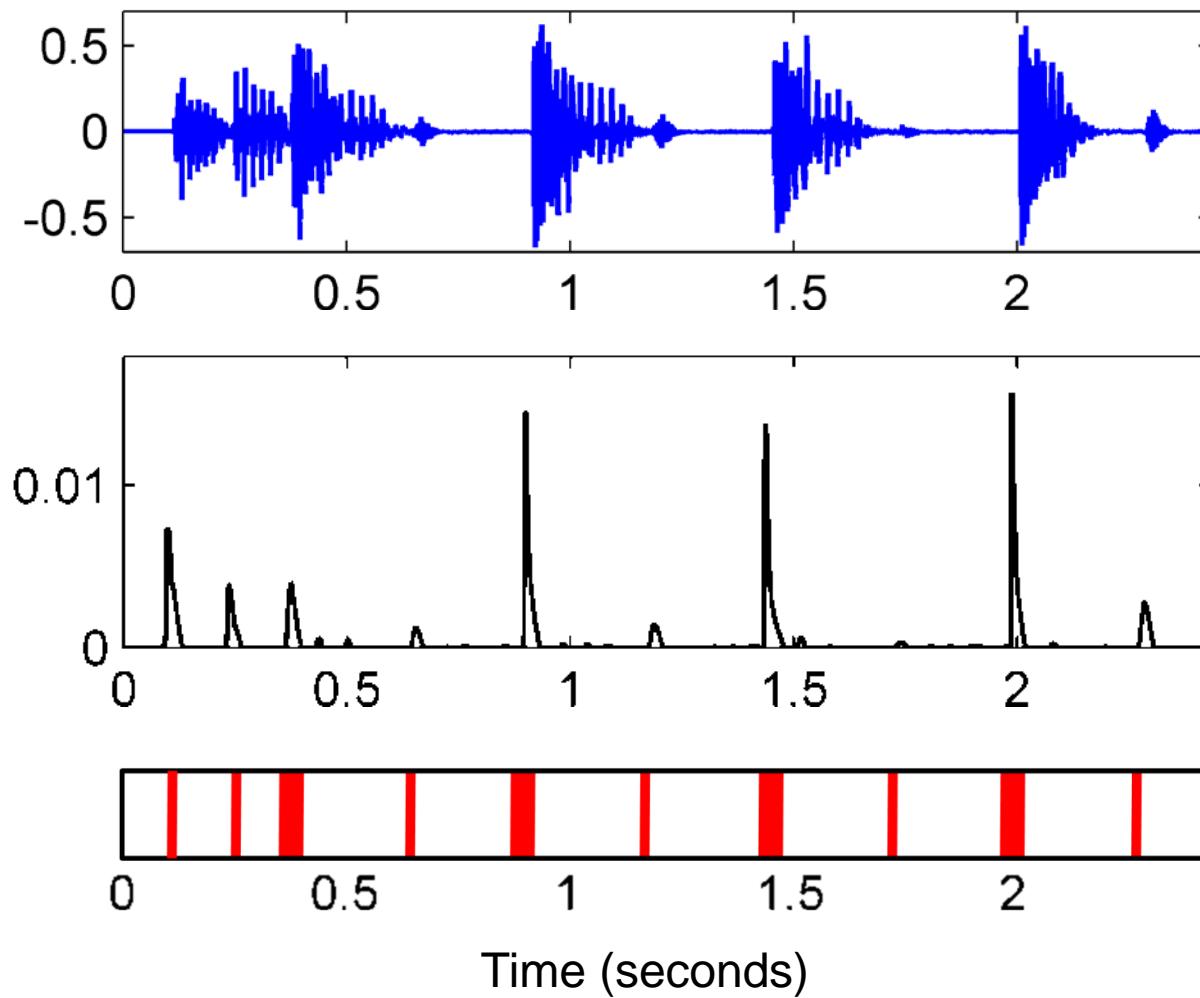
# 6.1 Onset Detection

Fig. 6.3



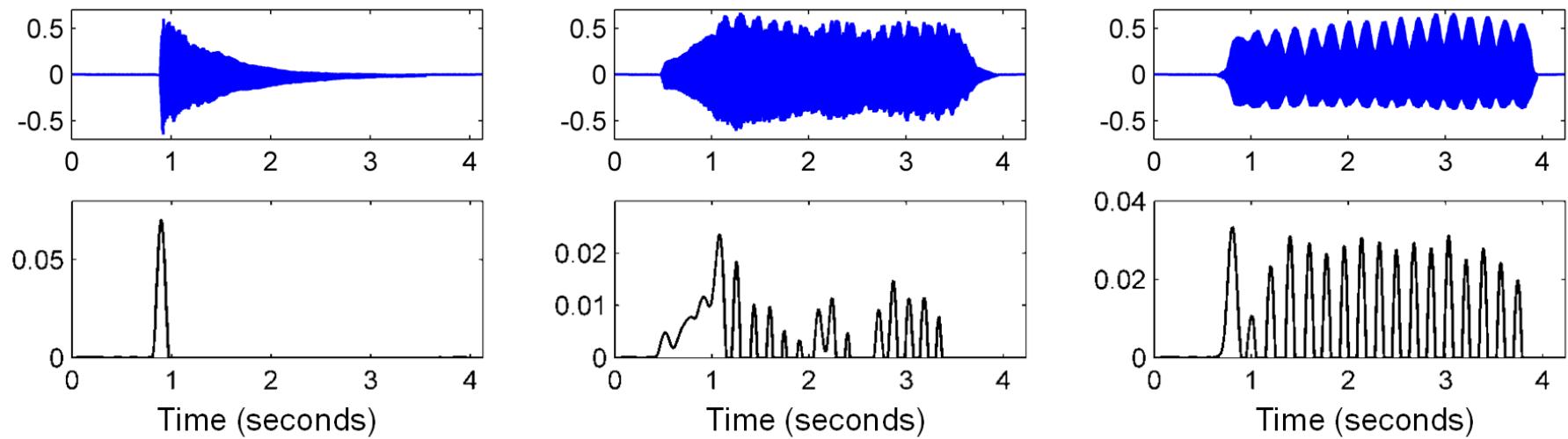
# 6.1 Onset Detection

Fig. 6.3



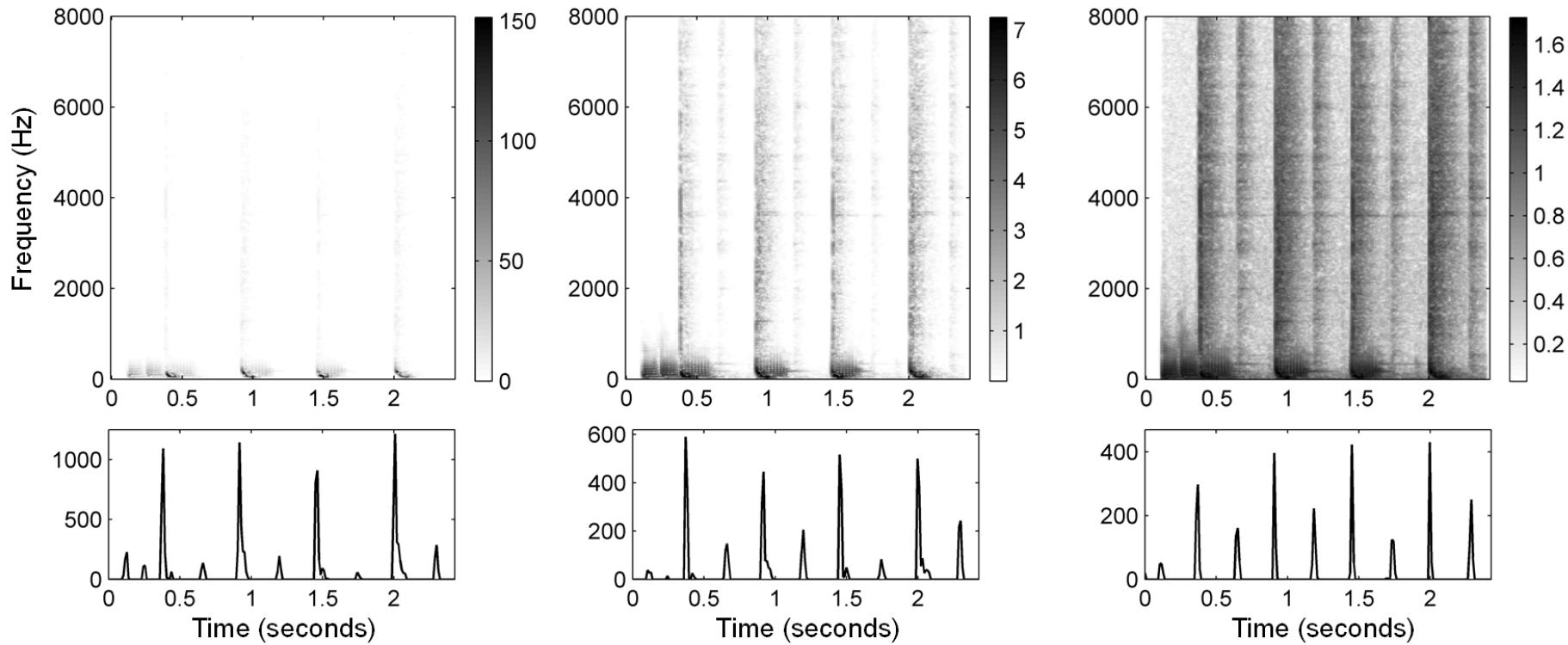
# 6.1 Onset Detection

Fig. 6.4



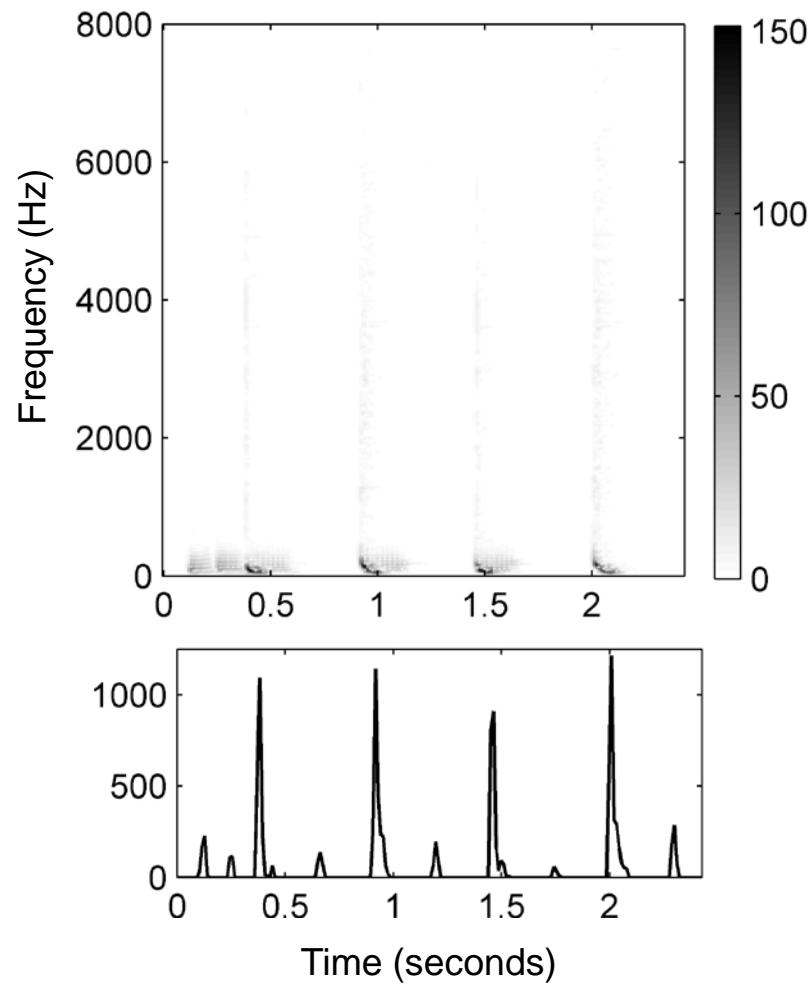
# 6.1 Onset Detection

Fig. 6.5



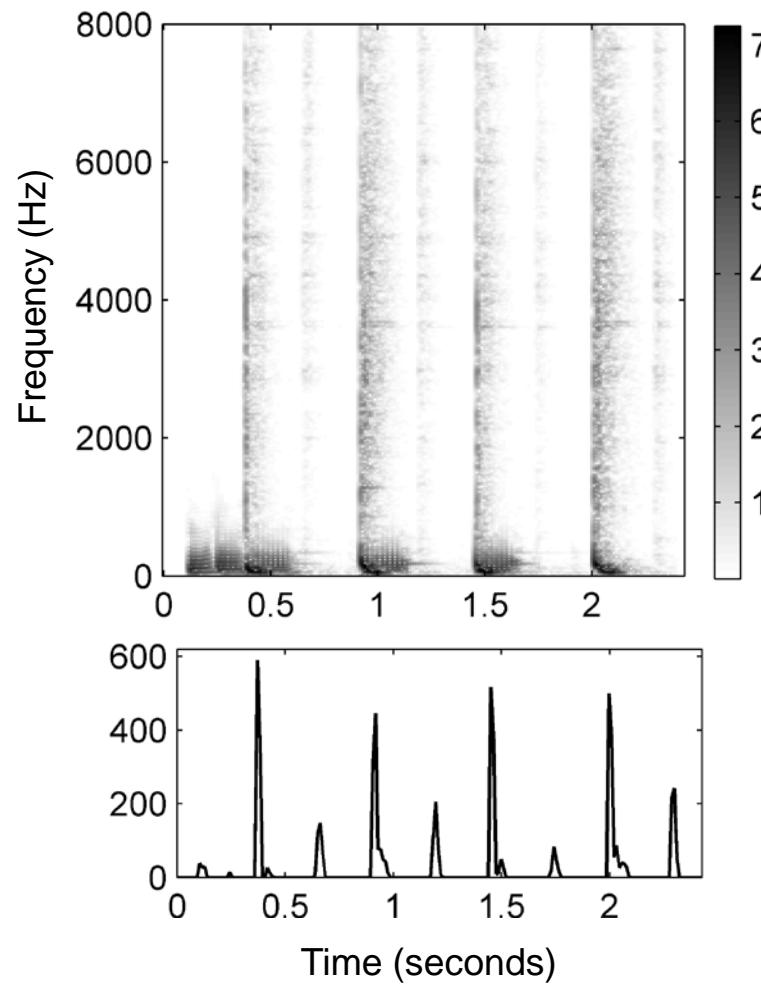
# 6.1 Onset Detection

Fig. 6.5



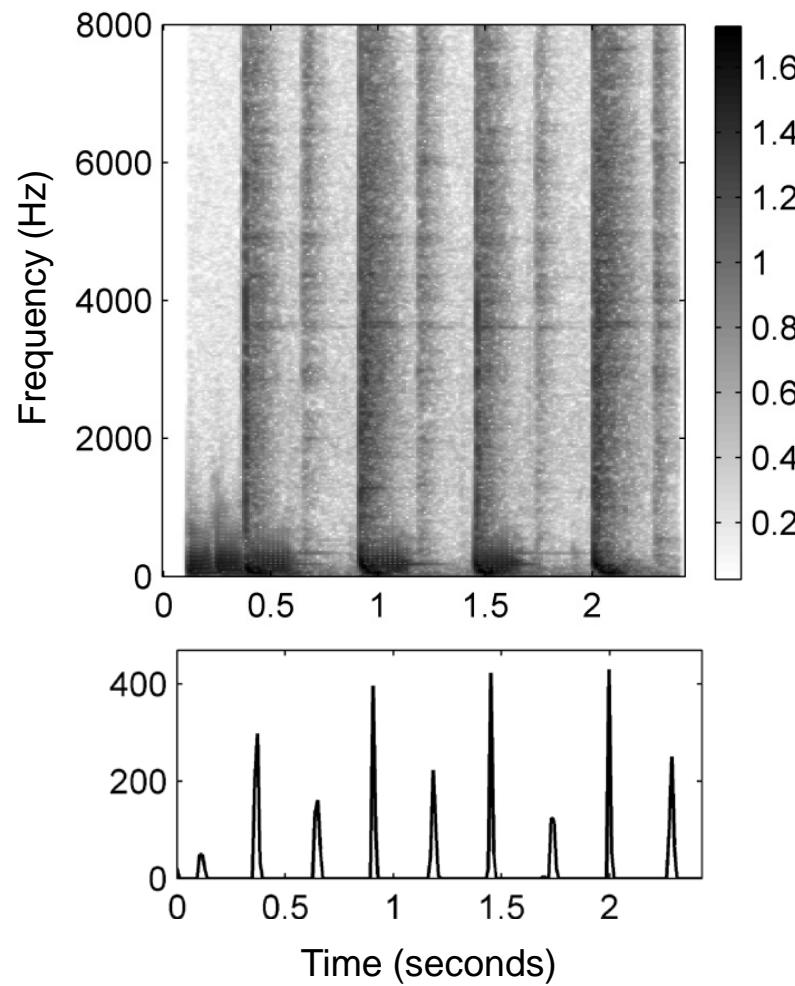
# 6.1 Onset Detection

Fig. 6.5



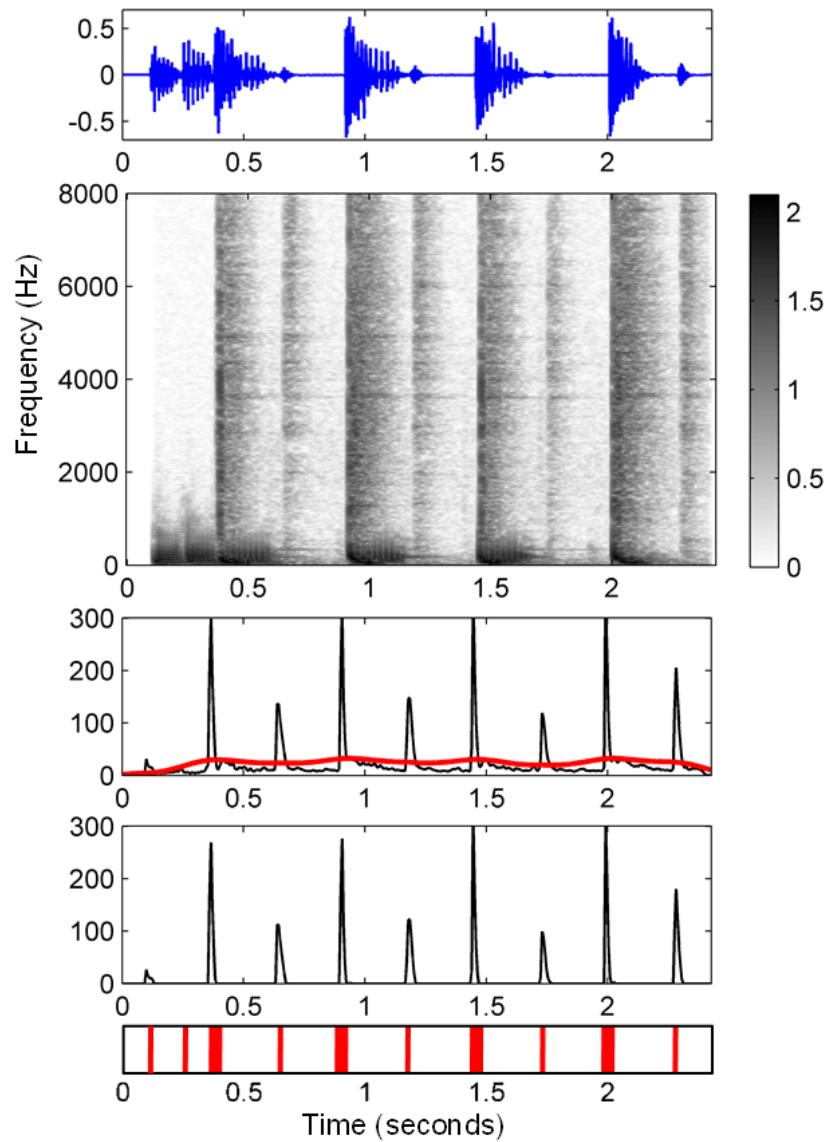
# 6.1 Onset Detection

Fig. 6.5



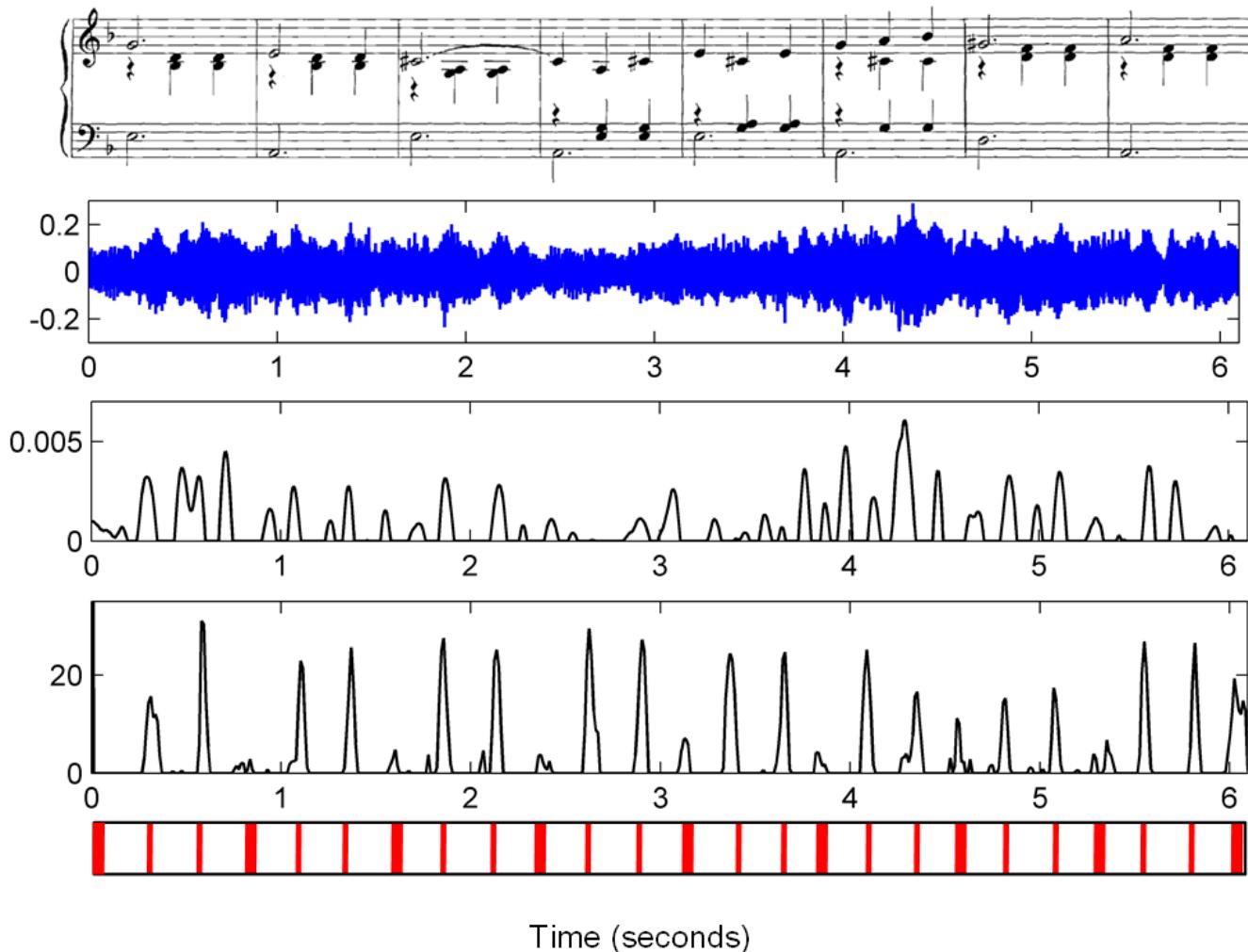
# 6.1 Onset Detection

Fig. 6.6



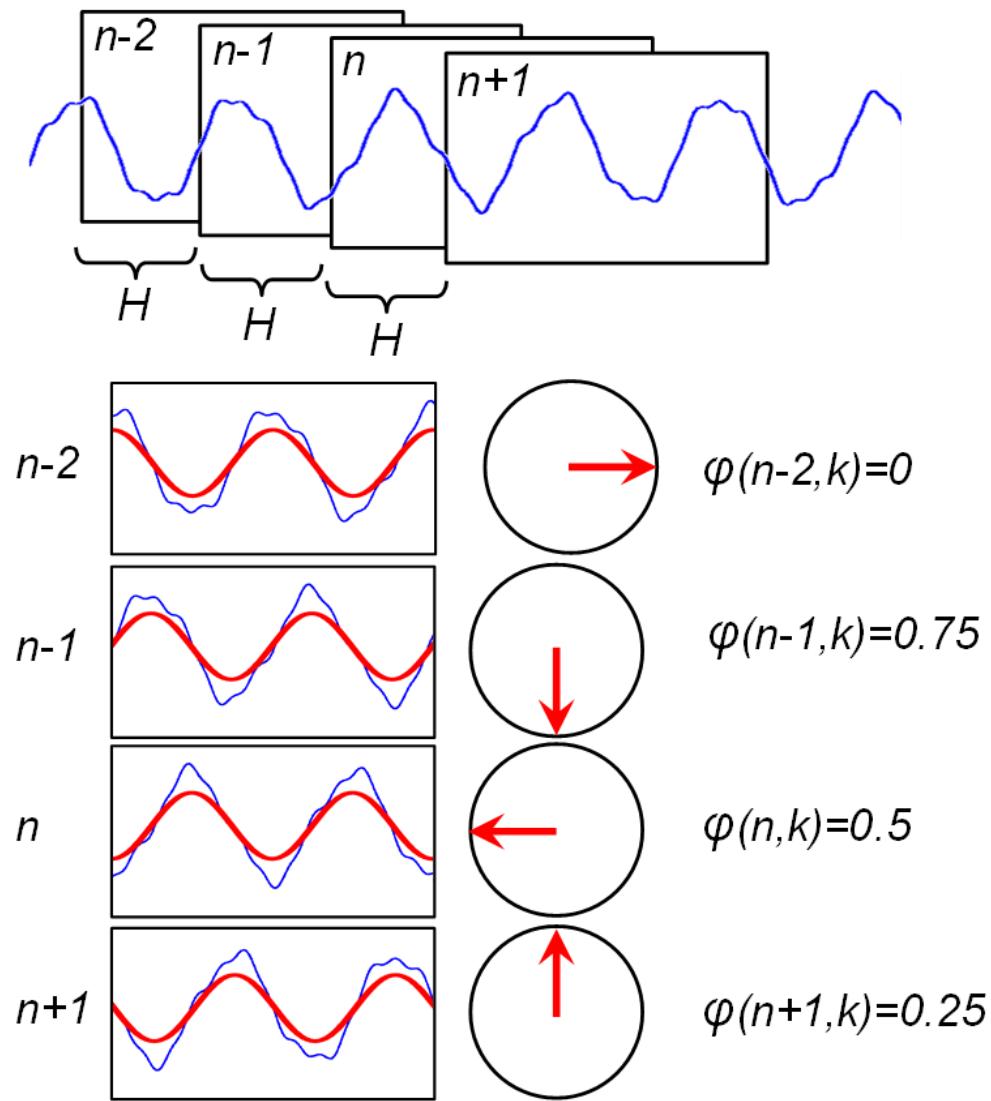
# 6.1 Onset Detection

Fig. 6.7



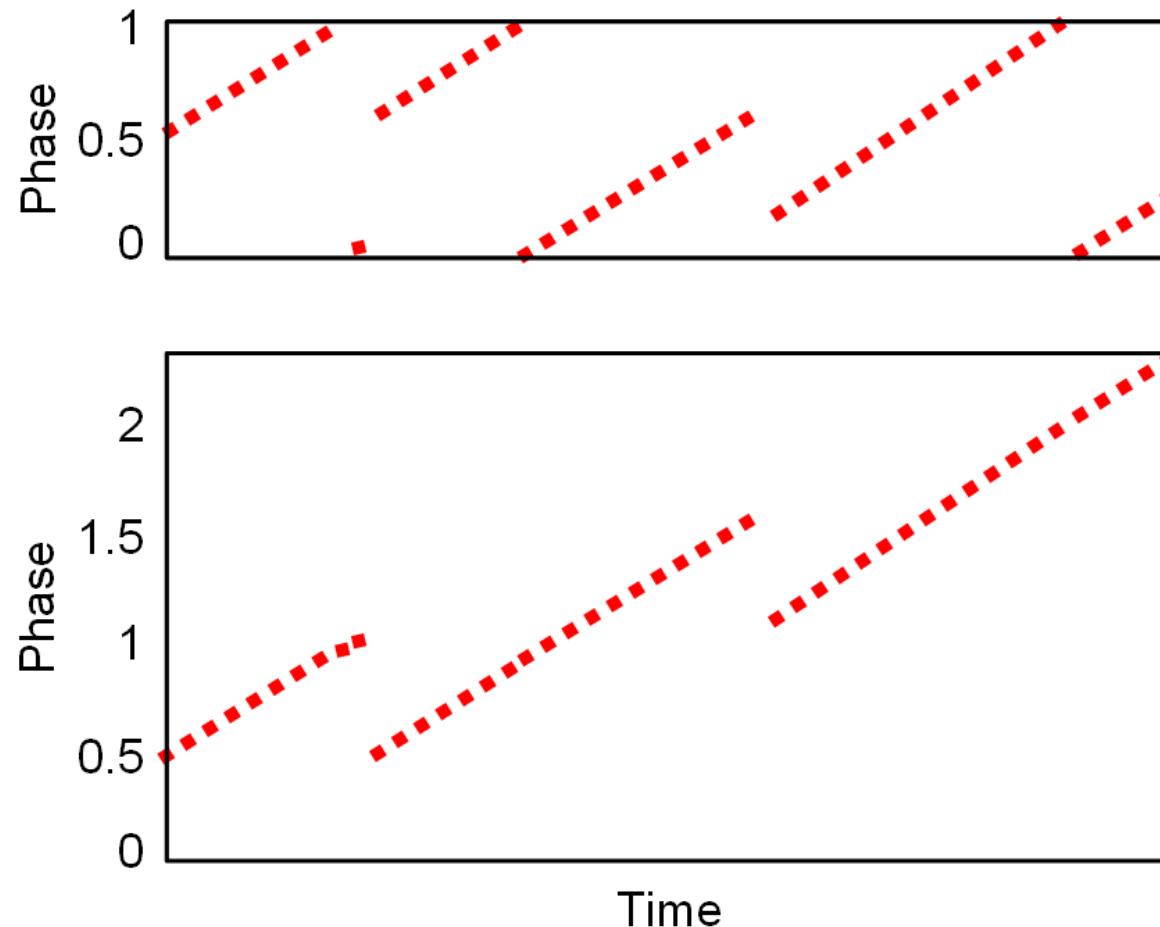
# 6.1 Onset Detection

Fig. 6.8



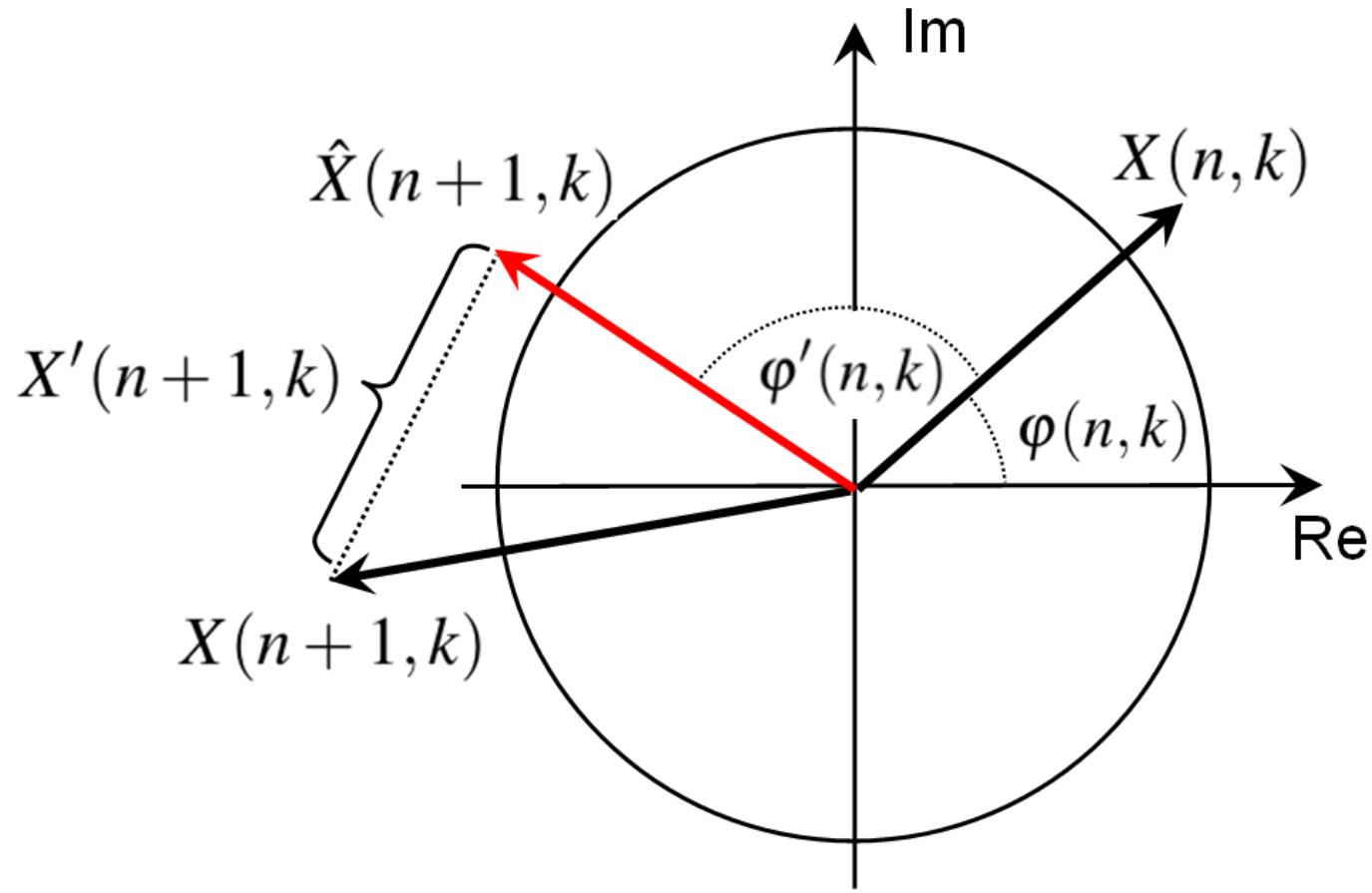
## 6.1 Onset Detection

Fig. 6.9



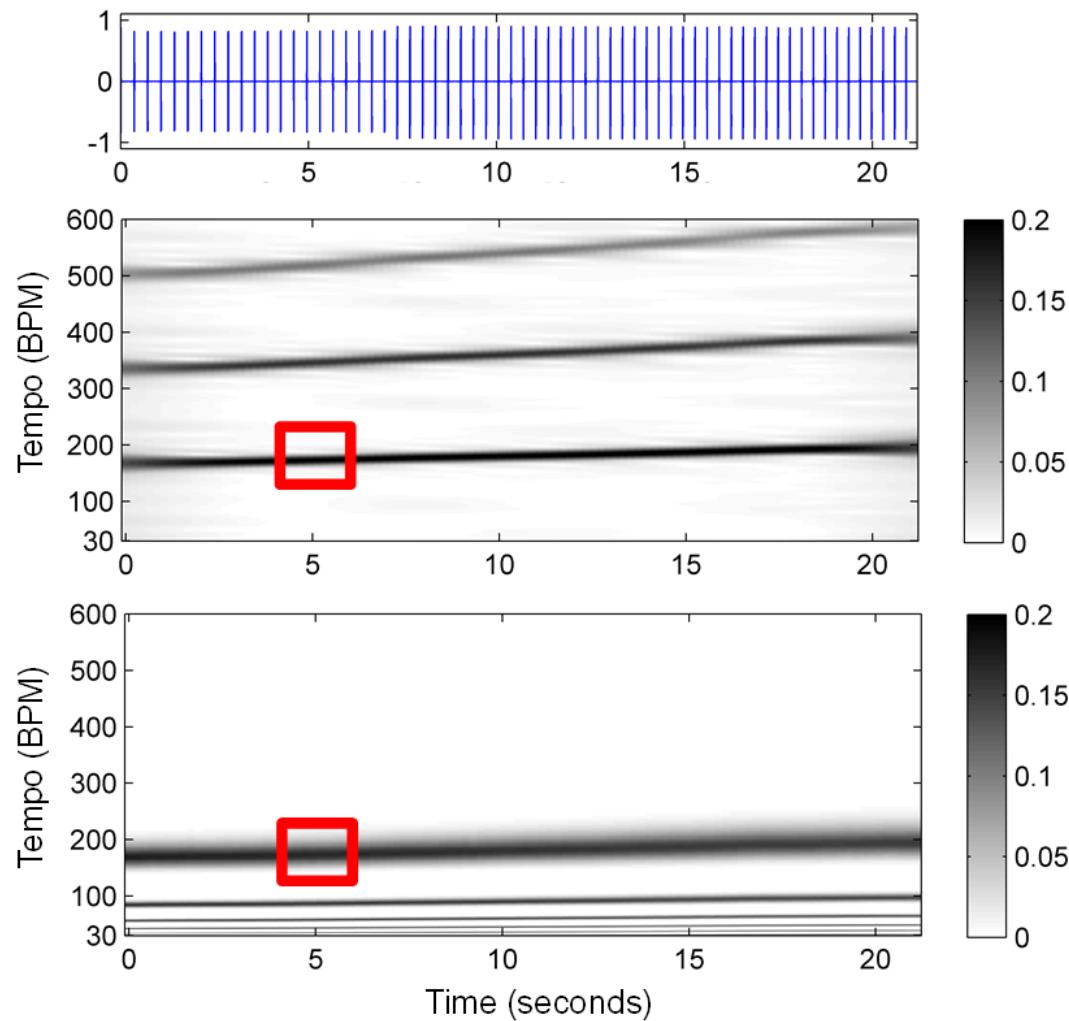
## 6.1 Onset Detection

Fig. 6.10



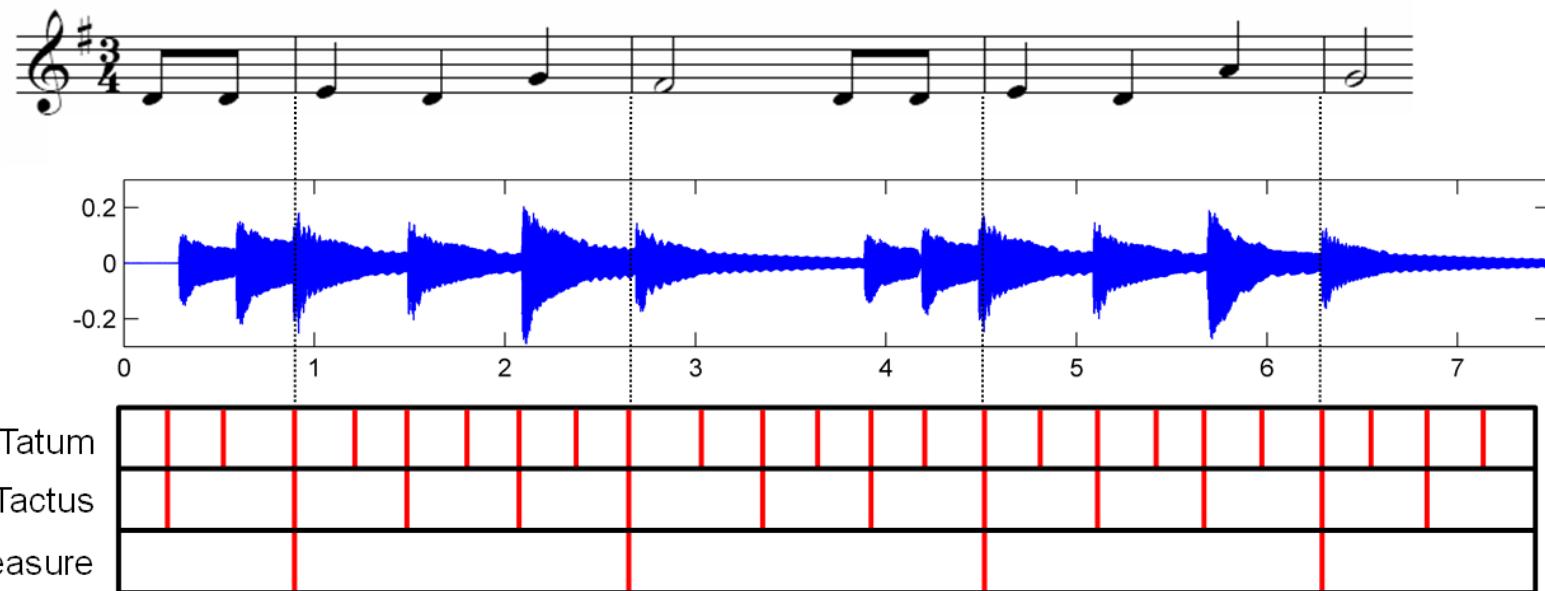
# 6.1 Onset Detection

Fig. 6.11



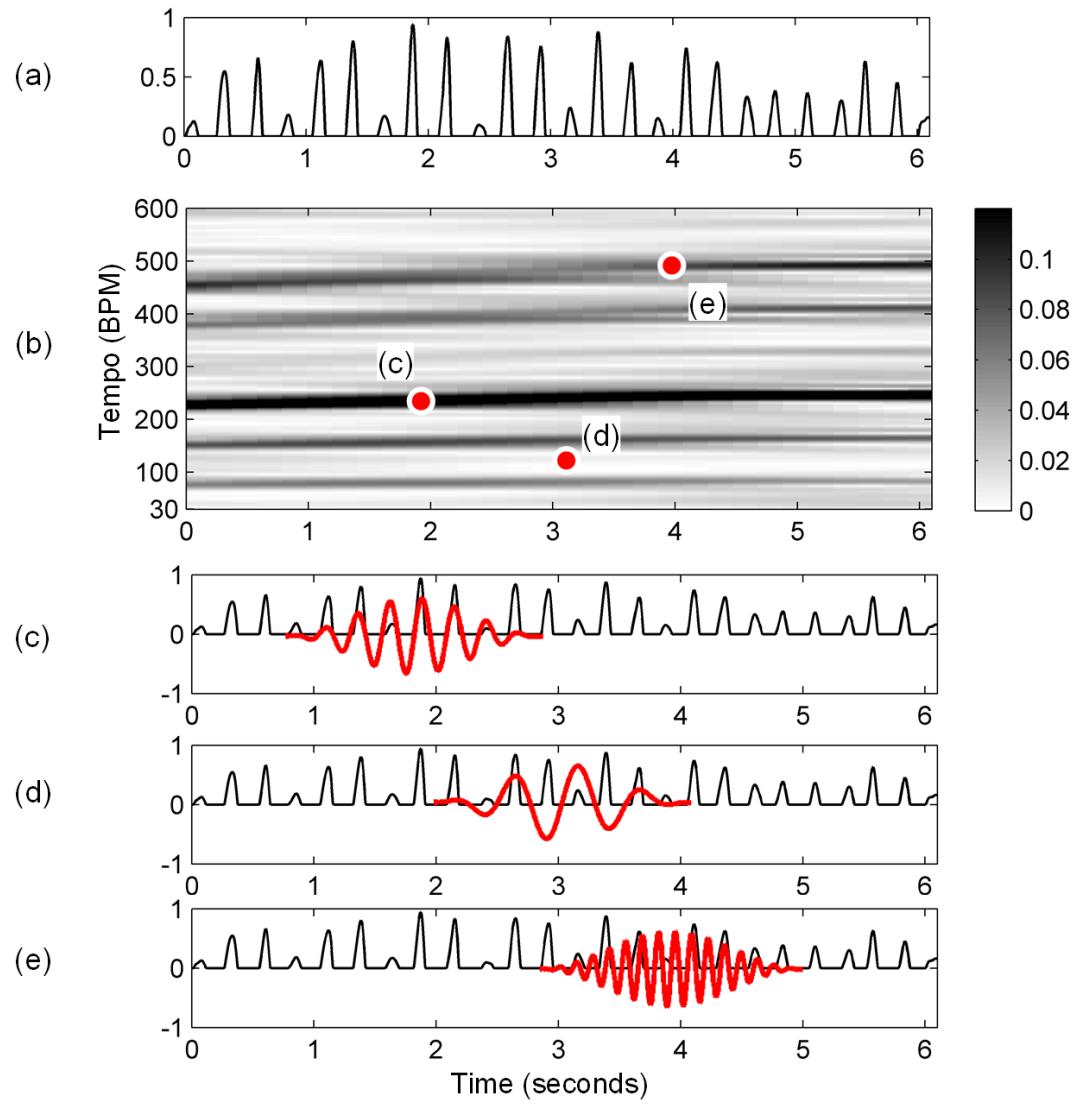
## 6.2 Tempo Analysis

Fig. 6.12



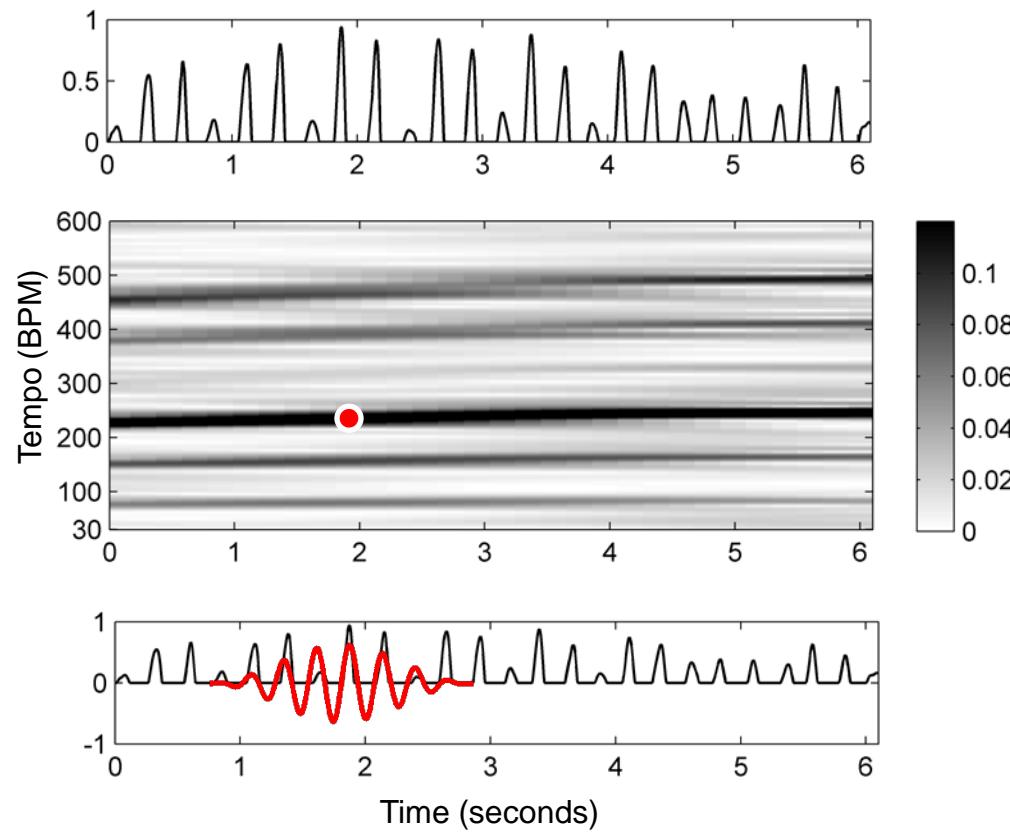
## 6.2 Tempo Analysis

Fig. 6.13



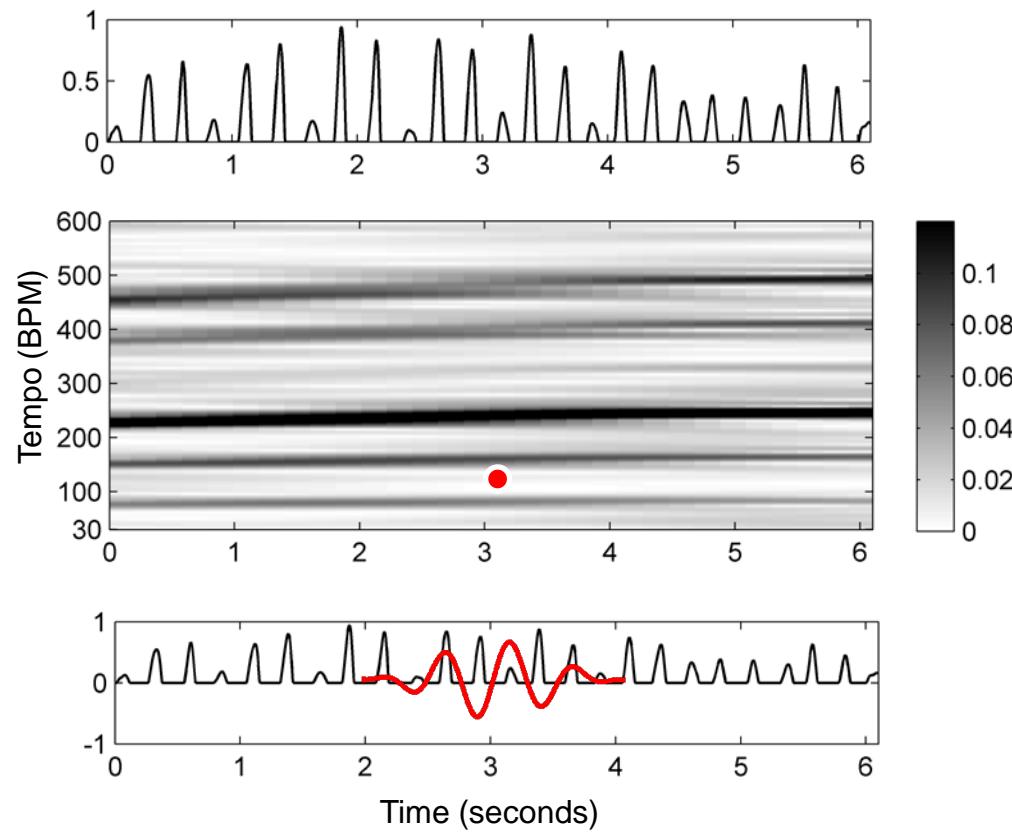
## 6.2 Tempo Analysis

Fig. 6.13



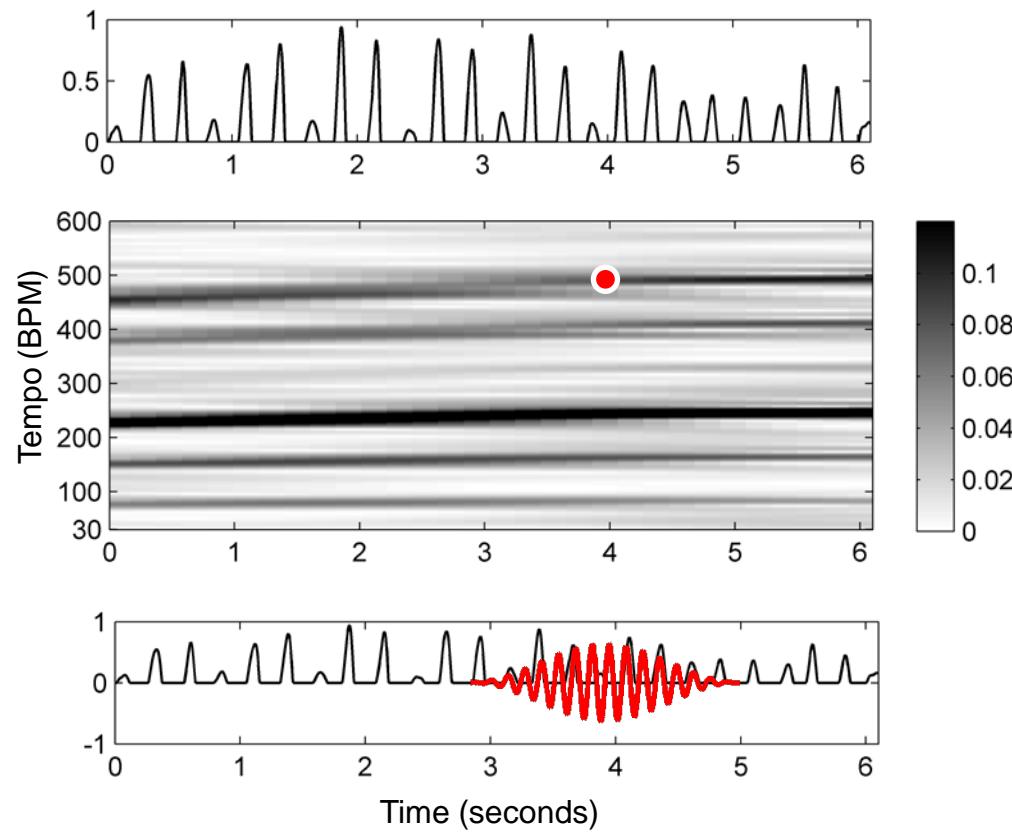
## 6.2 Tempo Analysis

Fig. 6.13



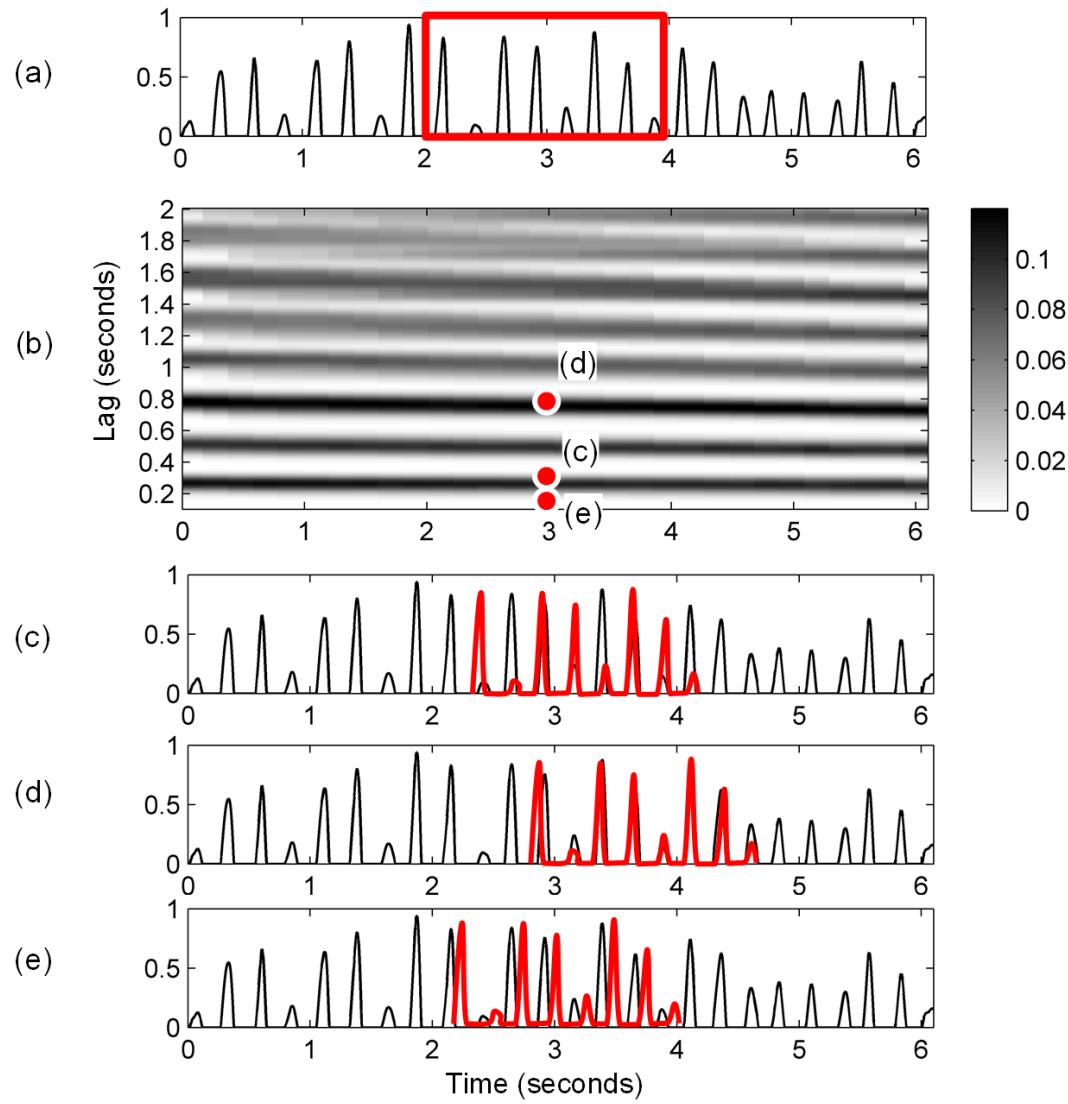
## 6.2 Tempo Analysis

Fig. 6.13



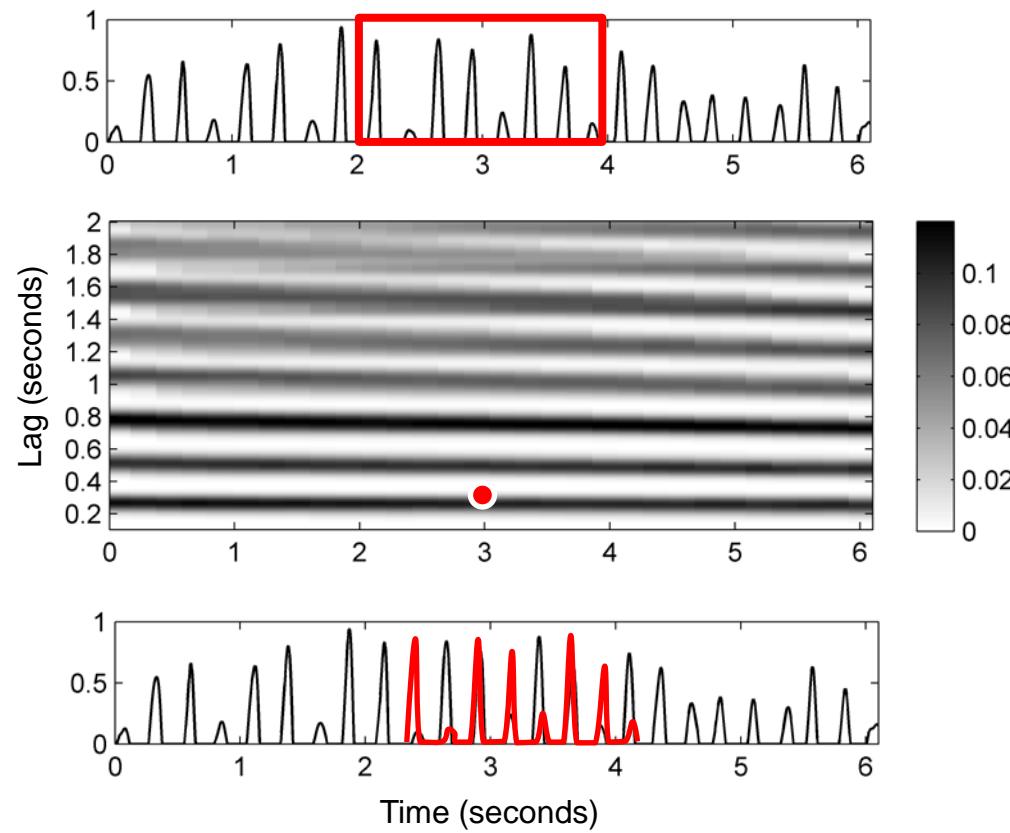
## 6.2 Tempo Analysis

Fig. 6.13



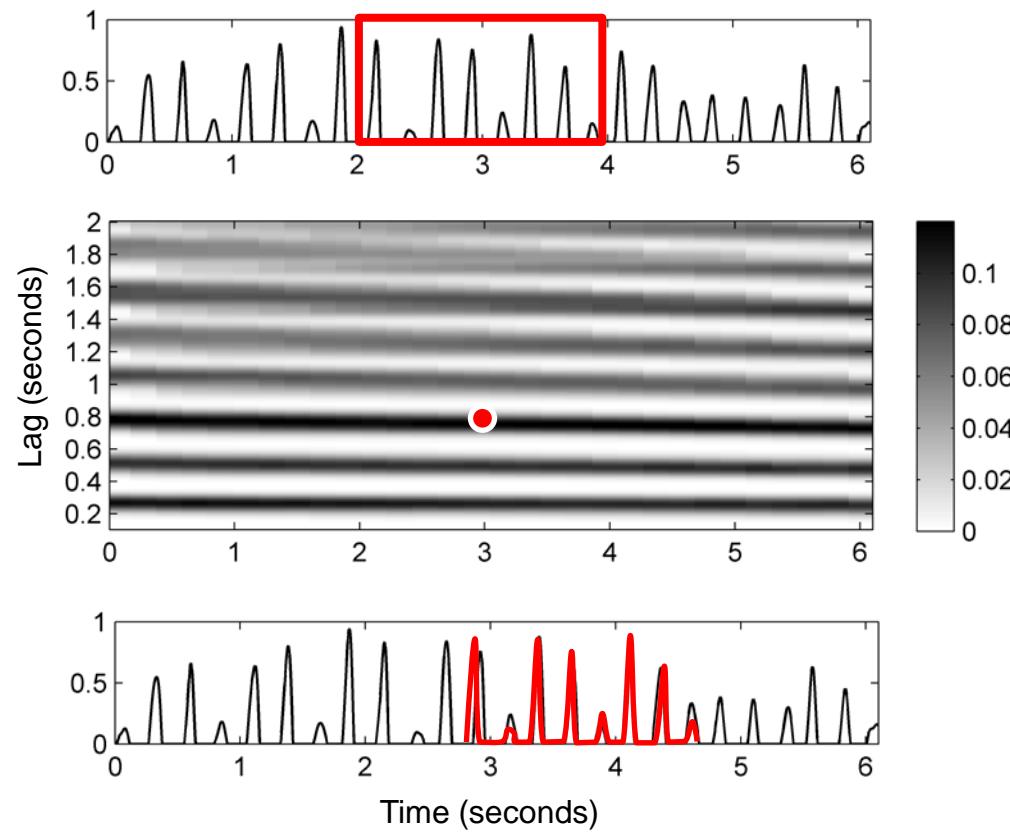
## 6.2 Tempo Analysis

Fig. 6.14



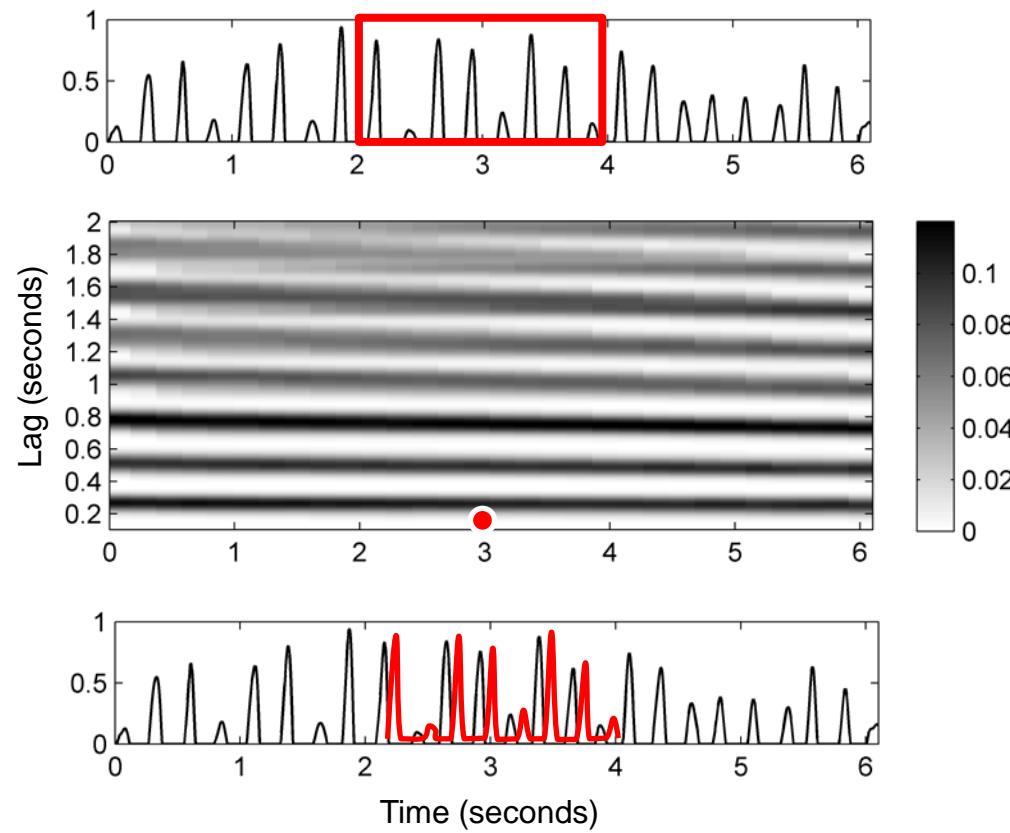
## 6.2 Tempo Analysis

Fig. 6.14



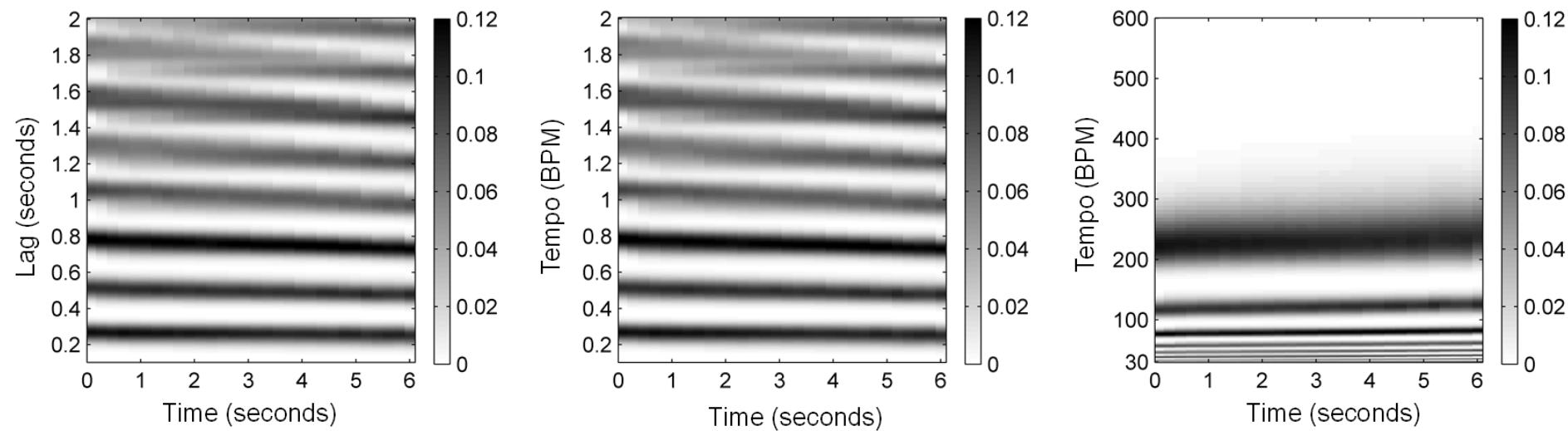
## 6.2 Tempo Analysis

Fig. 6.14



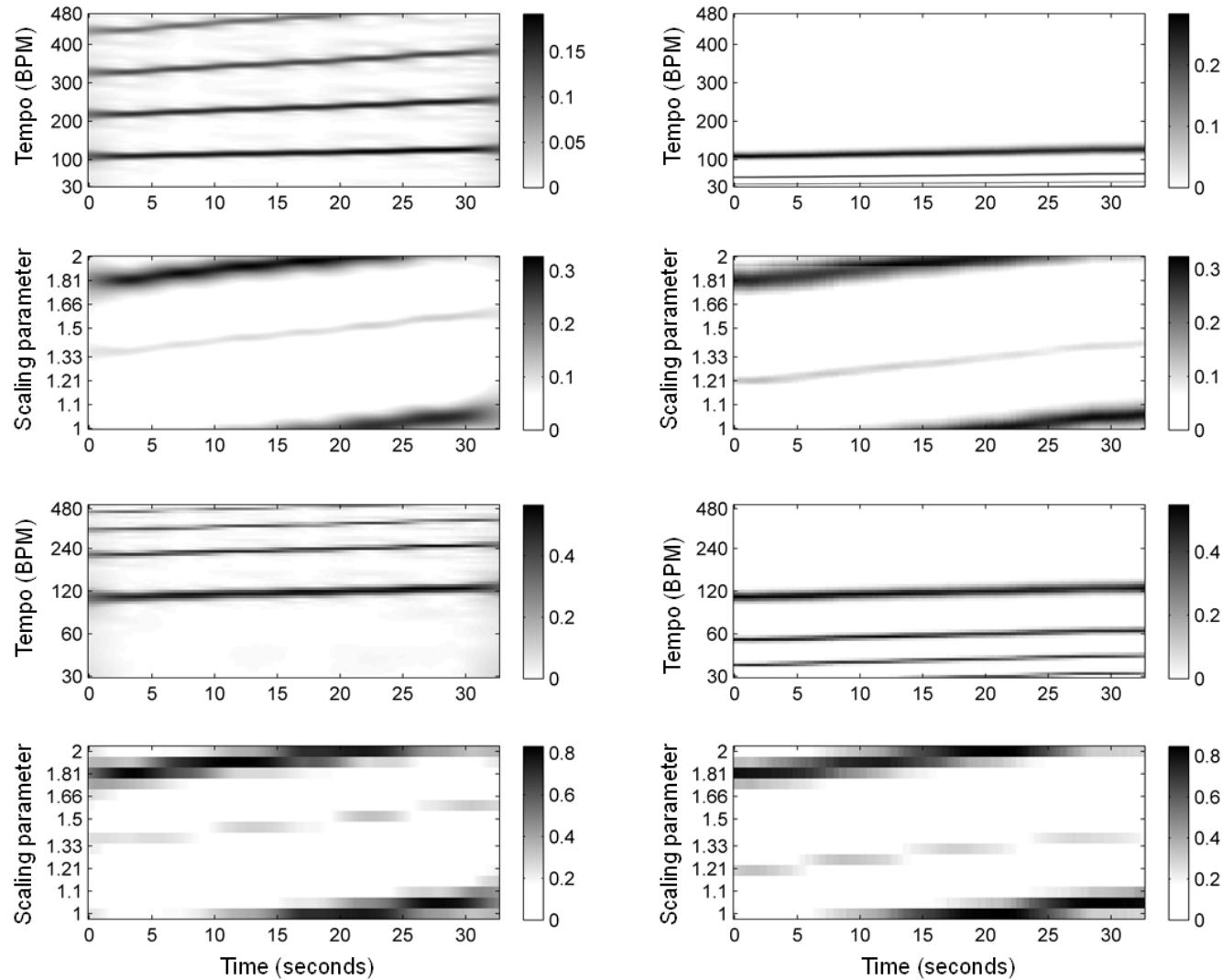
## 6.2 Tempo Analysis

Fig. 6.15



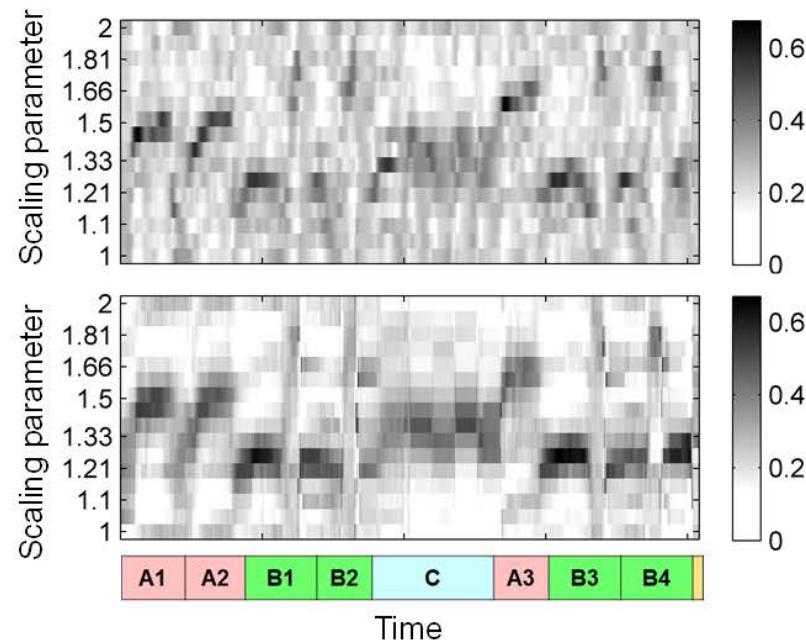
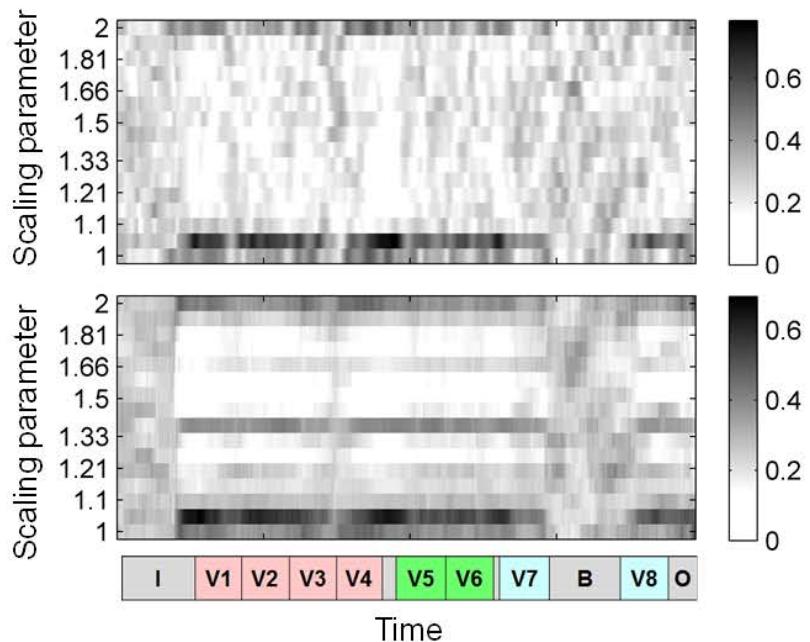
## 6.2 Tempo Analysis

Fig. 6.16



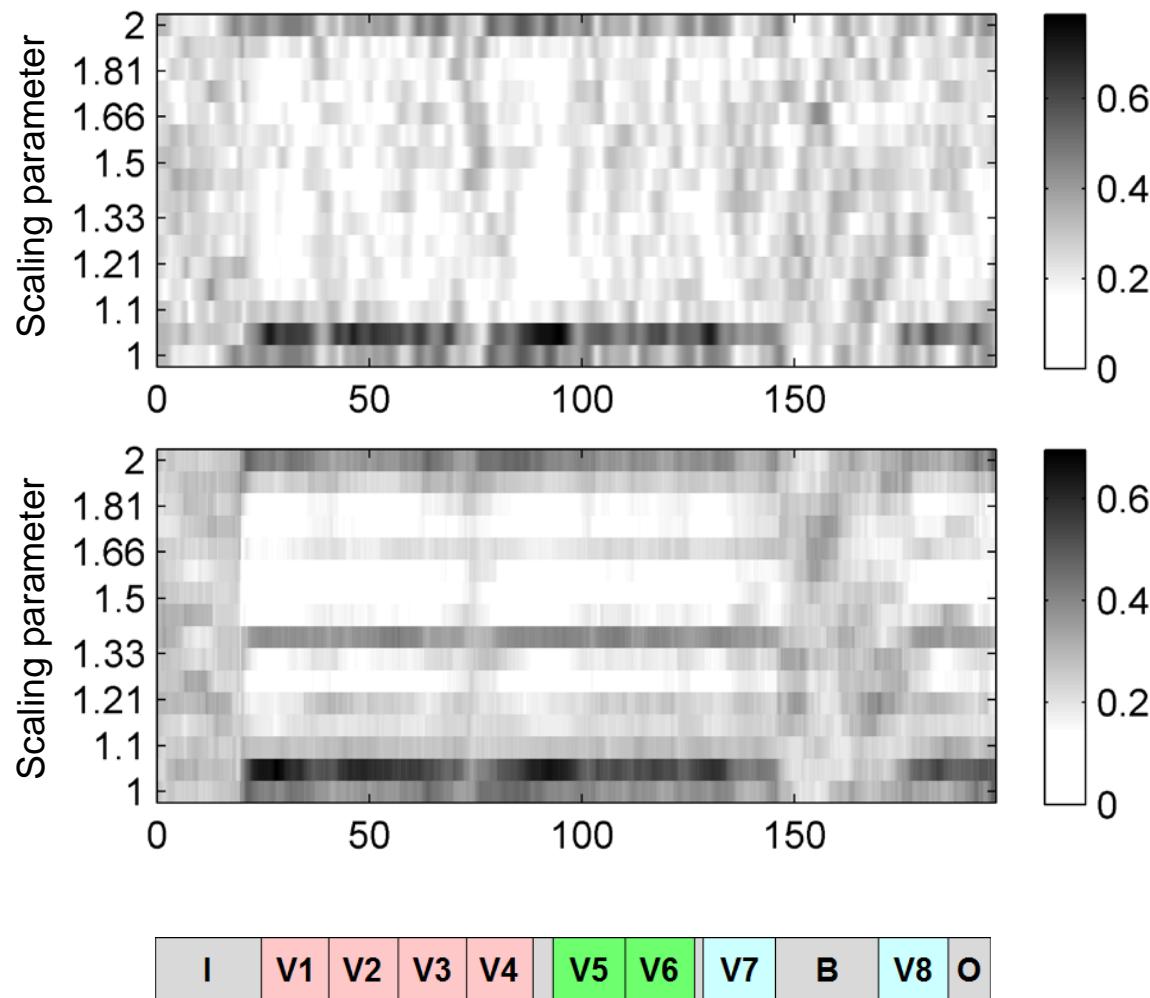
## 6.2 Tempo Analysis

Fig. 6.17



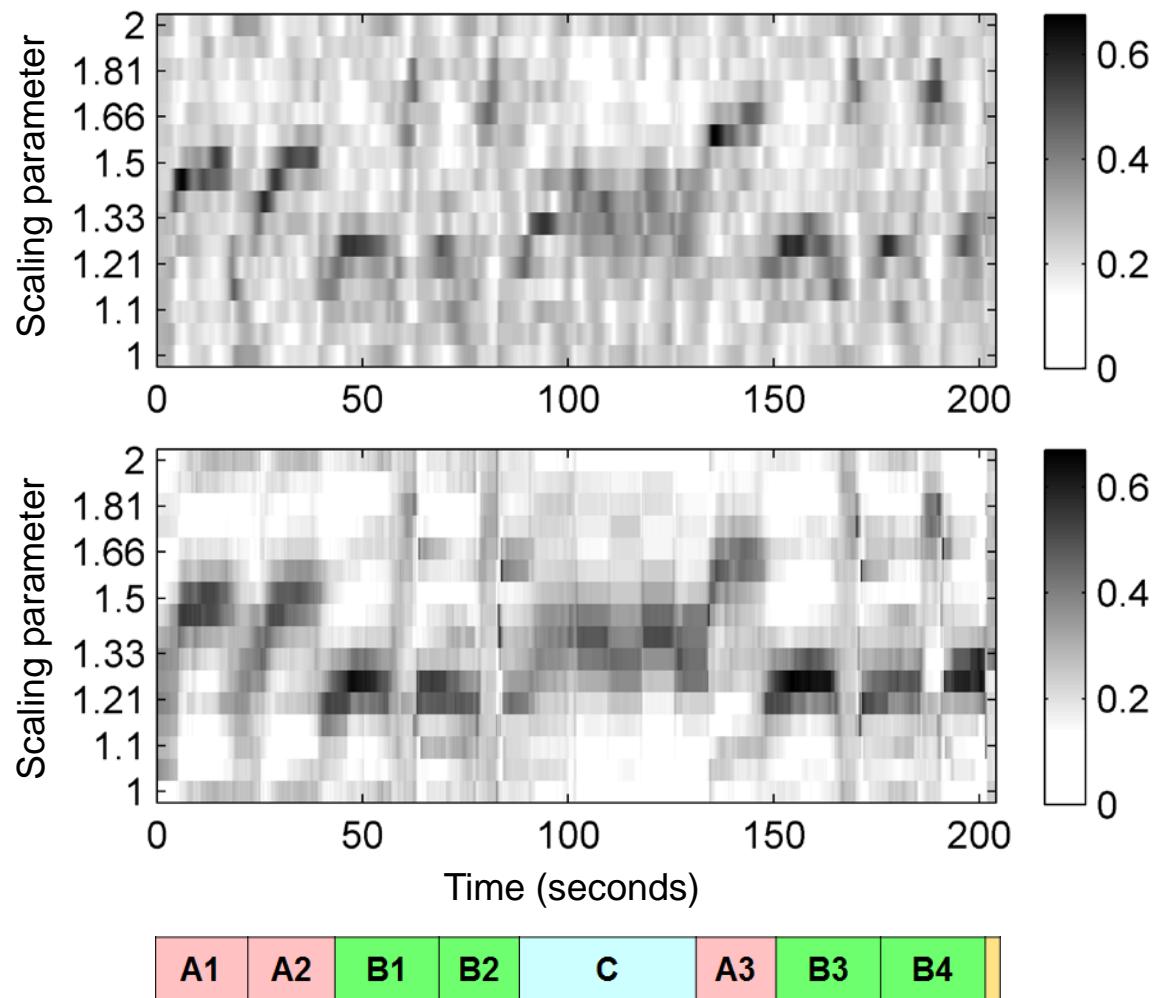
## 6.2 Tempo Analysis

Fig. 6.17



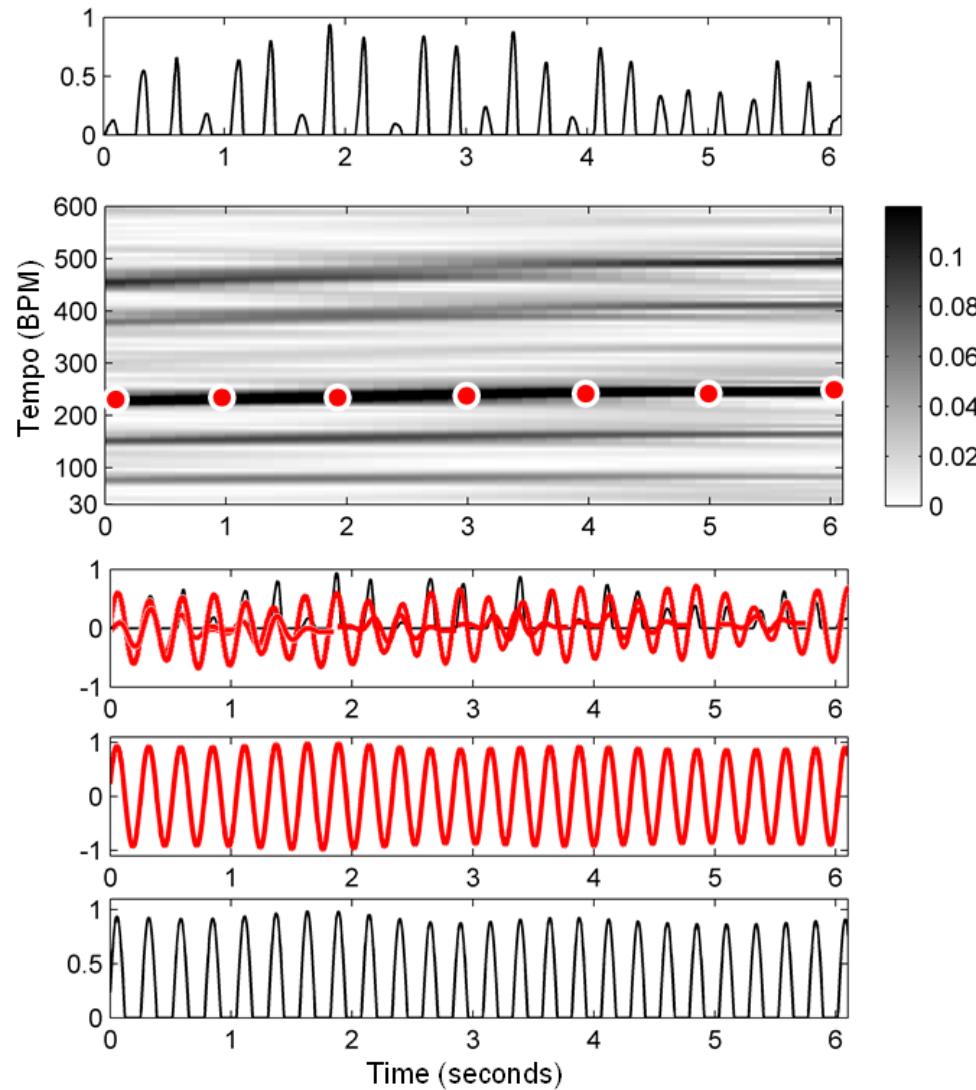
## 6.2 Tempo Analysis

Fig. 6.17



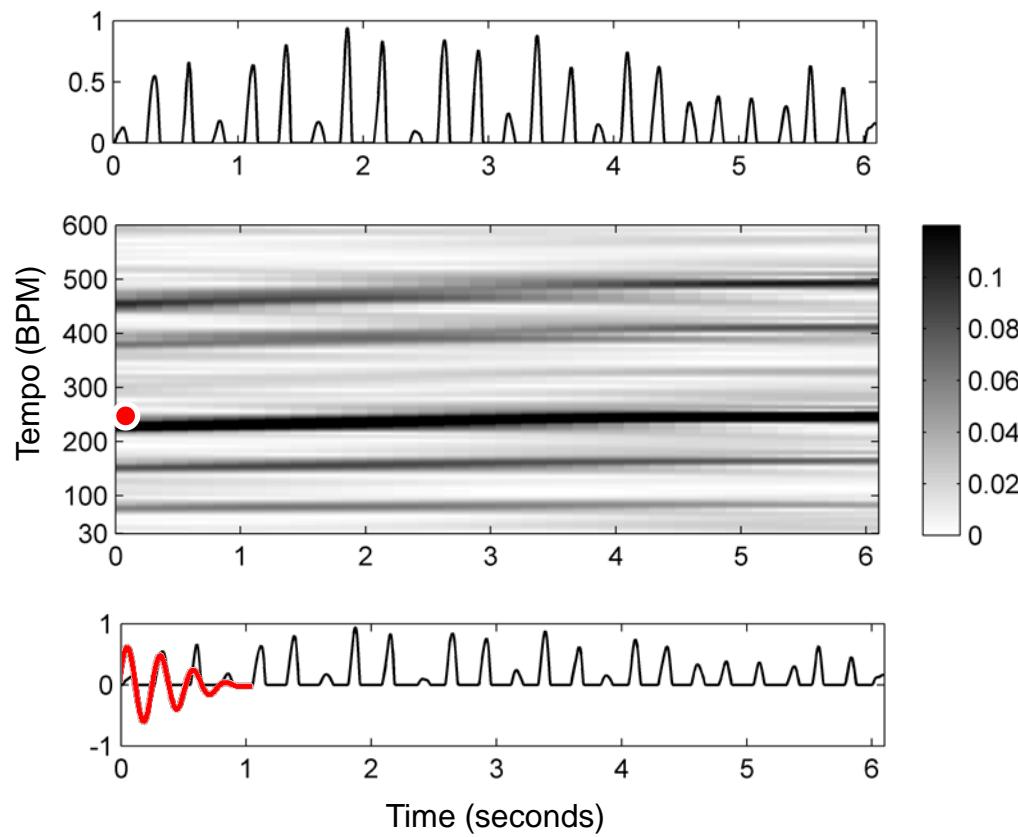
## 6.3 Beat and Pulse Tracking

Fig. 6.18



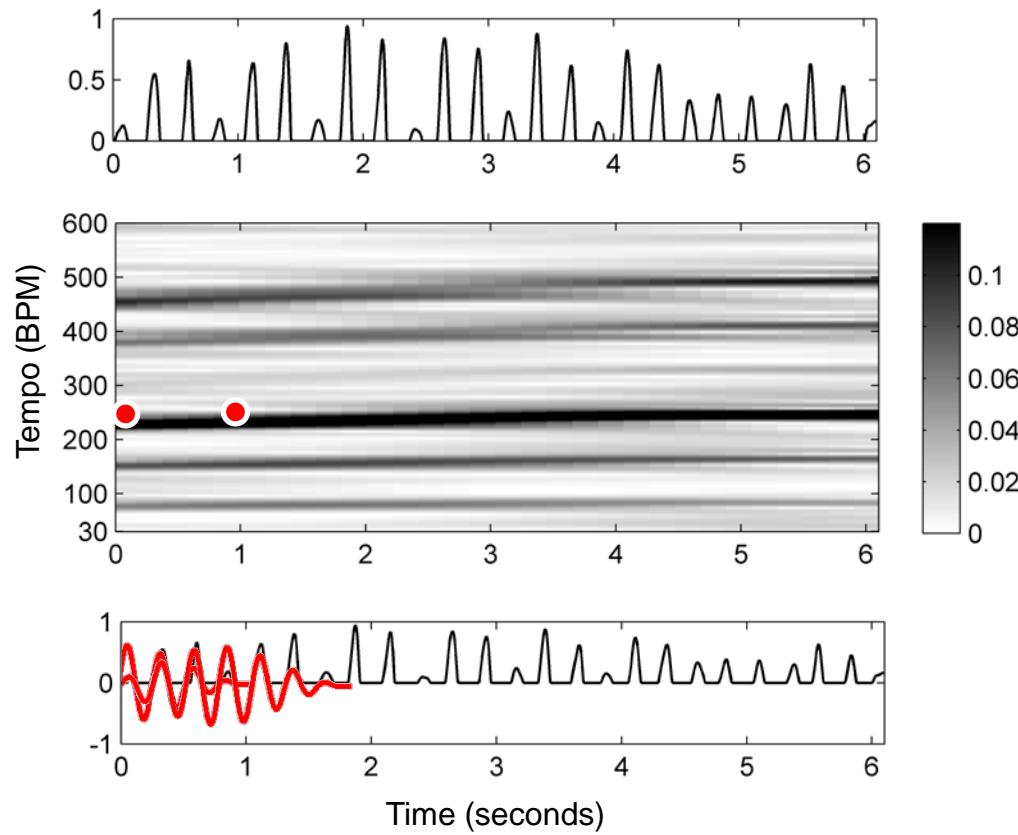
## 6.3 Beat and Pulse Tracking

Fig. 6.18



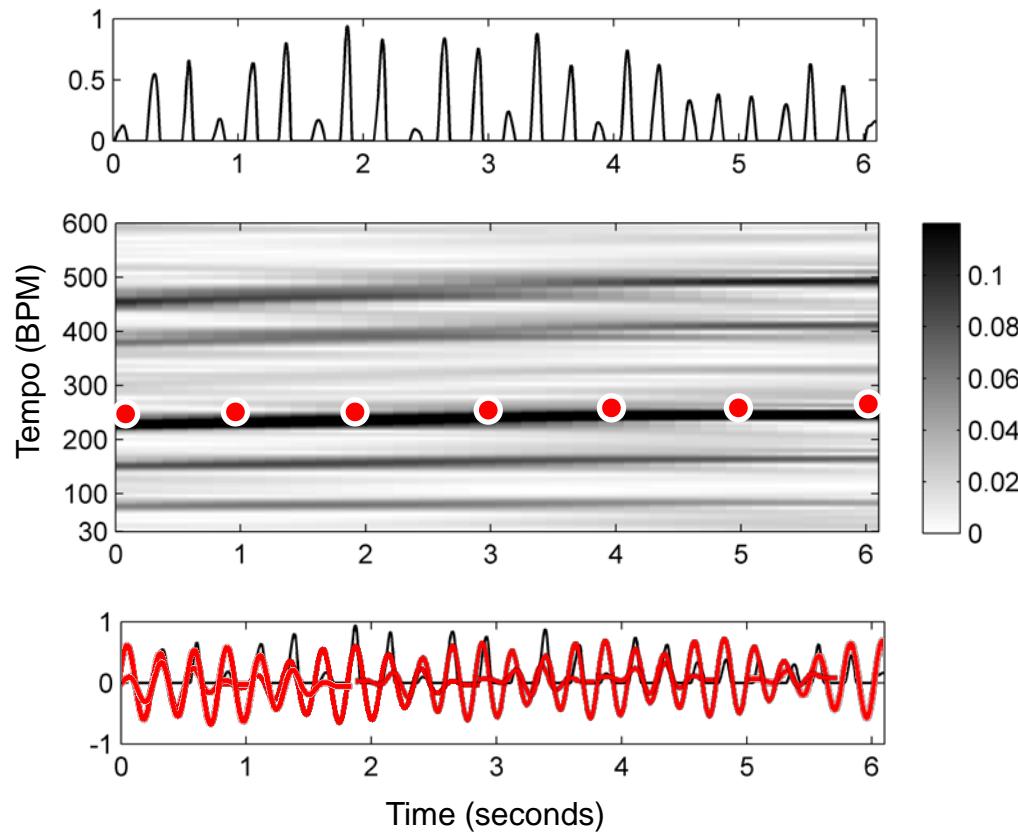
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Fig. 6.18



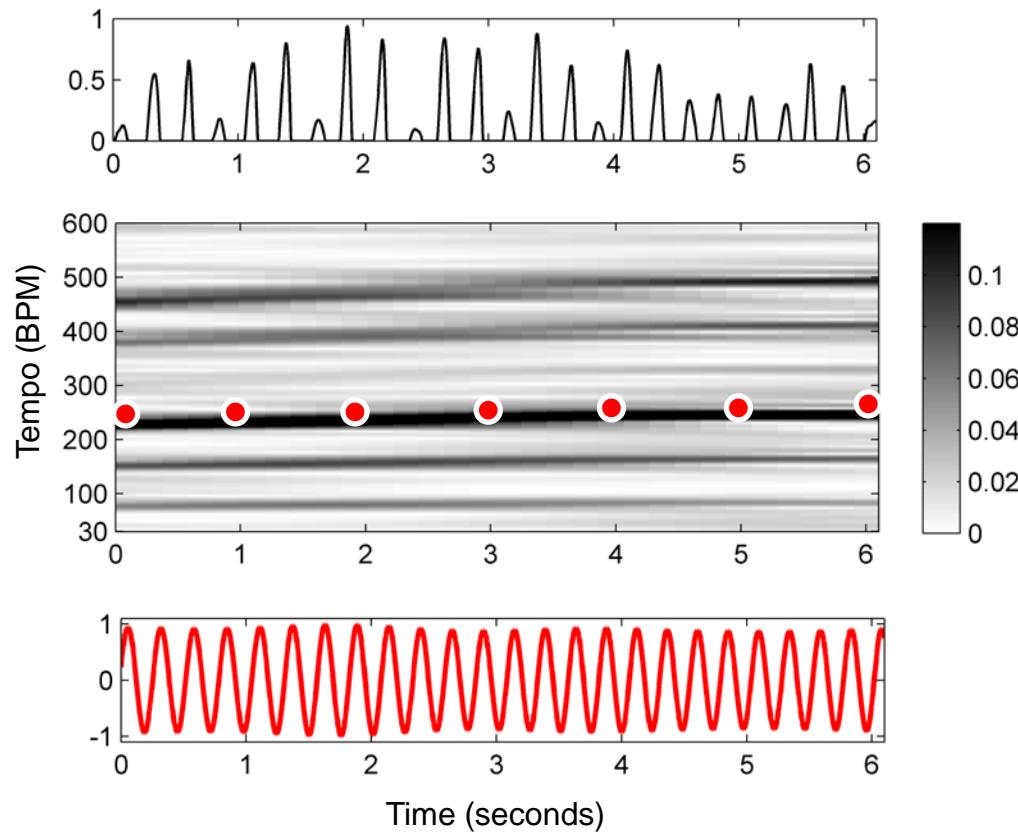
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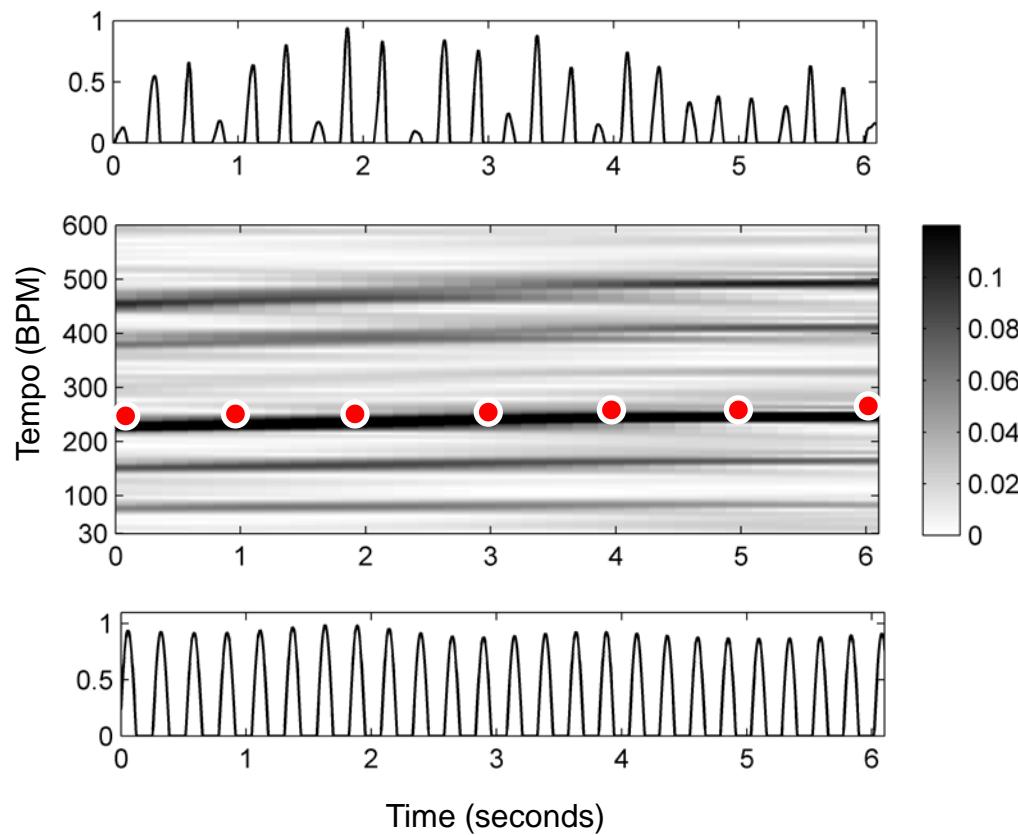
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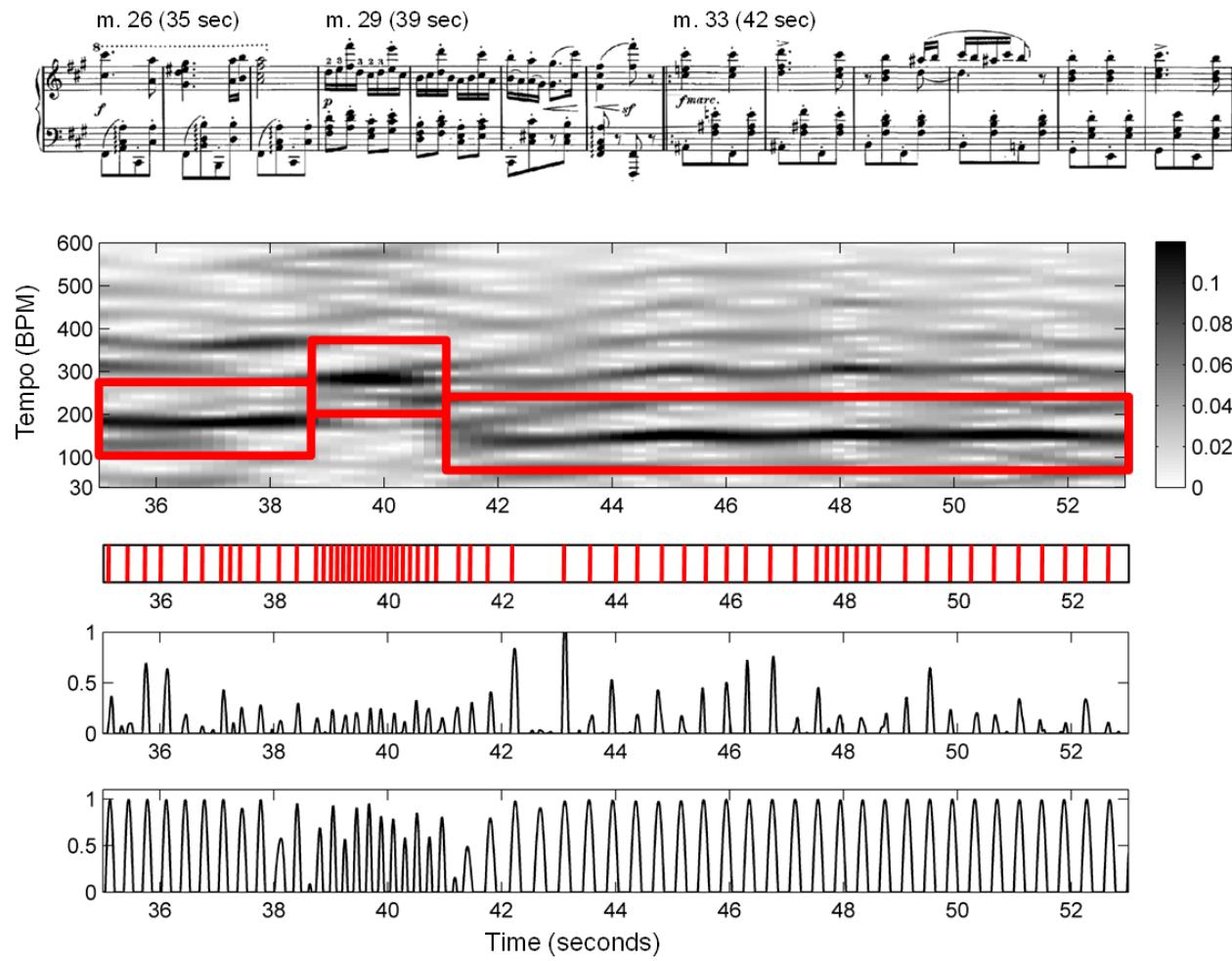
## 6.3 Beat and Pulse Tracking

Fig. 6.18



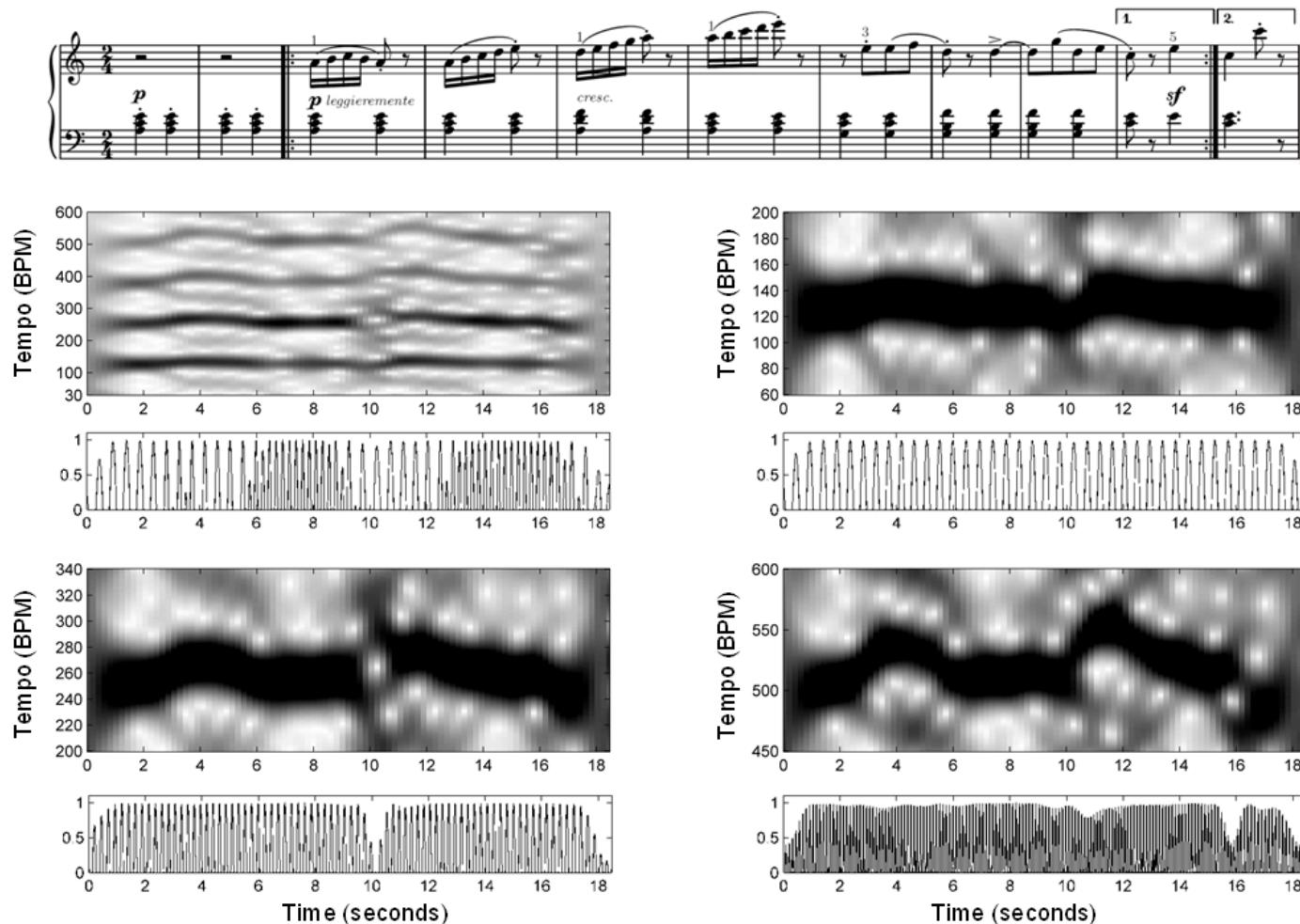
# 6.3 Beat and Pulse Tracking

Fig. 6.19



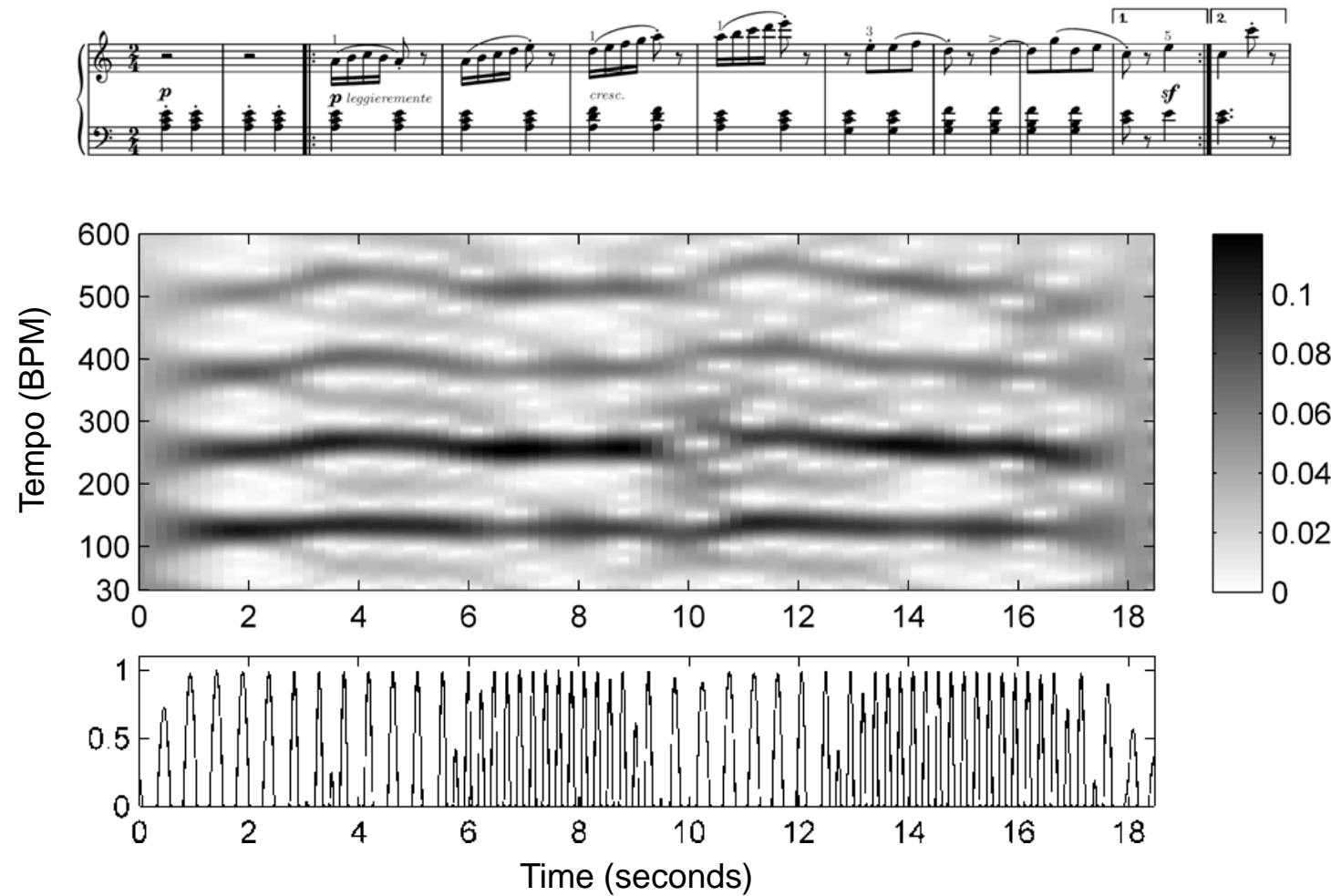
# 6.3 Beat and Pulse Tracking

Fig. 6.20



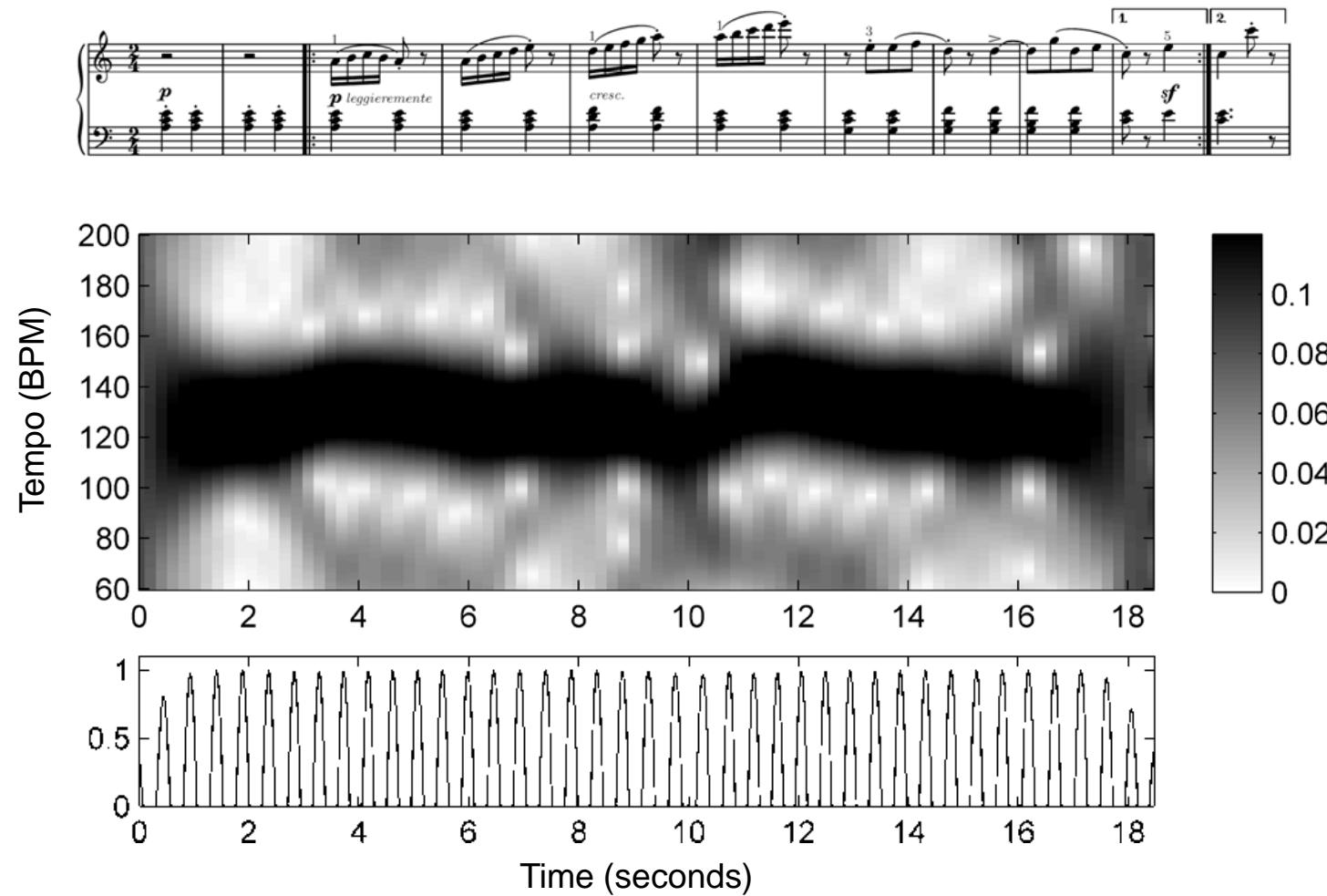
## 6.3 Beat and Pulse Tracking

Fig. 6.20



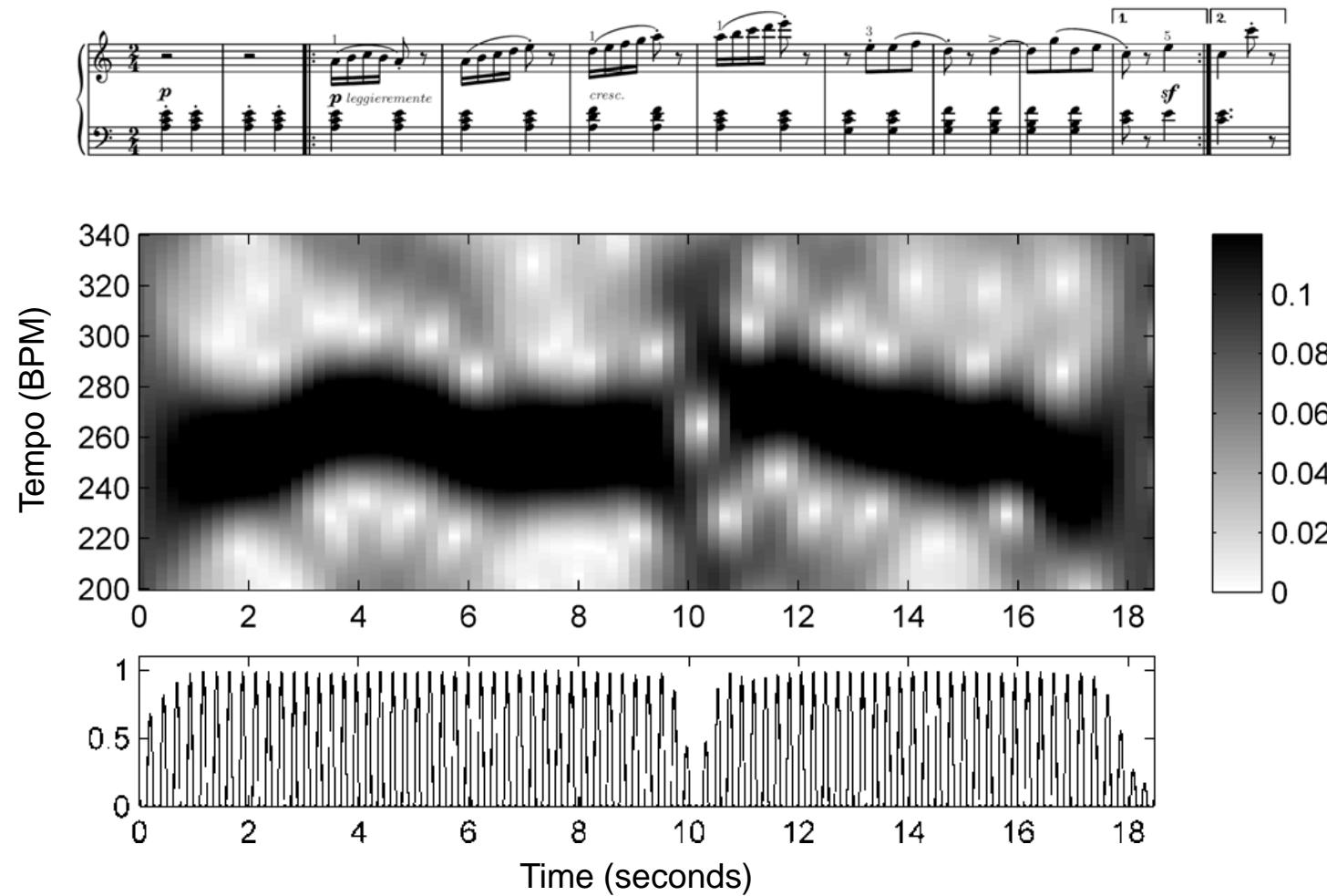
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Fig. 6.20



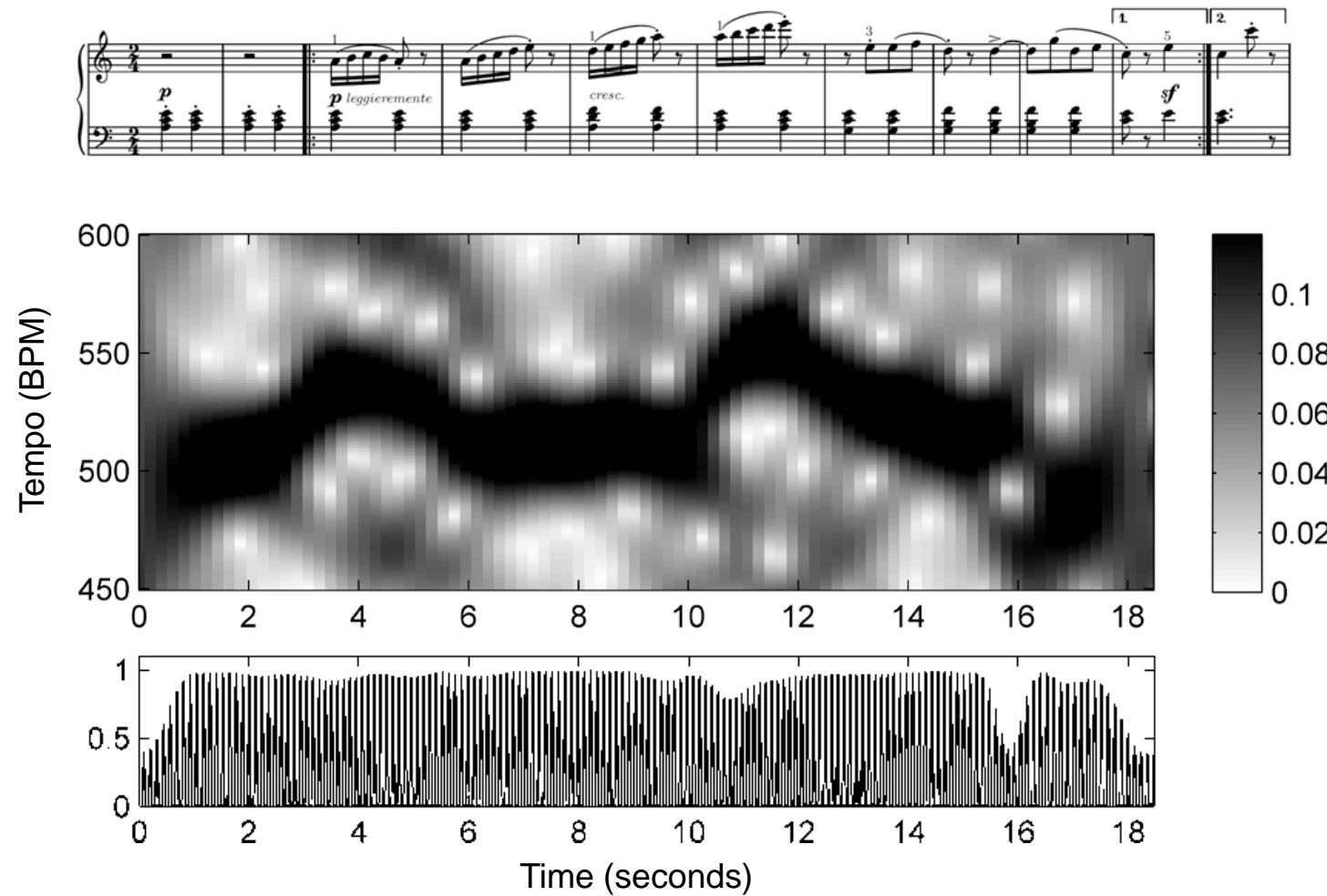
## 6.3 Beat and Pulse Tracking

Fig. 6.20



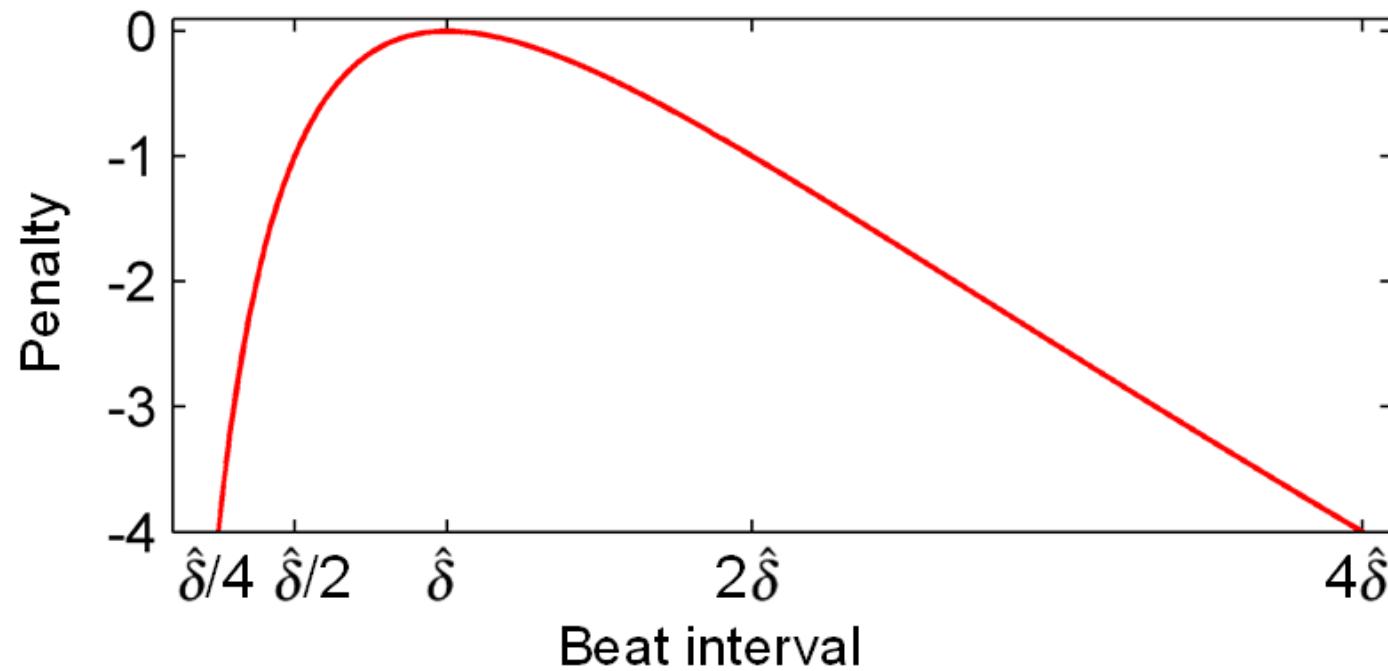
## 6.3 Beat and Pulse Tracking

Fig. 6.20



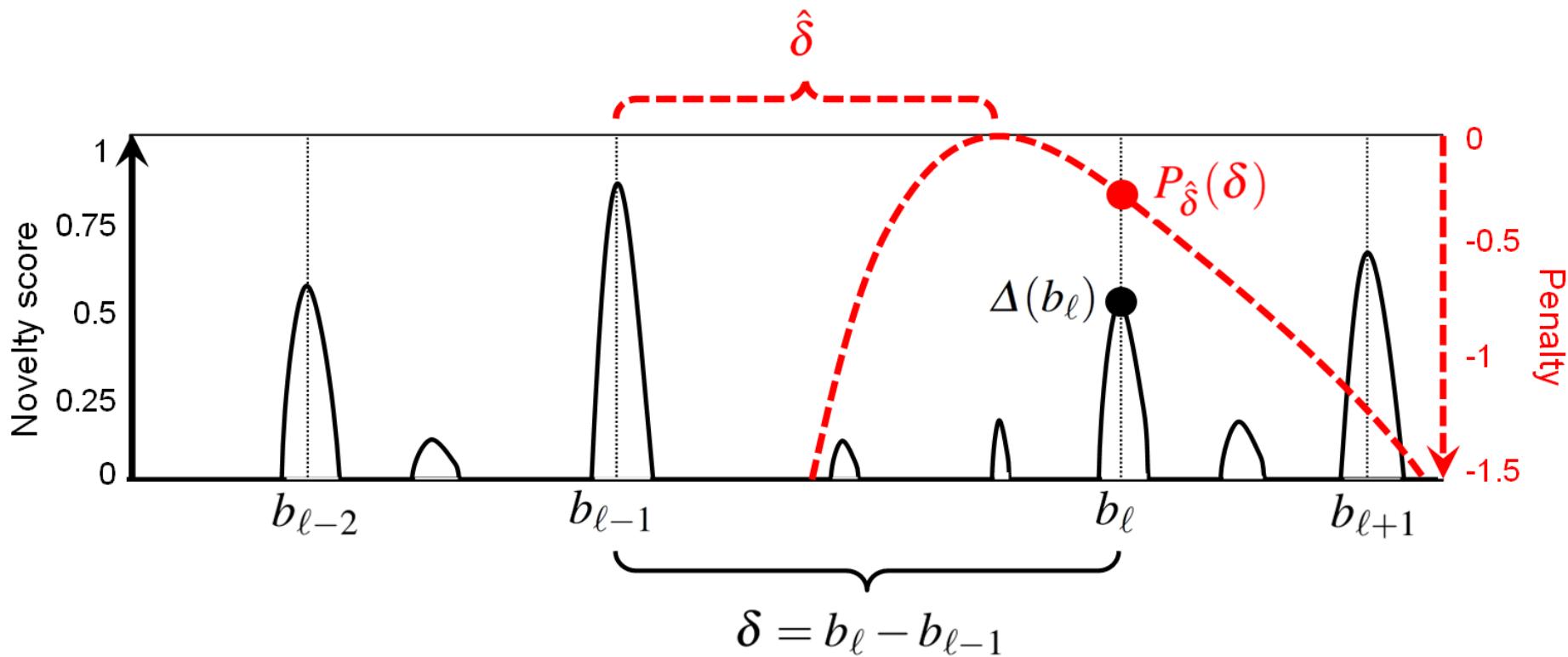
## 6.3 Beat and Pulse Tracking

Fig. 6.21



## 6.3 Beat and Pulse Tracking

Fig. 6.22



# 6.3 Beat and Pulse Tracking

Table 6.1

## Algorithm: OPTIMAL BEAT SEQUENCE

**Input:** Novelty function  $\Delta : [1 : N] \rightarrow \mathbb{R}$

Estimate  $\hat{\delta}$  for the beat period (given in samples)

Weight parameter  $\lambda \in \mathbb{R}$

**Output:** Optimal beat sequence  $B^* = (b_1, b_2, \dots, b_L)$

**Procedure:** Initialize  $\mathbf{D}(0) = 0$  and  $\mathbf{P}(0) = 0$ . Then compute in a loop for  $n = 1, \dots, N$ :

$$\mathbf{D}(n) = \Delta(n) + \max \left\{ 0, \max_{m \in [1:n-1]} \{ \mathbf{D}(m) + \lambda P_{\hat{\delta}}(n-m) \} \right\}$$

If  $\mathbf{D}(n) = \Delta(n)$  then set  $\mathbf{P}(n) = 0$ ,

otherwise set  $\mathbf{P}(n) = \operatorname{argmax}_{m \in [1:n-1]} \{ \mathbf{D}(m) + \lambda P_{\hat{\delta}}(n-m) \}$

Set  $\ell = 1$  and  $a_\ell = \operatorname{argmax}_{n \in [0:N]} \mathbf{D}(n)$ . Then repeat the following steps until  $\mathbf{P}(a_\ell) = 0$ :

Increase  $\ell$  by one.

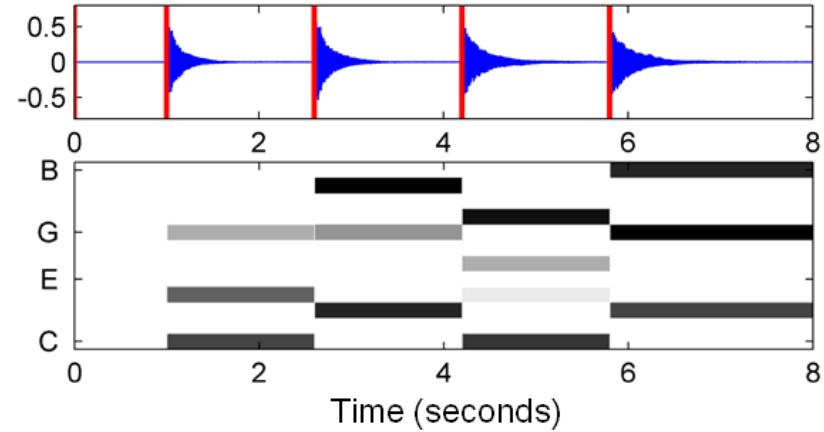
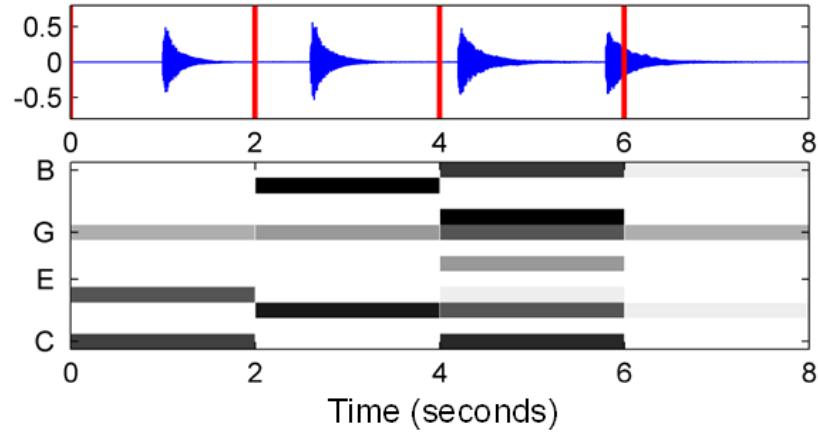
Set  $a_\ell = \mathbf{P}(a_{\ell-1})$ .

If  $a_\ell = 0$ , then set  $L = 0$  and return  $B^* = \emptyset$ .

Otherwise let  $L = \ell$  and return  $B^* = (a_L, a_{L-1}, \dots, a_1)$ .

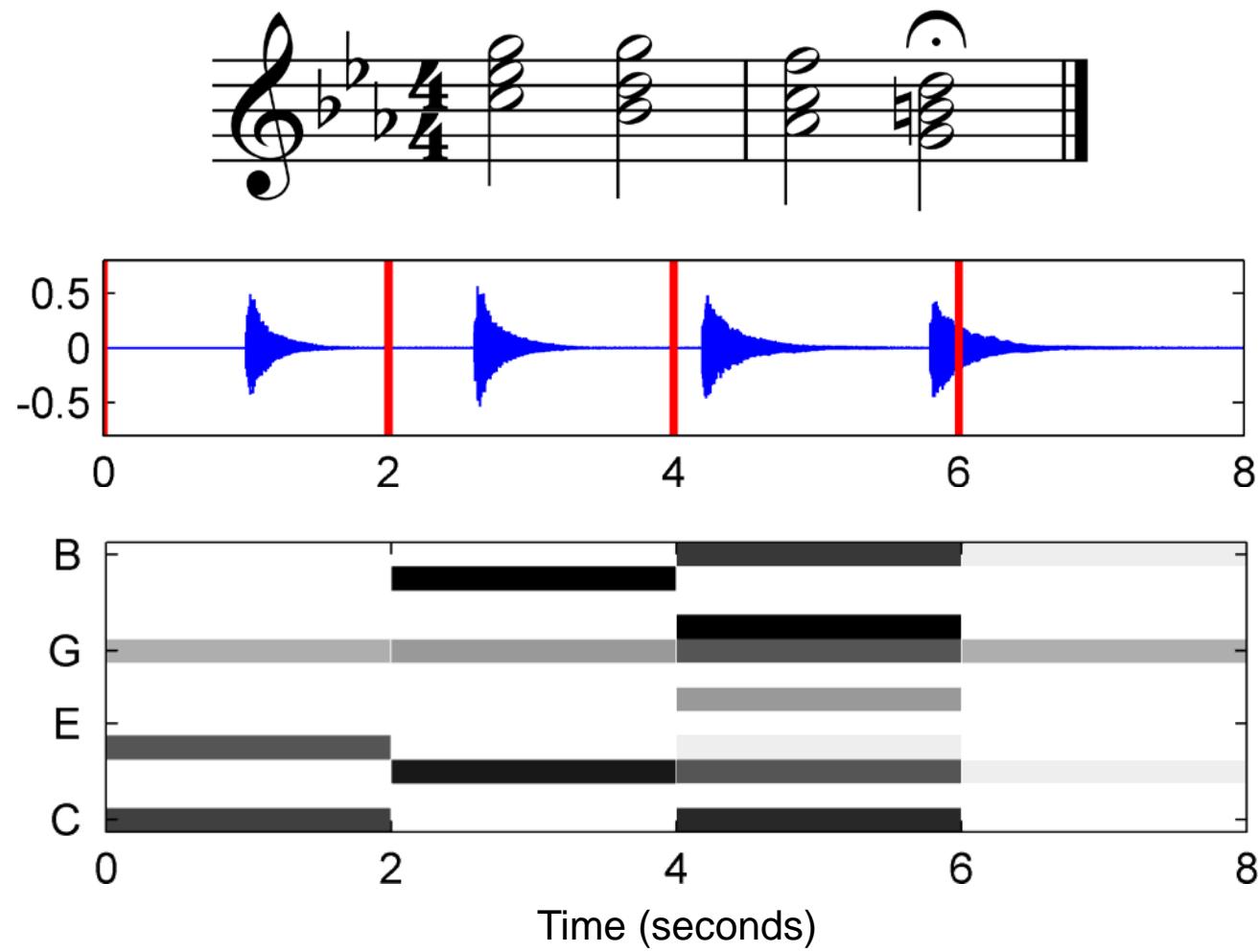
## 6.3 Beat and Pulse Tracking

Fig. 6.23



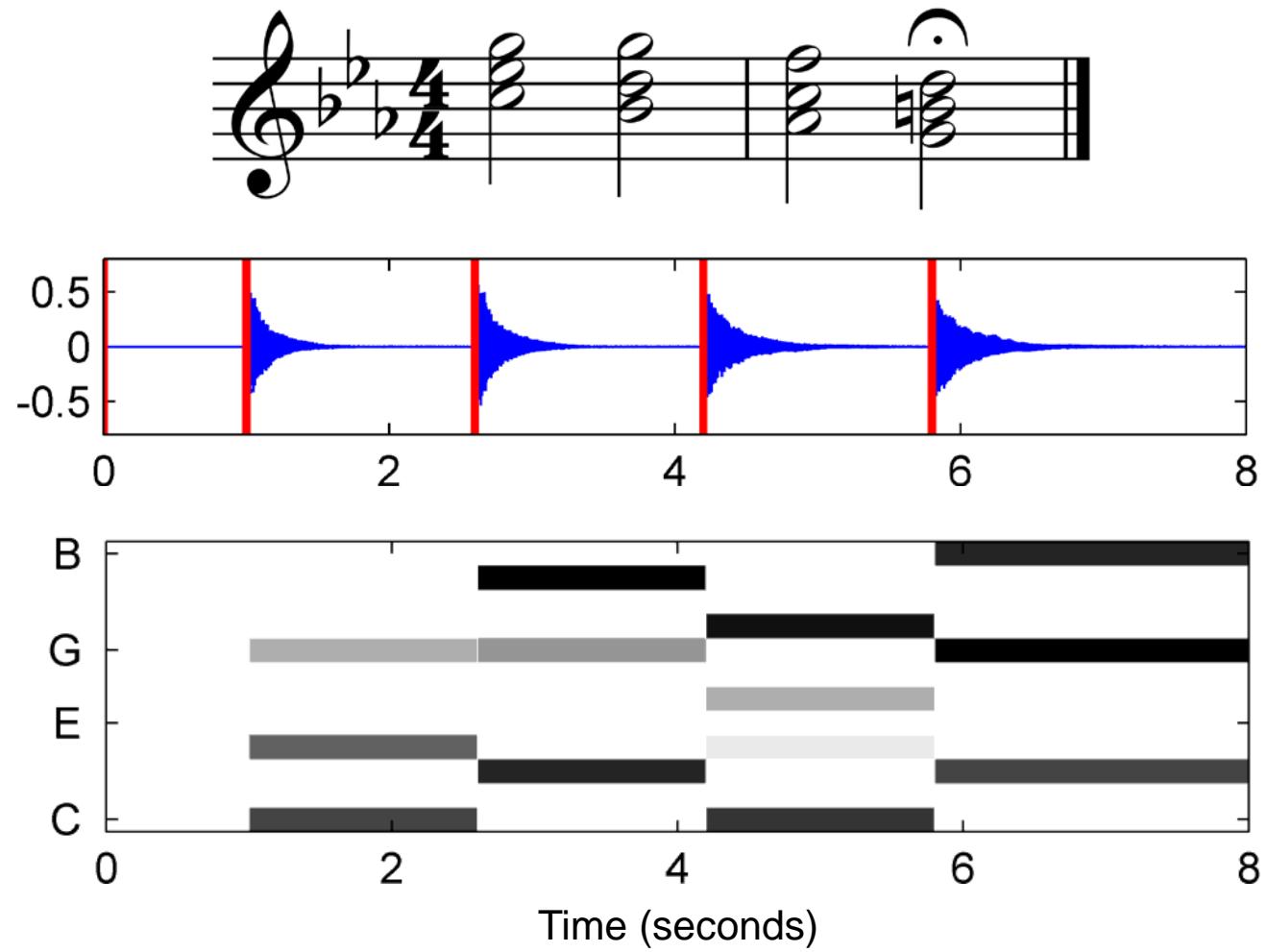
## 6.3 Beat and Pulse Tracking

Fig. 6.23



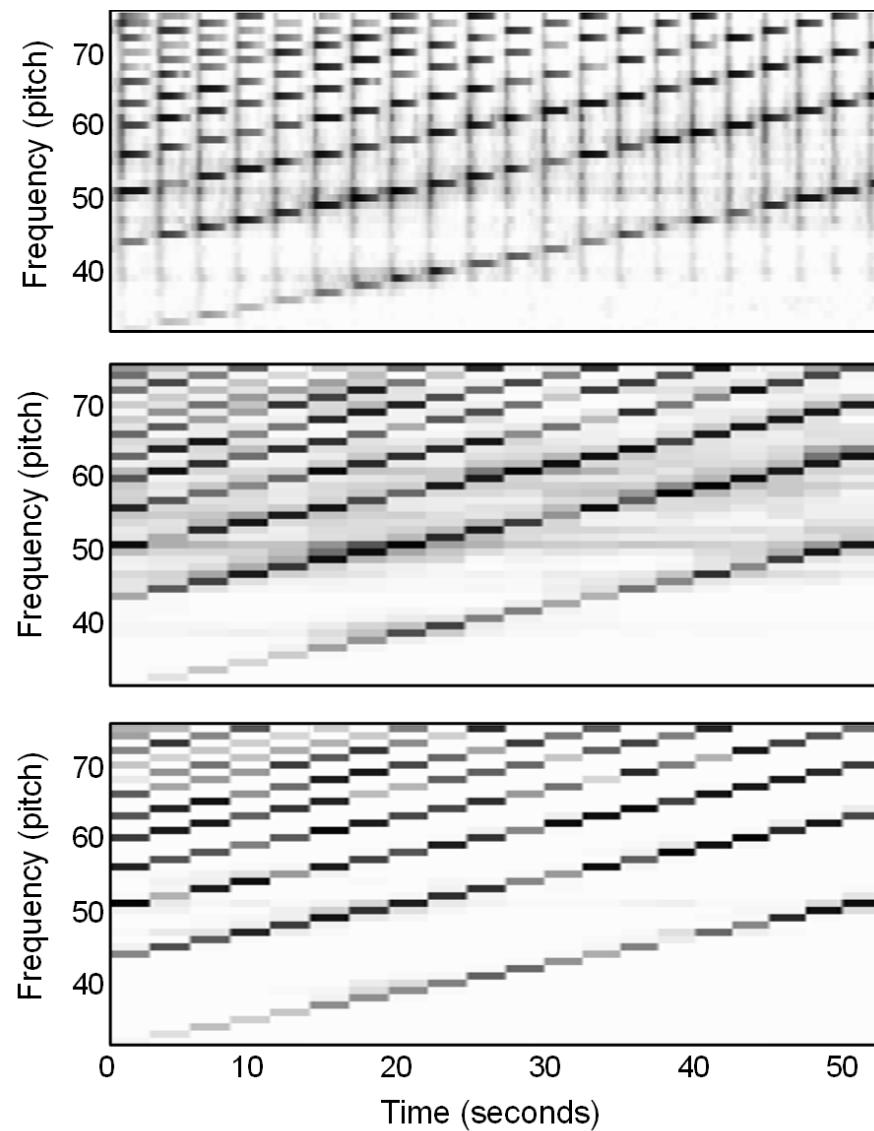
## 6.3 Beat and Pulse Tracking

Fig. 6.23



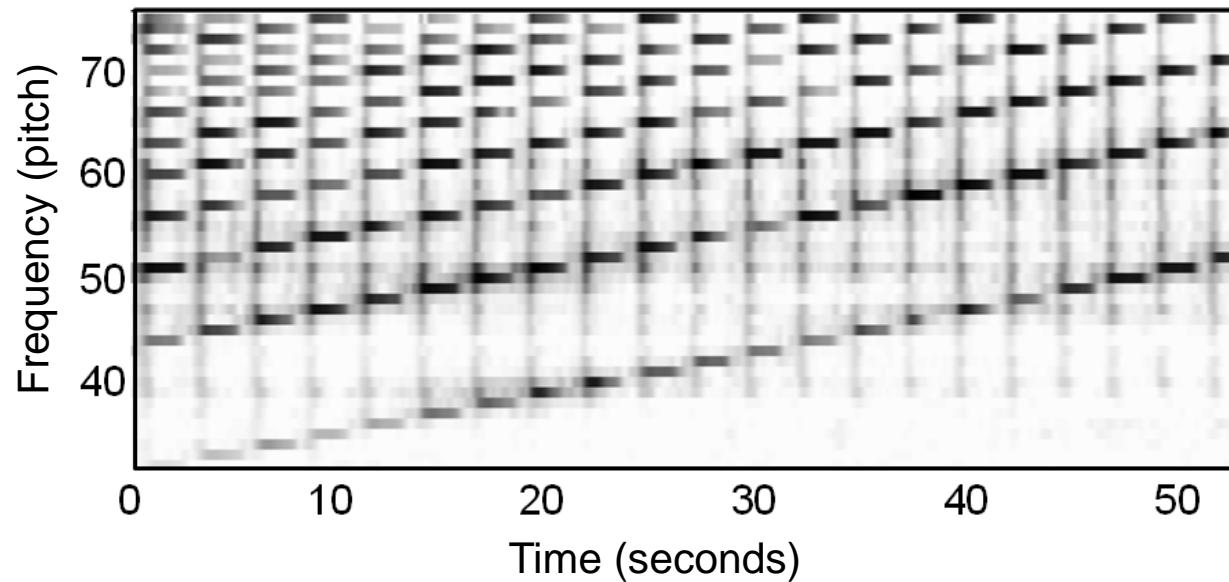
## 6.4 Further Notes

Fig. 6.24



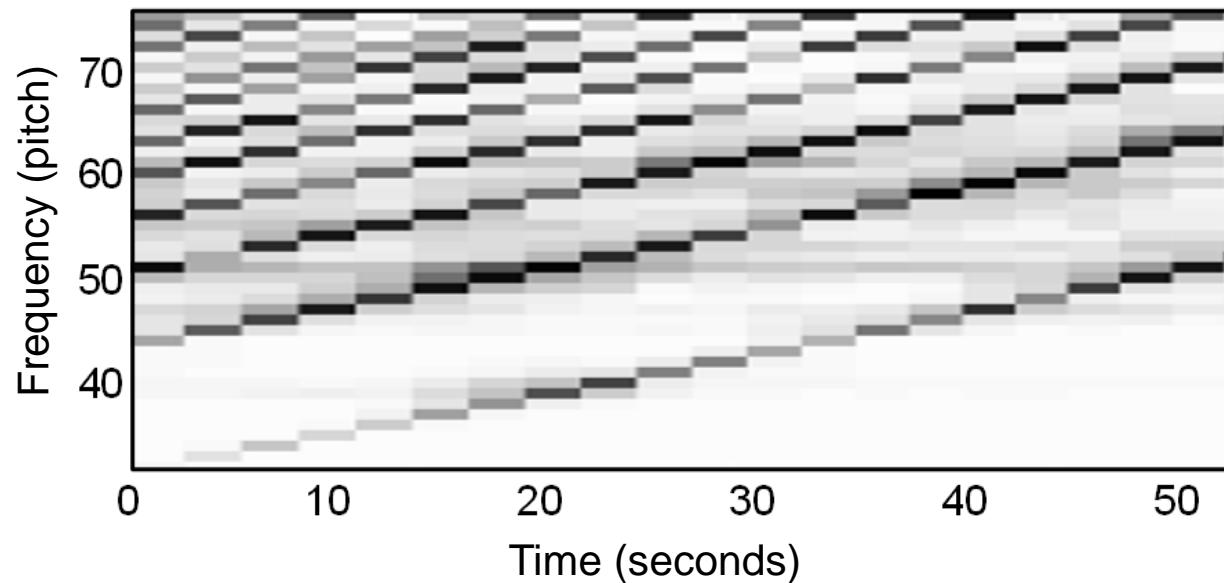
## 6.4 Further Notes

Fig. 6.24



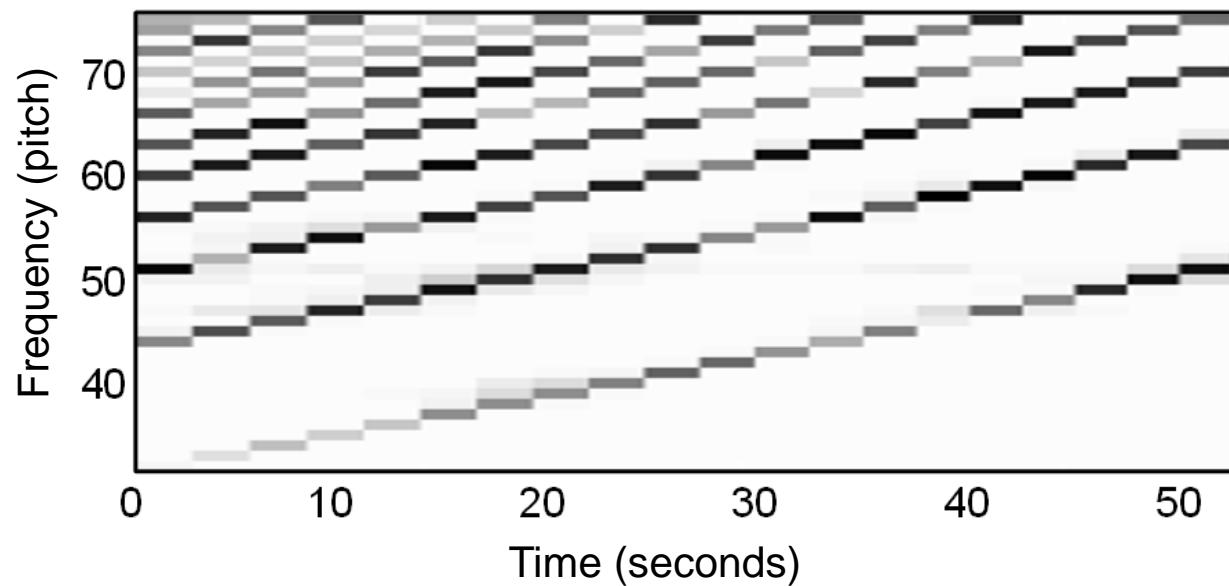
## 6.4 Further Notes

Fig. 6.24



## 6.4 Further Notes

Fig. 6.24



## 6.4 Further Notes

Table 6.2

Fourier tempogram	Autocorrelation tempogram
Comparison of novelty curve with windowed sinusoids with each sinusoid representing a tempo.	Comparison of novelty curve with time-shifted windowed sections of itself with each lag representing a tempo.
Conversion of frequency (Hertz) into tempo (BPM).	Conversion of lag (seconds) into tempo (BPM).
Measurement of novelty periodicities.	Measurement of novelty self-similarities.
Emphasis of tempo harmonics.	Emphasis of tempo subharmonics.
Suitable to analyze tempo on tactus and tatum level.	Suitable to analyze tempo on tactus and measure level.

## 6.4 Further Notes

Table 6.3

Steps	Chromagram	Cyclic tempogram
1.	Analysis of waveform.	Analysis of novelty curve.
2.	Computation of spectrogram.	Computation of tempogram.
3.	Usage of log-frequency axis.	Usage of log-tempo axis.
4.	Cyclic projection.	Cyclic projection.