

**Selected Topics in Deep Learning for  
Audio, Speech, and Music Processing**

# **Nonnegative Autoencoders with Applications to Music Audio Decomposing**

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# Thanks

- Tim Zunner (Master Thesis 2021)
- Edgar Suárez Guarnizo (Master Thesis 2020)
- Christian Dittmar (PhD 2018, Fraunhofer IIS)
- Michael Krause (PhD student)
- Yigitcan Özer (PhD student)

# Literature

- Daniel Lee and Sebastian Seung: **Algorithms for Non-Negative Matrix Factorization**. Proc. NIPS, 2000.
- Sebastian Ewert and Meinard Müller: **Using Score-Informed Constraints for NMF-Based Source Separation**. Proc. ICASSP, 2012.
- Paris Smaragdis and Shrikant Venkataramani: **A Neural Network Alternative to Non-Negative Audio Models**. Proc. ICASSP, 2017.
- Sebastian Ewert and Mark B. Sandler: **Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation**. Proc. ICASSP, 2017.
- Tim Zunner: **Neural Networks with Nonnegativity Constraints for Decomposing Music Recordings**. Master Thesis, FAU, 2021.
- Edgar Andrés Suárez Guarnizo: **DNN-Based Matrix Factorization with Applications to Drum Sound Decomposition**. Master Thesis, FAU, 2020.

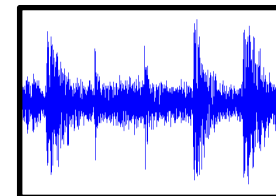
# Score-Informed Source Separation

Exploit musical score to support decomposition process

Musical  
Information



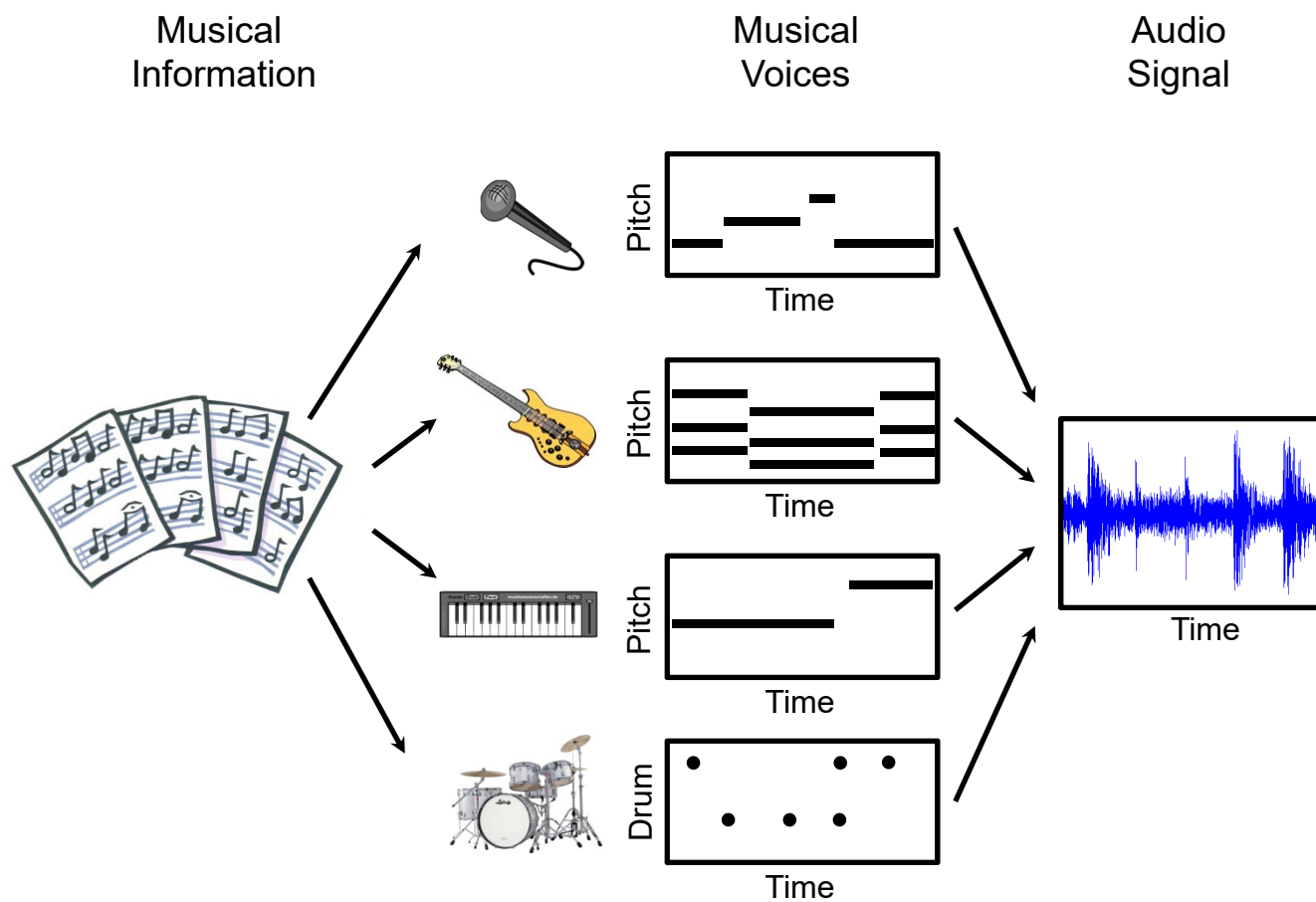
Audio  
Signal



Time

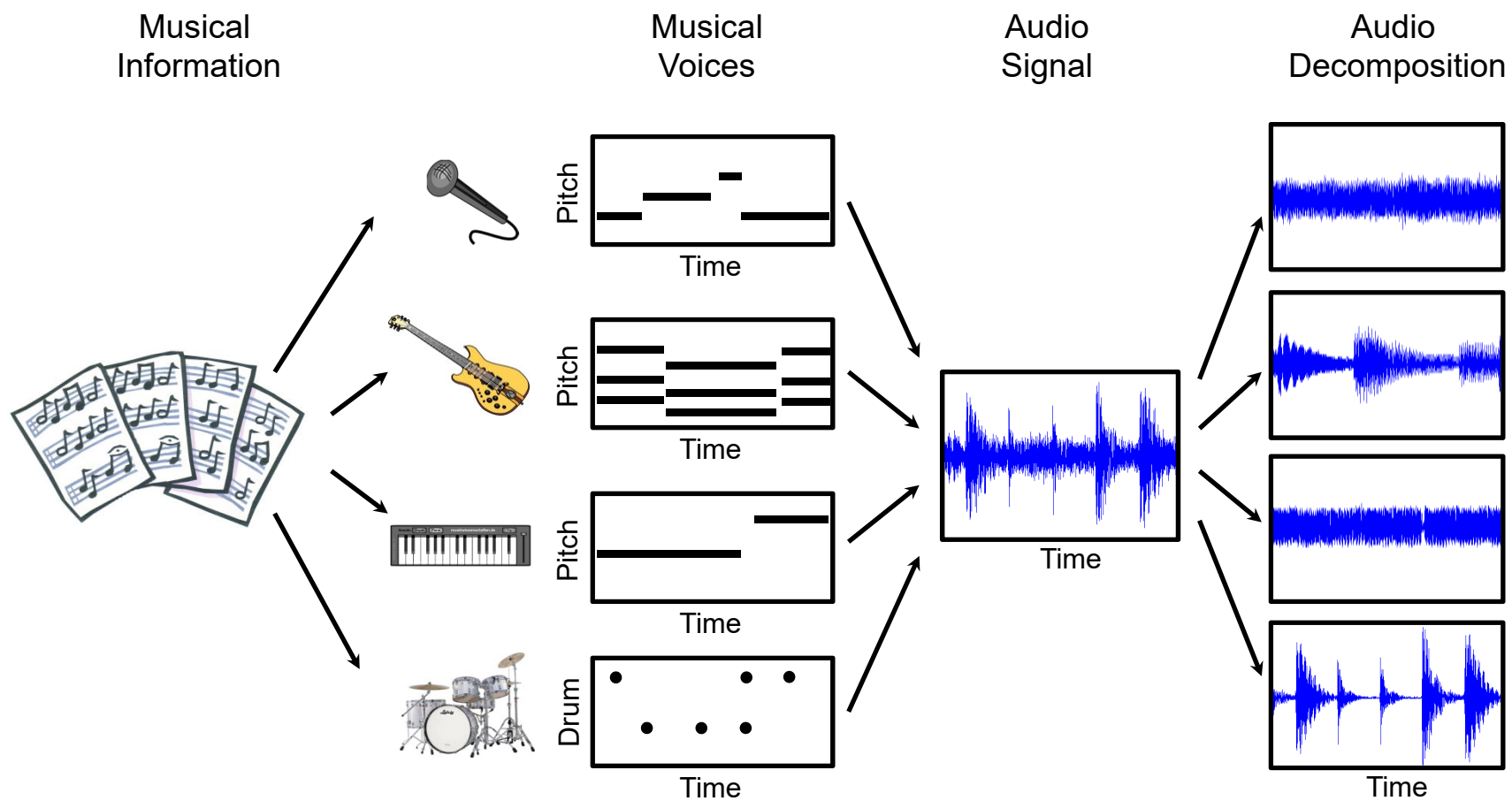
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Exploit musical score to support decomposition process



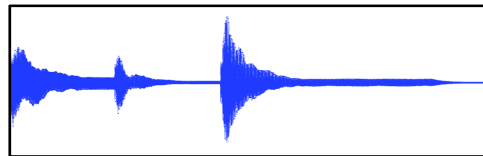
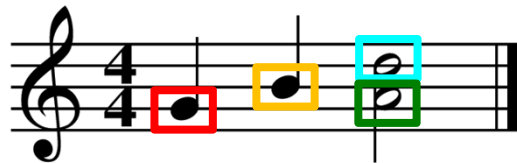
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Exploit musical score to support decomposition process



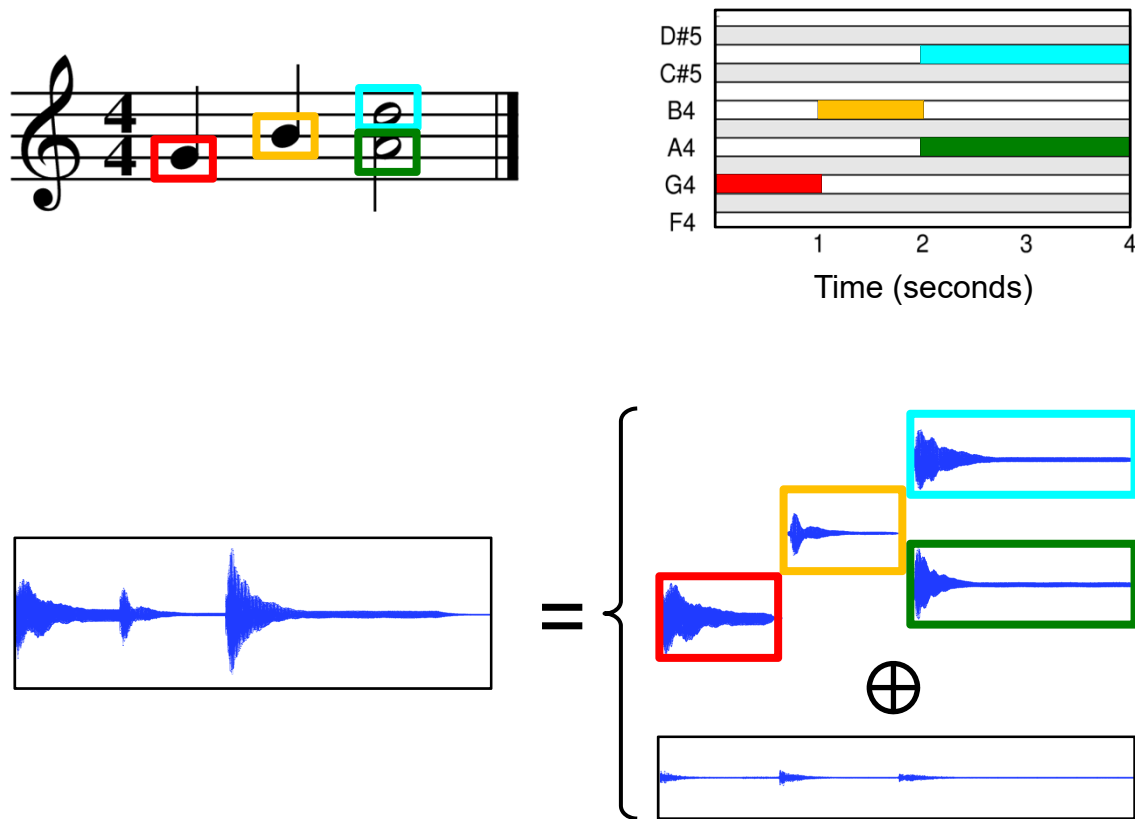
# Score-Informed Audio Decomposition

Notewise decomposition



# Score-Informed Audio Decomposition

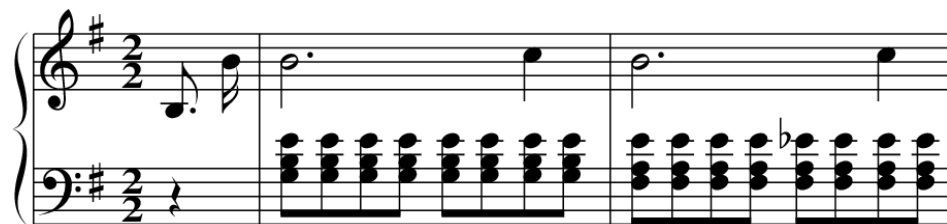
## Notewise decomposition



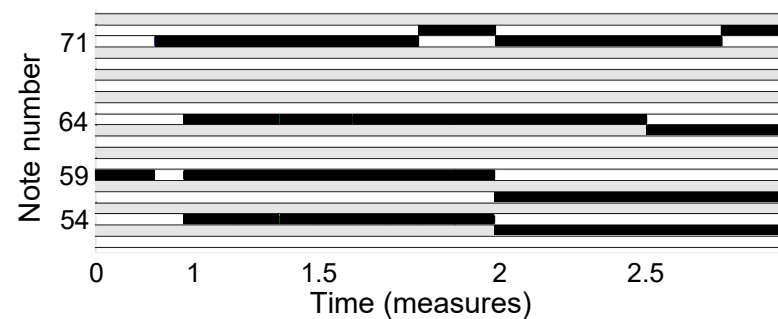


# Score-Informed Audio Decomposition

Sheet music

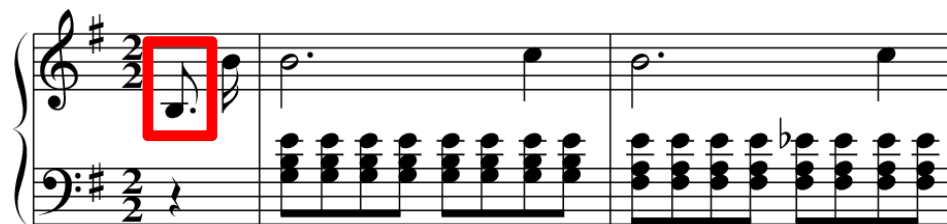


Piano roll



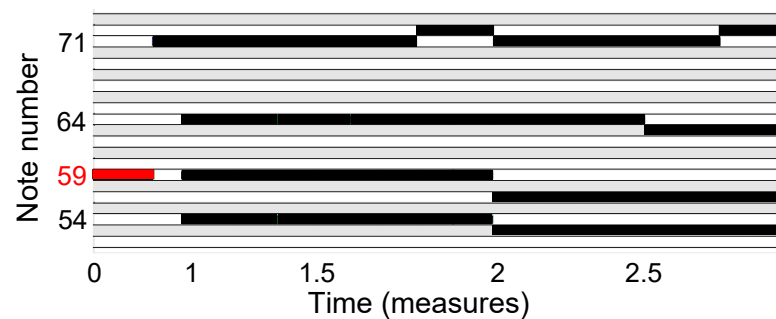
# Score-Informed Audio Decomposition

Sheet music



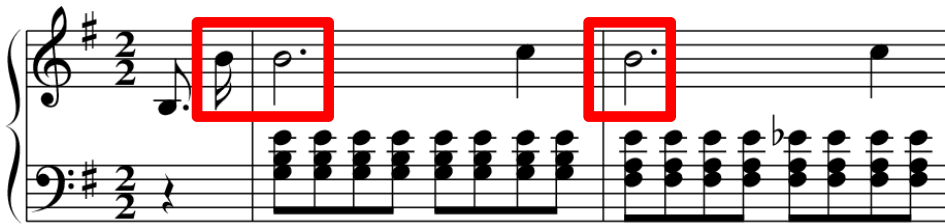
$p = 59$

Piano roll



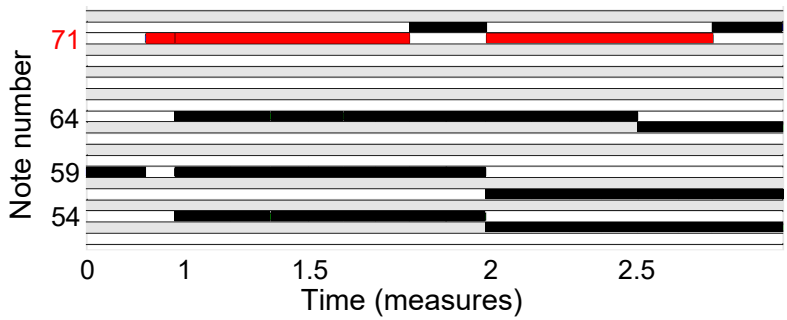
# Score-Informed Audio Decomposition

Sheet music



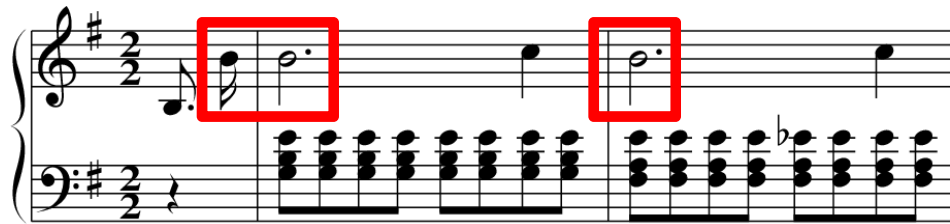
$p = 71$

Piano roll



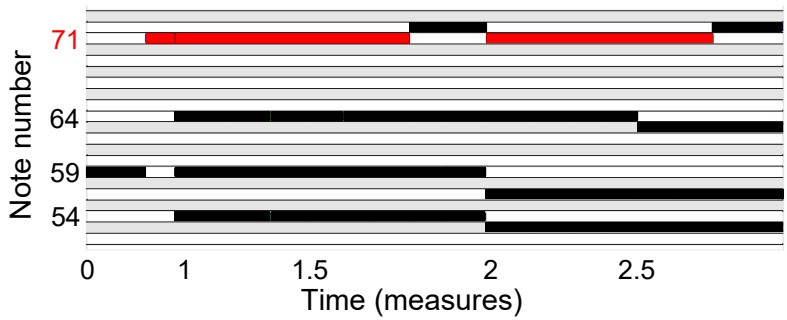
# Score-Informed Audio Decomposition

Sheet music

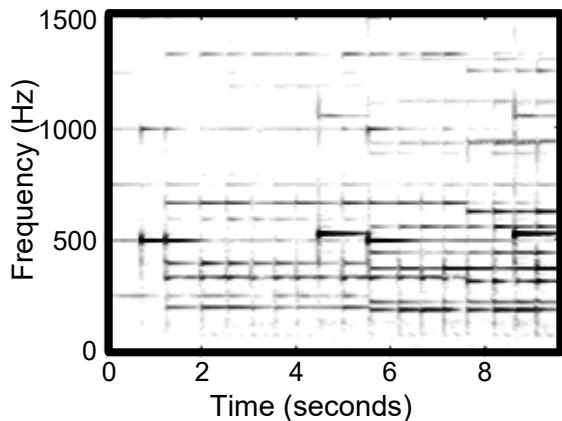


$p = 71$

Piano roll

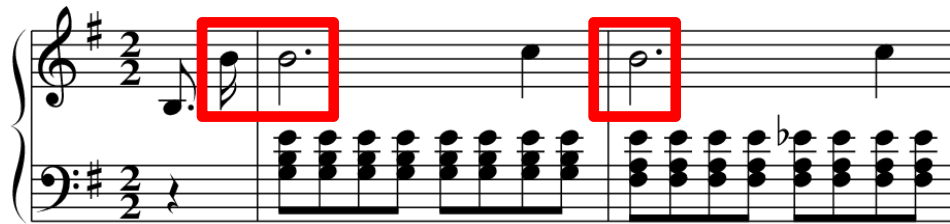


Spectrogram



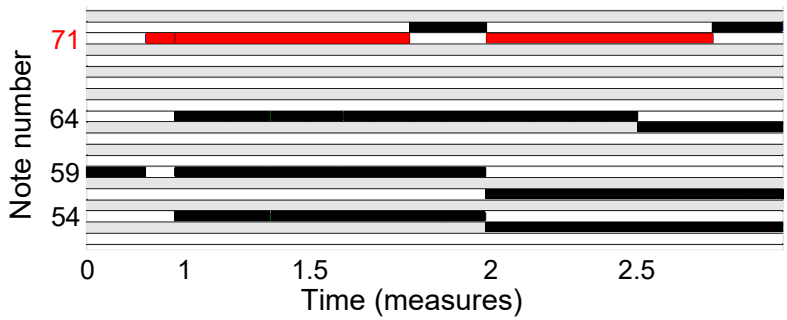
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Sheet music

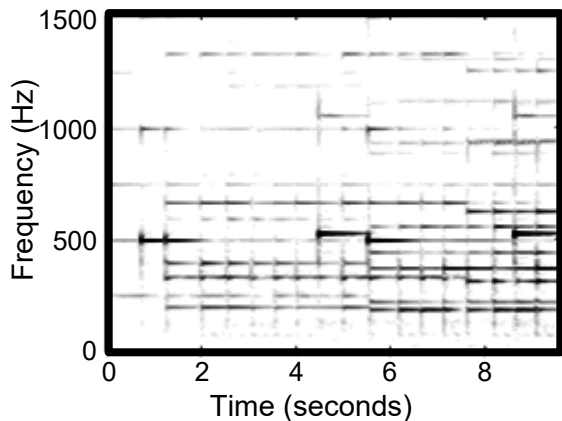


$p = 71$

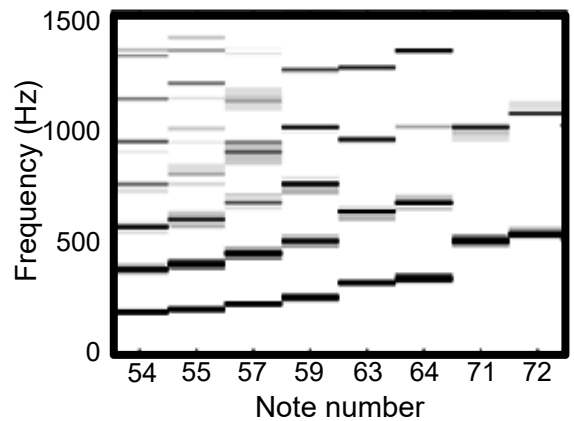
Piano roll



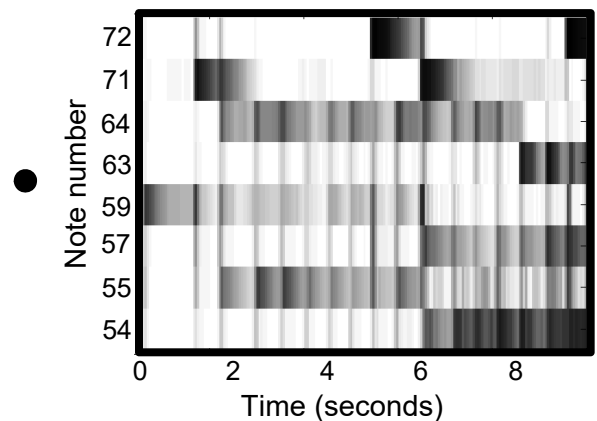
Spectrogram



Spectral patterns

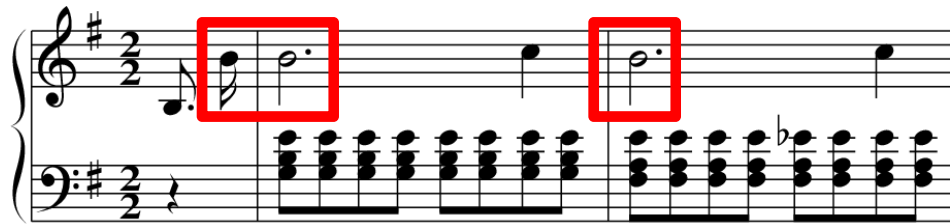


Activity patterns



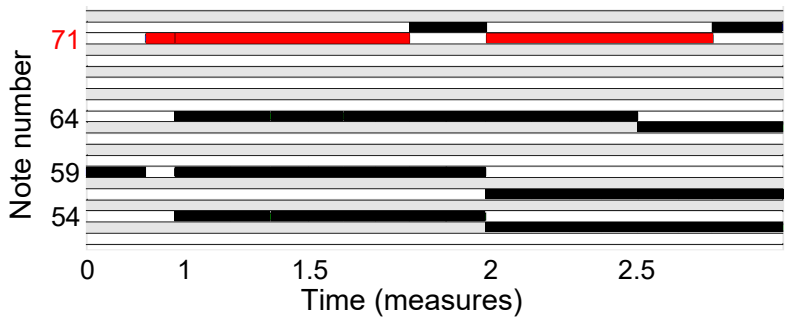
# Score-Informed Audio Decomposition

Sheet music

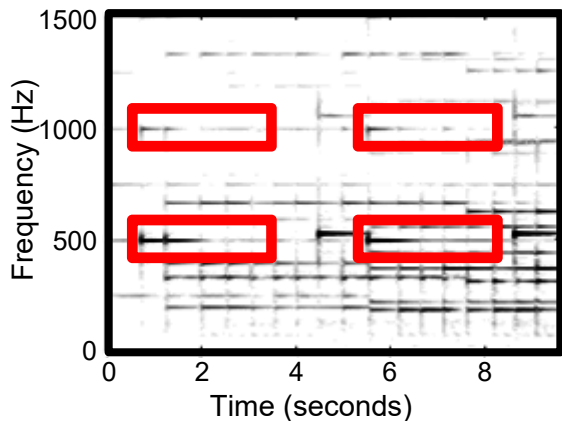


$p = 71$

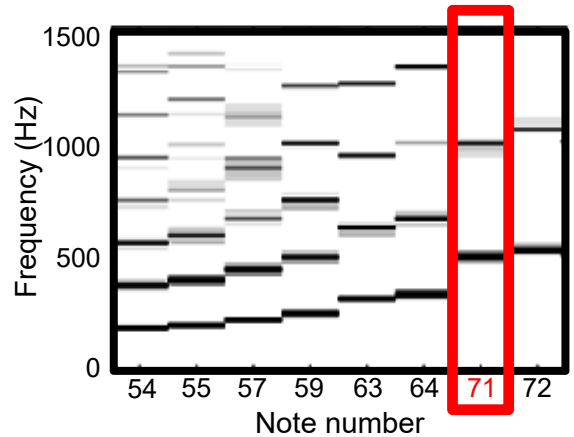
Piano roll



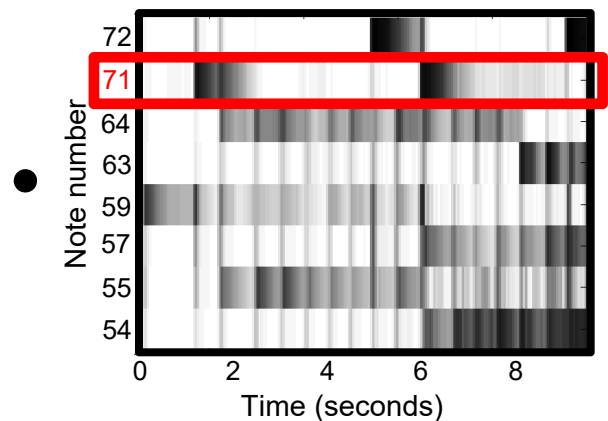
Spectrogram



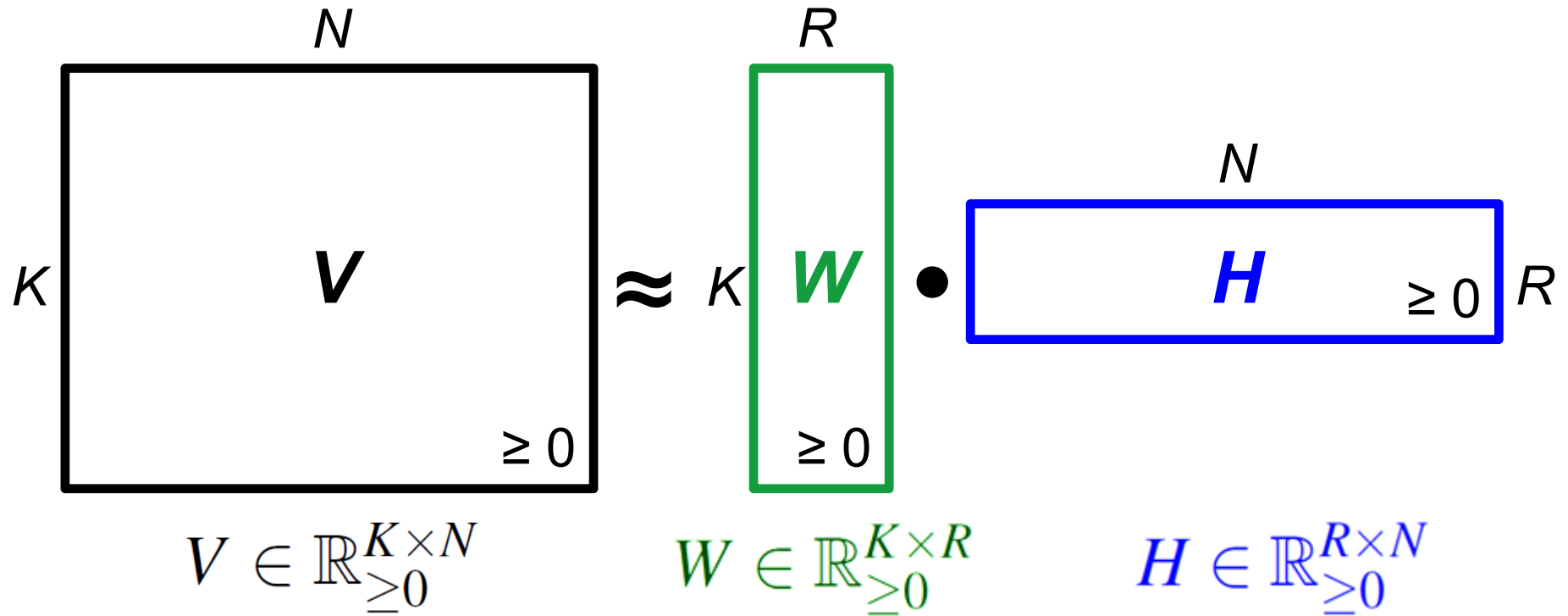
Spectral patterns



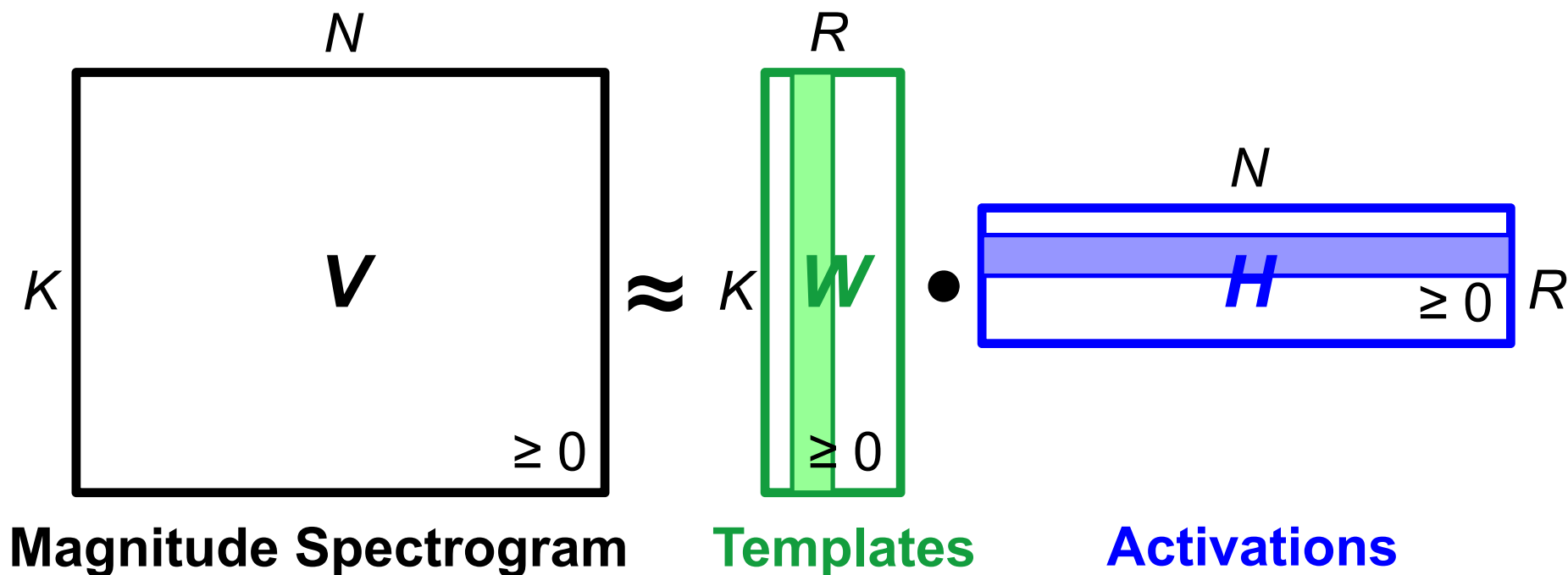
Activity patterns



# Nonnegative Matrix Factorization (NMF)



# Nonnegative Matrix Factorization (NMF)



**Templates:** Pitch + Timbre

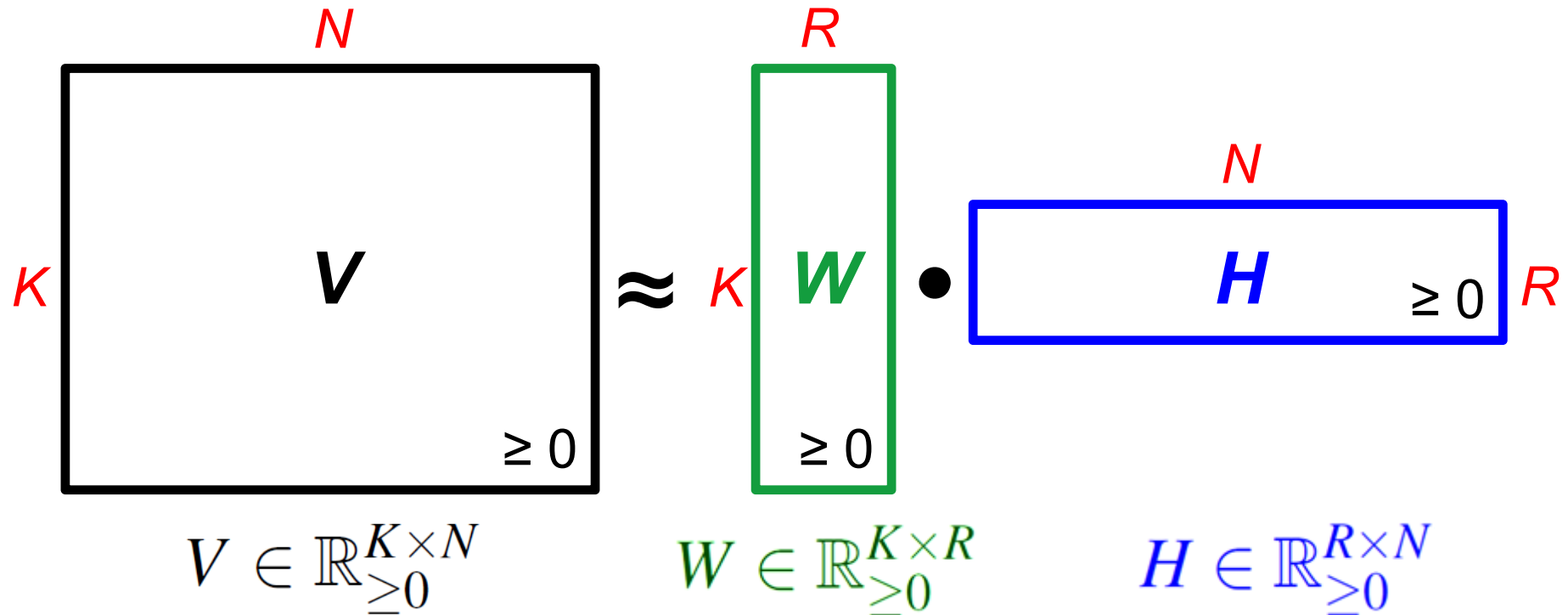
“How does it sound”

**Activations:** Onset time + Duration

“When does it sound”



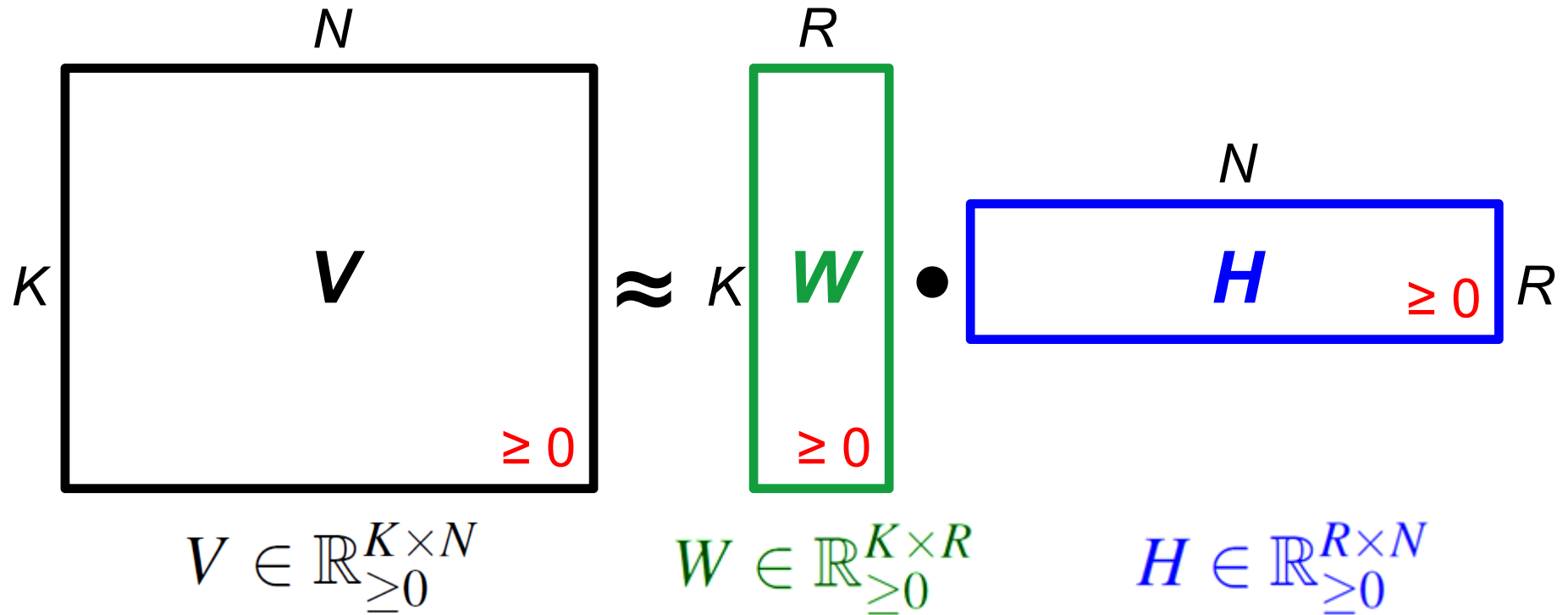
# Nonnegative Matrix Factorization (NMF)



## Dimensionality reduction

- $K, N$  typically much larger than  $R$  (maximal rank)
- Example:  $N = 1000, K = 500, R = 20$   
 $K \times N = 500,000, \quad K \times R = 10,000, \quad R \times N = 20,000$

# Nonnegative Matrix Factorization (NMF)



## Nonnegativity:

- Prevents mutual cancellation of template vectors
- Encourages semantically meaningful decomposition

# NMF Optimization

Optimization problem:

Given  $V \in \mathbb{R}_{\geq 0}^{K \times N}$  and rank parameter  $R$  minimize

$$\|V - WH\|^2$$

with respect to  $W \in \mathbb{R}_{\geq 0}^{K \times R}$  and  $H \in \mathbb{R}_{\geq 0}^{R \times N}$ .

Optimization not easy:

- Nonnegativity constraints
- Nonconvexity when jointly optimizing  $W$  and  $H$

Strategy: Iteratively optimize  $W$  and  $H$  via gradient descent

# NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

$$\begin{aligned} D &:= RN \\ \varphi^W &: \mathbb{R}^D \rightarrow \mathbb{R} \\ \varphi^W(H) &:= \|V - WH\|^2 \end{aligned} \quad \frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}}$$

### Variables

$$H \in \mathbb{R}^{R \times N}$$

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# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

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$$= \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

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$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Summand that does not depend on  $H_{\rho v}$  must be zero

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

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$$= \sum_{k=1}^K 2 \left( V_{kv} - \sum_{r=1}^R W_{kr} H_{rv} \right) \cdot (-W_{k\rho})$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

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Apply chain rule  
from calculus

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

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Rearrange  
summands



# NMF Optimization

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$$= 2 \left( \sum_{r=1}^R \left( \sum_{k=1}^K W_{\rho k}^\top W_{kr} \right) H_{rv} - \sum_{k=1}^K W_{\rho k}^\top V_{kv} \right)$$

Introduce  
transposed  $W^\top$

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

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$$= 2 \left( \sum_{r=1}^R \left( \sum_{k=1}^K W_{\rho k}^\top W_{kr} \right) H_{rv} - \sum_{k=1}^K W_{\rho k}^\top V_{kv} \right)$$

$$= 2 \left( (W^\top W H)_{\rho v} - (W^\top V)_{\rho v} \right).$$

# NMF Optimization

## Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^\top W H^{(\ell)})_{rn} - (W^\top V)_{rn} \right)$$

with suitable learning rate  $\gamma_{rn}^{(\ell)} \geq 0$

# NMF Optimization

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with suitable learning rate  $\gamma_{rn}^{(\ell)} \geq 0$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

# NMF Optimization

## Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := \frac{H_{rn}^{(\ell)}}{(W^T W H^{(\ell)})_{rn}}$$

$$\begin{aligned} H_{rn}^{(\ell+1)} &= H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^T W H^{(\ell)})_{rn} - (W^T V)_{rn} \right) \\ &= H_{rn}^{(\ell)} \cdot \frac{(W^T V)_{rn}}{(W^T W H^{(\ell)})_{rn}} \end{aligned}$$

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## Gradient descent

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Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := \frac{H_{rn}^{(\ell)}}{(W^\top W H^{(\ell)})_{rn}}$$

- Update rule become multiplicative
- Nonnegative values stay nonnegative

# NMF Optimization

**Algorithm:** NMF ( $V \approx WH$ )

**Input:** Nonnegative matrix  $V$  of size  $K \times N$   
Rank parameter  $R \in \mathbb{N}$   
Threshold  $\varepsilon$  used as stop criterion

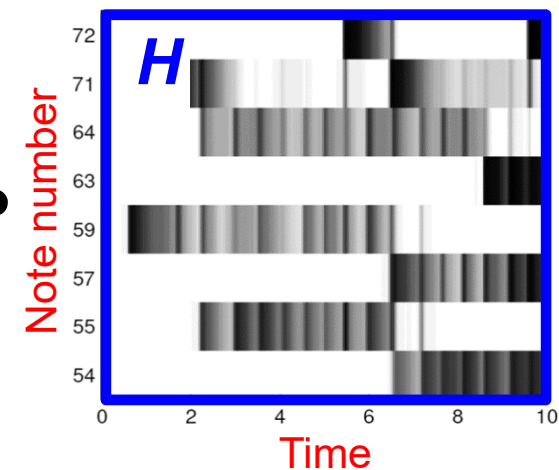
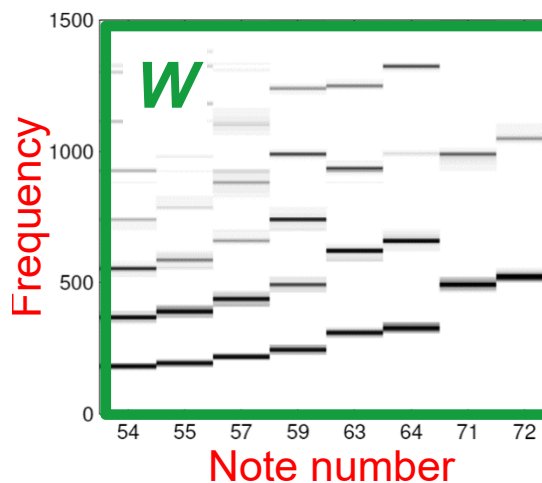
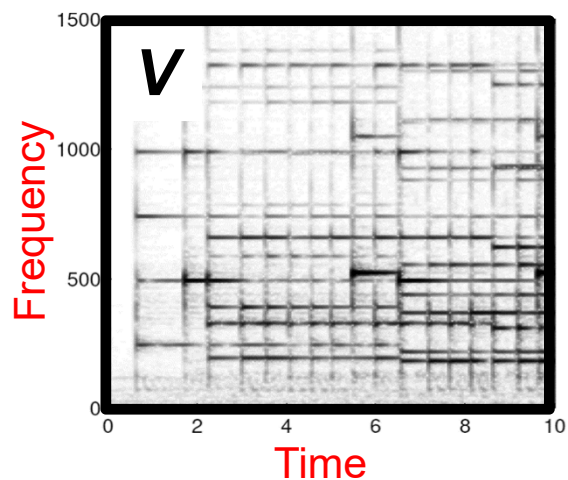
**Output:** Nonnegative template matrix  $W$  of size  $K \times R$   
Nonnegative activation matrix  $H$  of size  $R \times N$

**Procedure:** Define nonnegative matrices  $W^{(0)}$  and  $H^{(0)}$  by some random or informed initialization. Furthermore set  $\ell = 0$ . Apply the following update rules (written in matrix notation):

- (1)  $H^{(\ell+1)} = H^{(\ell)} \odot (((W^{(\ell)})^\top V) \oslash ((W^{(\ell)})^\top W^{(\ell)} H^{(\ell)}))$
- (2)  $W^{(\ell+1)} = W^{(\ell)} \odot ((V(H^{(\ell+1)})^\top) \oslash (W^{(\ell)} H^{(\ell+1)} (H^{(\ell+1)})^\top))$
- (3) Increase  $\ell$  by one.

Repeat the steps (1) to (3) until  $\|H^{(\ell)} - H^{(\ell-1)}\| \leq \varepsilon$  and  $\|W^{(\ell)} - W^{(\ell-1)}\| \leq \varepsilon$  (or until some other stop criterion is fulfilled). Finally, set  $H = H^{(\ell)}$  and  $W = W^{(\ell)}$ .

# NMF-based Spectrogram Decomposition



Templates: Pitch + Timbre

“How does it sound”

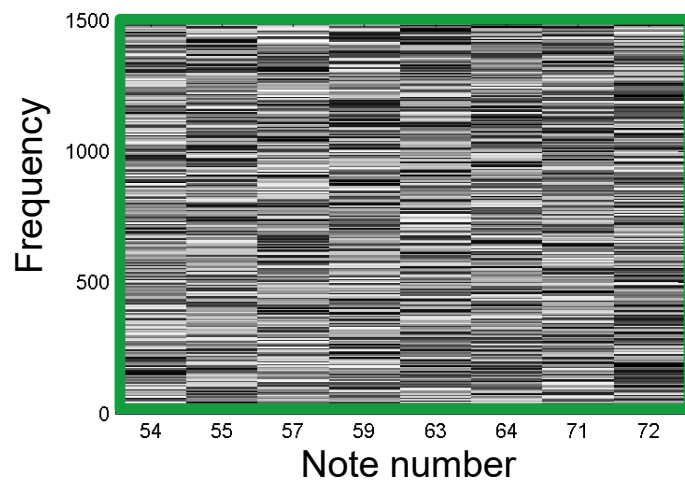
Activations: Onset time + Duration

“When does it sound”

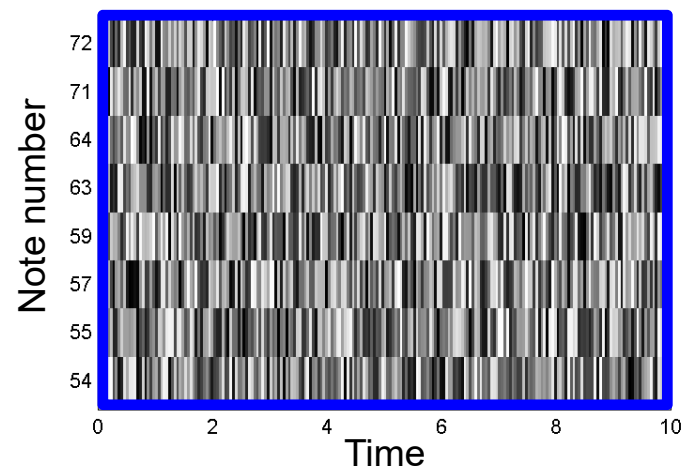


# NMF-based Spectrogram Decomposition

Template initialization



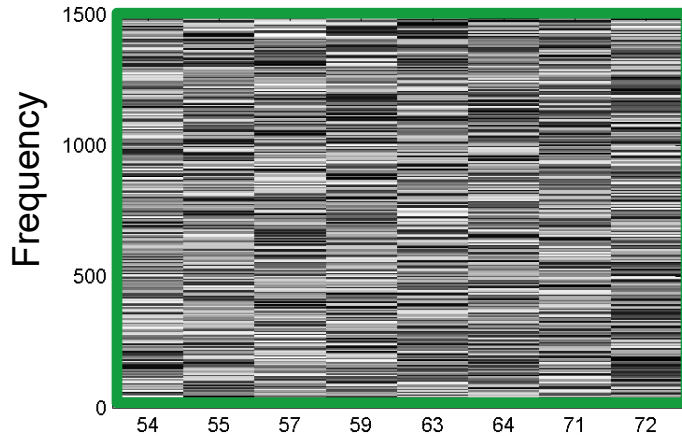
Activation initialization



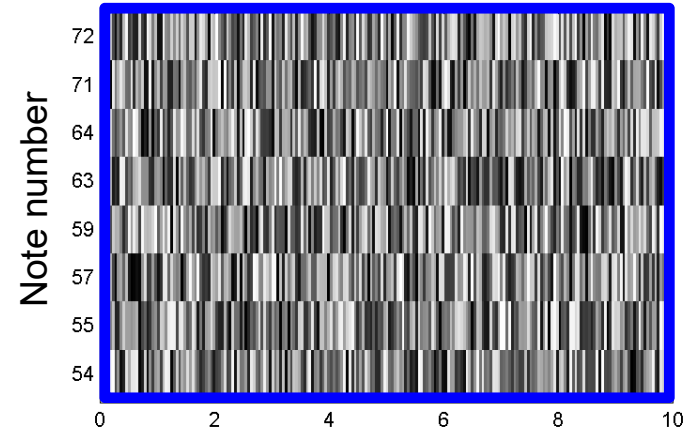
Random initialization

# NMF-based Spectrogram Decomposition

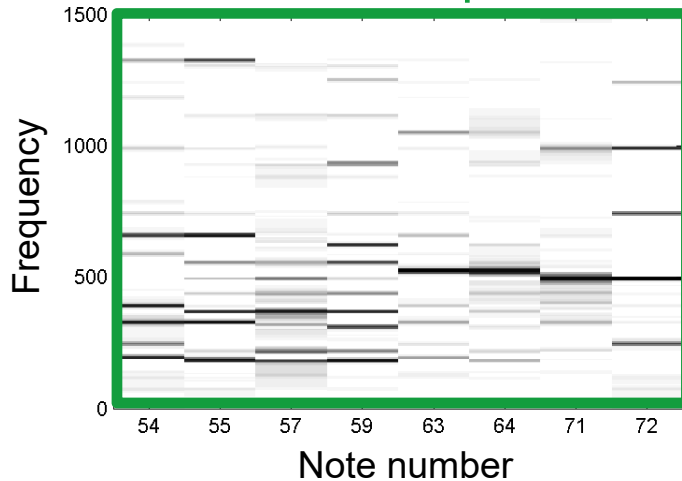
Template initialization



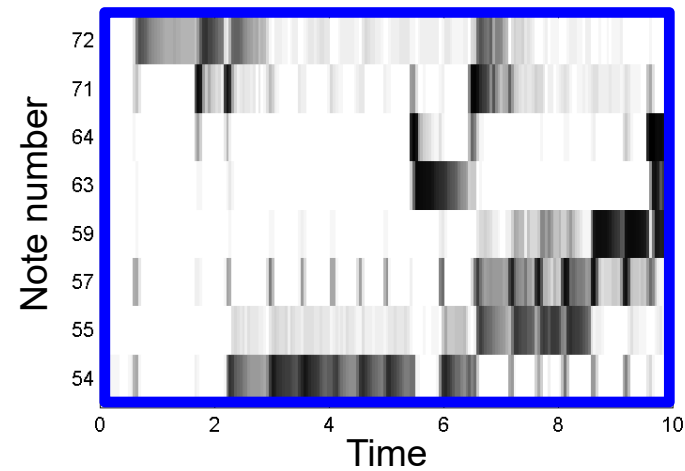
Activation initialization



Learnt templates



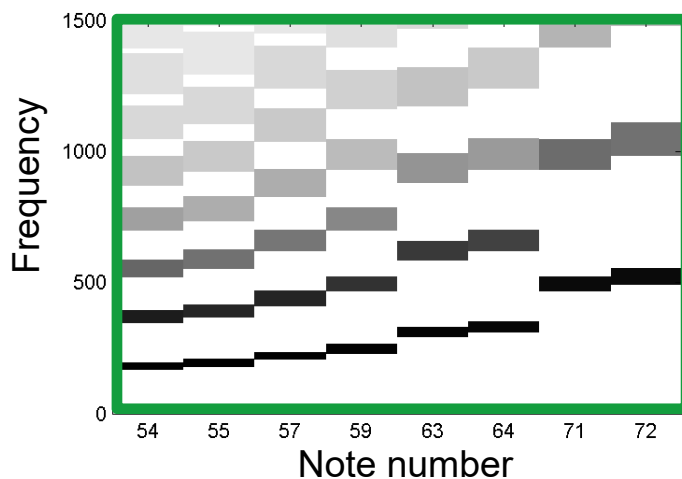
Learnt activations



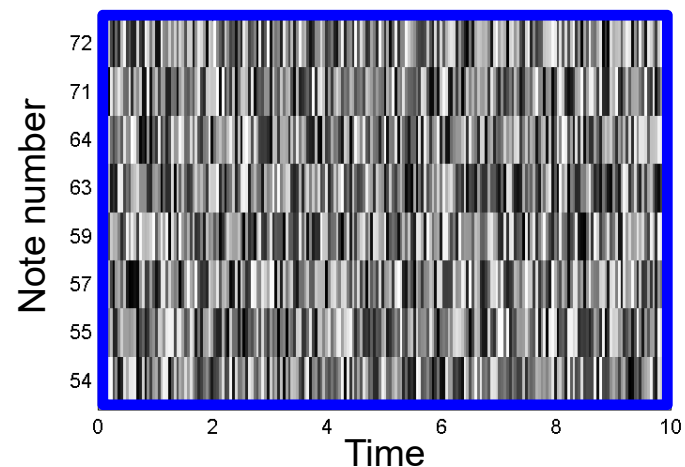
Random initialization → No semantic meaning

# Constrained NMF: Templates

Template initialization



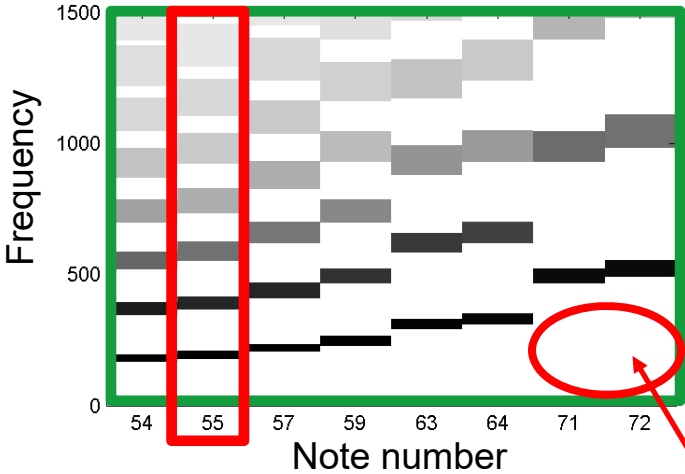
Activation initialization



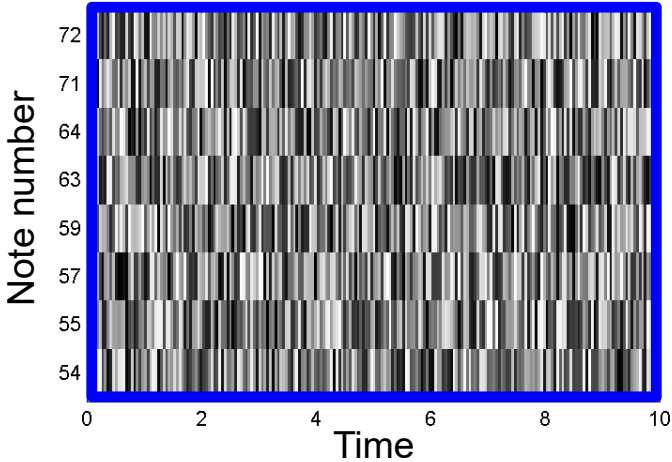
Enforce harmonic structure with zero-valued entries

# Constrained NMF: Templates

Template initialization



Activation initialization

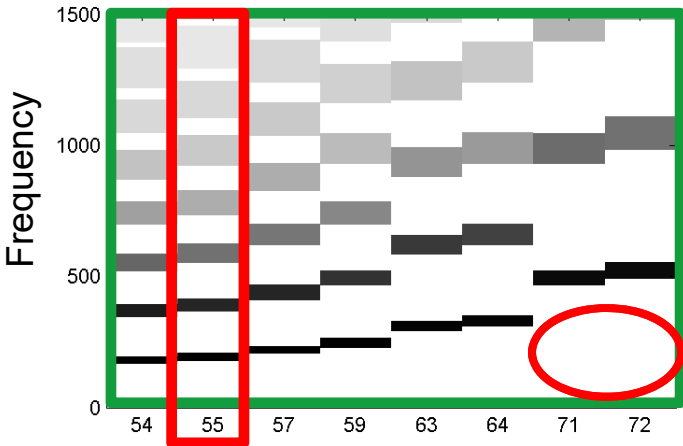


Template constraint for  $p=55$

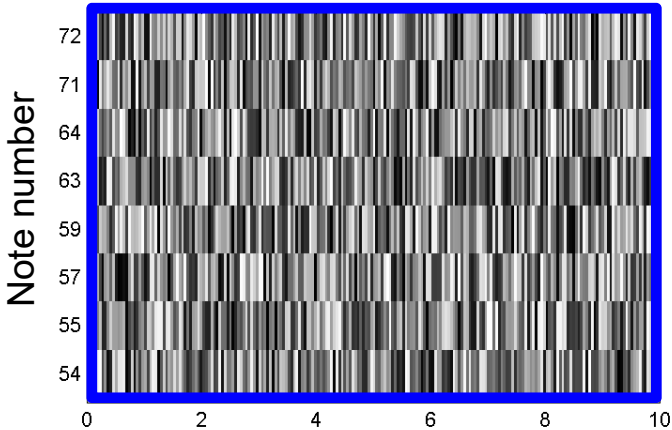
Enforce harmonic structure with zero-valued entries

# Constrained NMF: Templates

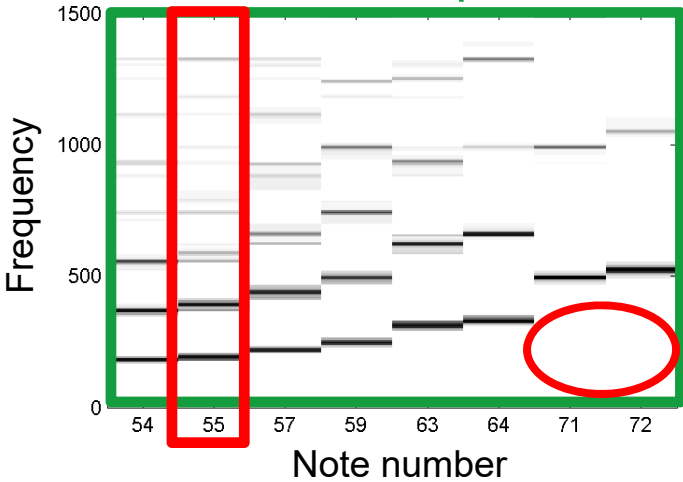
Template initialization



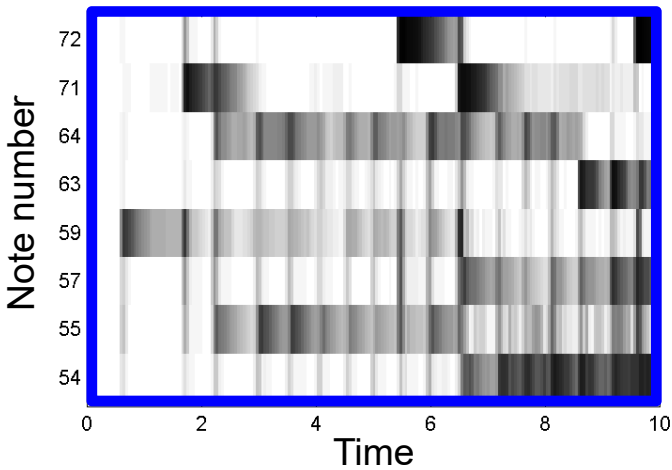
Activation initialization



Learnt templates



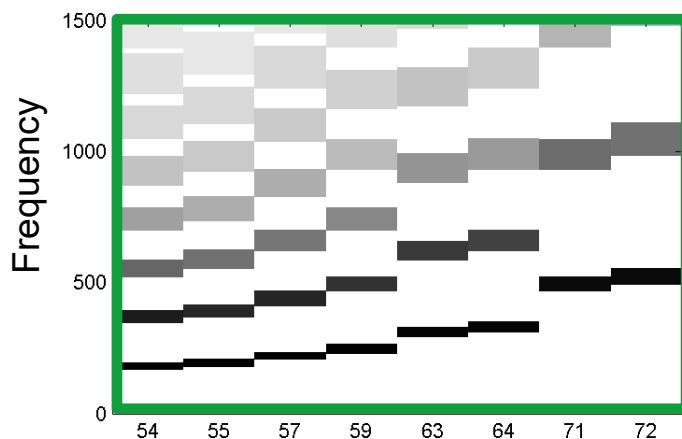
Learnt activations



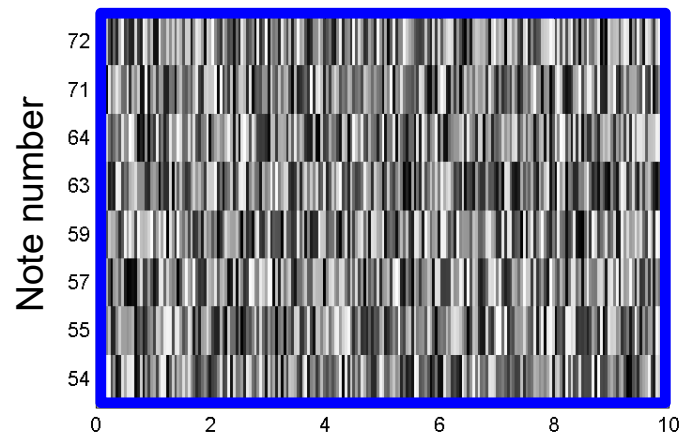
Zero-valued entries remain zero-valued entries!

# Constrained NMF: Templates

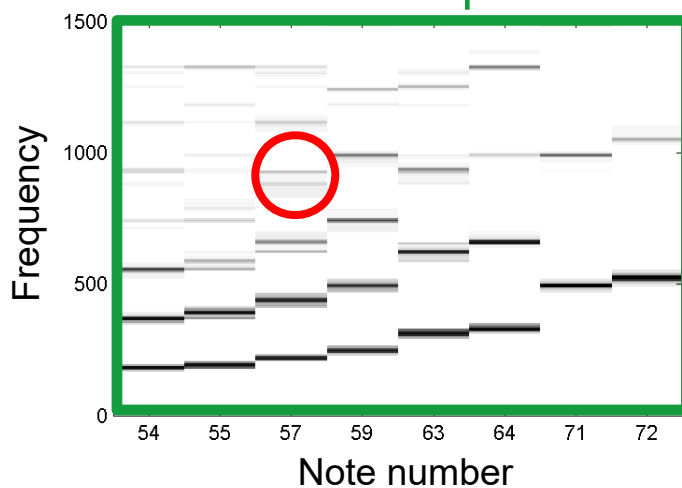
Template initialization



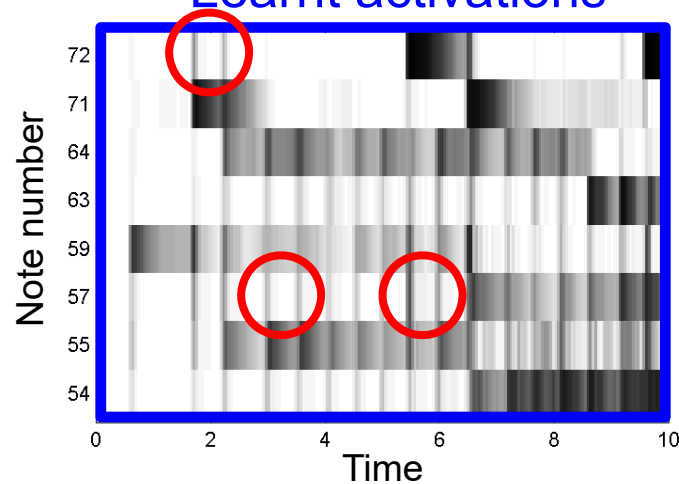
Activation initialization



Learnt templates



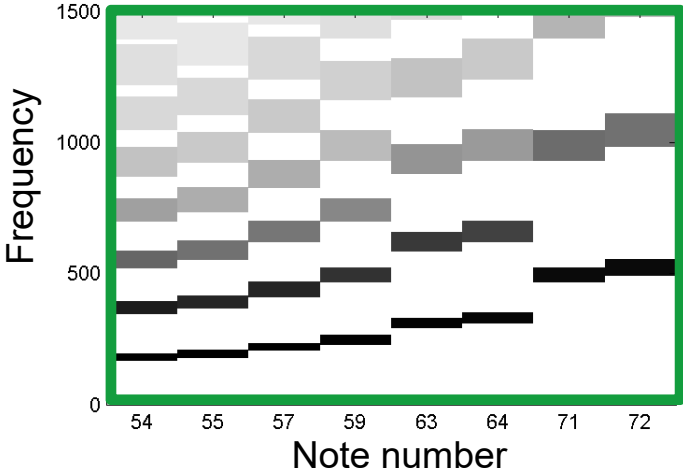
Learnt activations



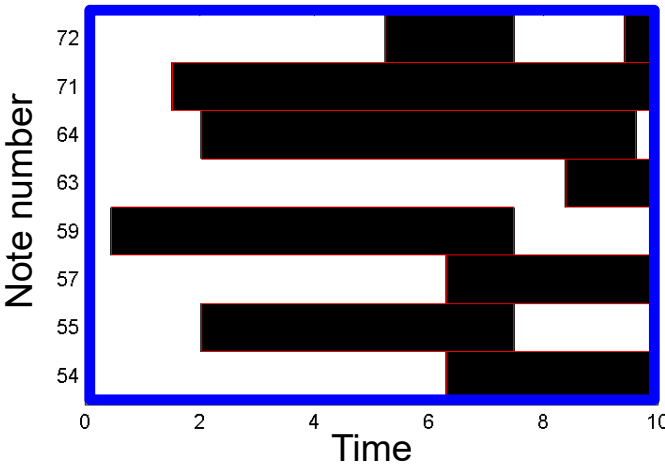
Pitch templates misused to represent onsets

# Constrained NMF: Double Constraints

Template initialization

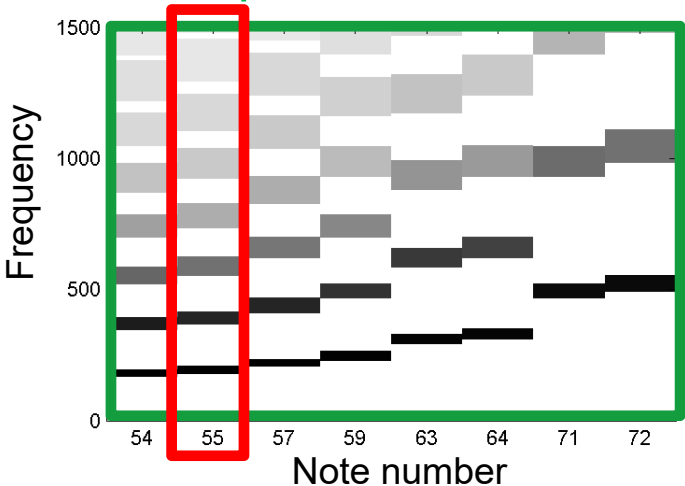


Activation initialization



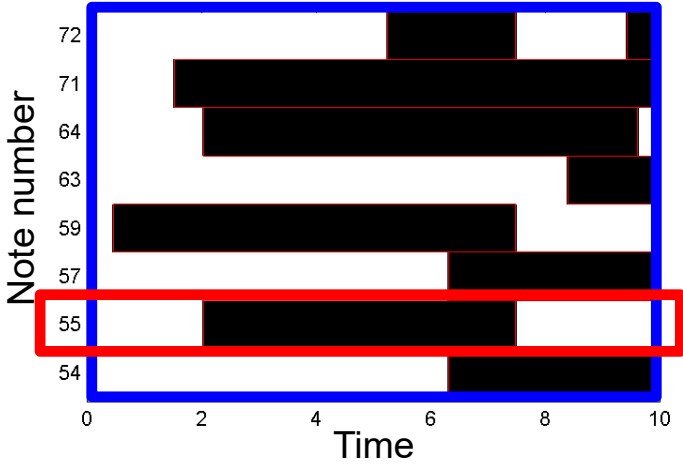
# Constrained NMF: Double Constraints

Template initialization



Template constraint for  $p=55$

Activation initialization

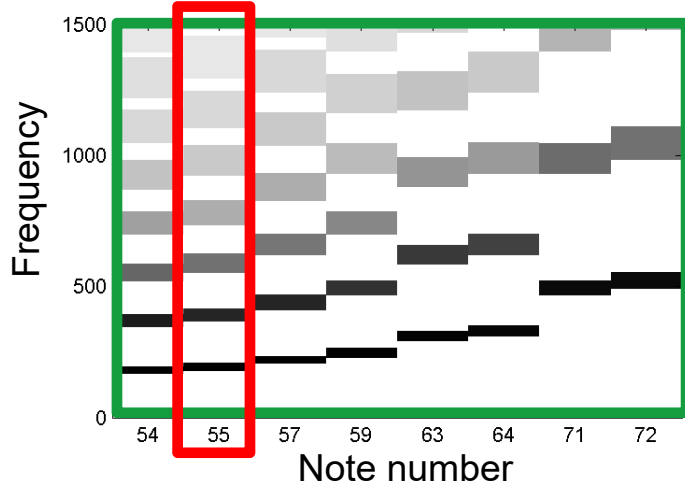


Activation constraints for  $p=55$



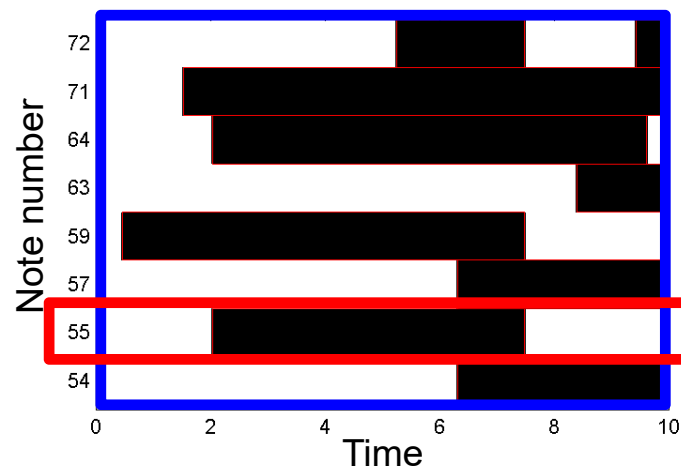
# Constrained NMF: Double Constraints

Template initialization



Template constraint for  $p=55$

Activation initialization

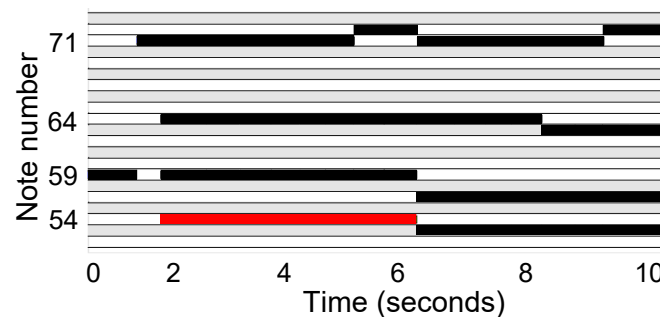
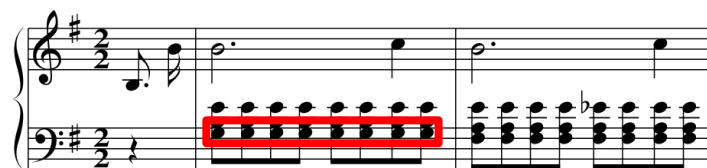


Activation constraints for  $p=55$

Such information may come from a synchronized score

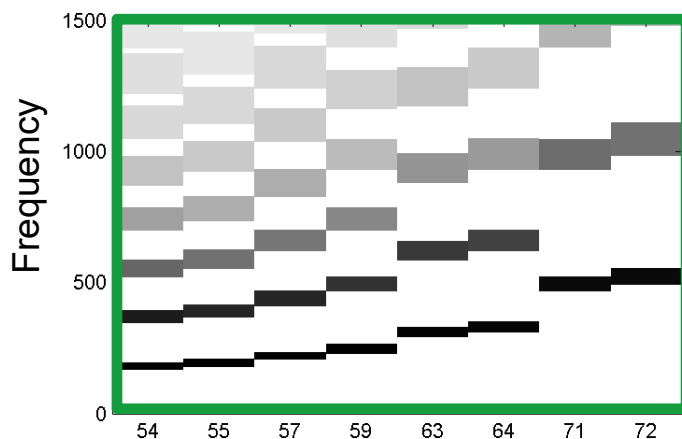


Sheet music

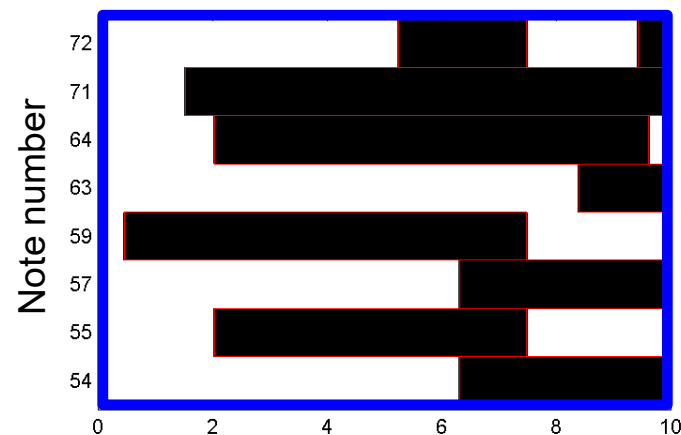


# Constrained NMF: Double Constraints

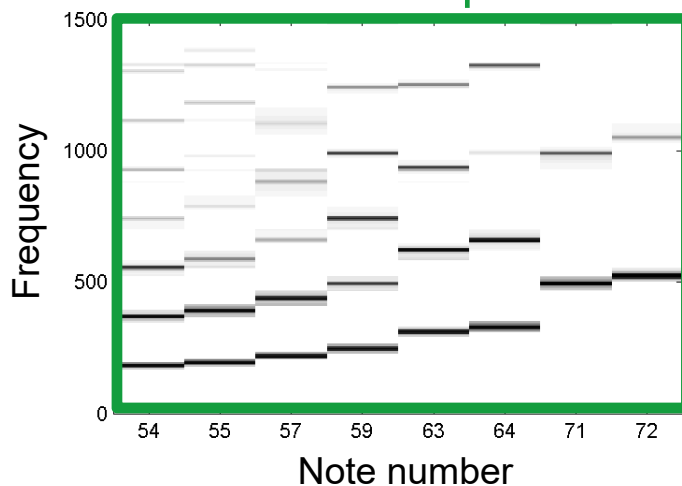
Template initialization



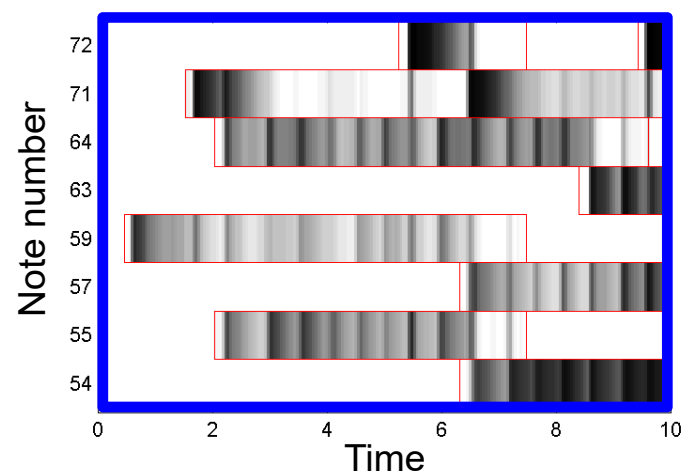
Activation initialization



Learnt templates



Learnt activations



Original



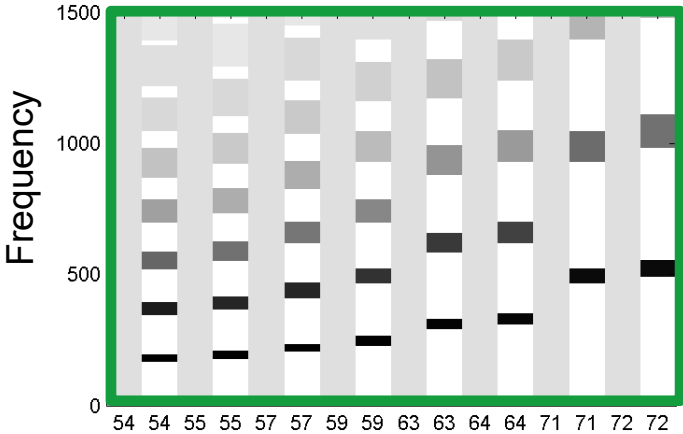
Model



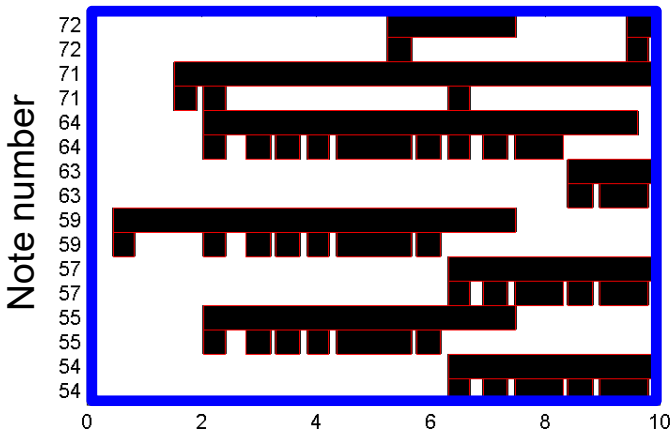
Significant gain in structure, but onsets are missing

# Constrained NMF: Onset Templates

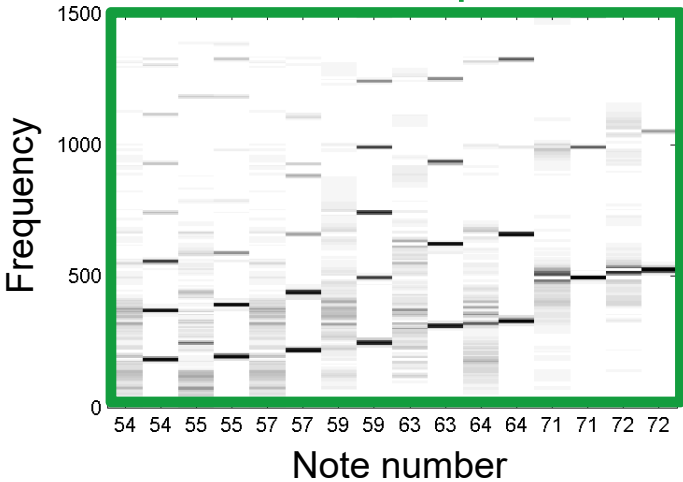
Template initialization



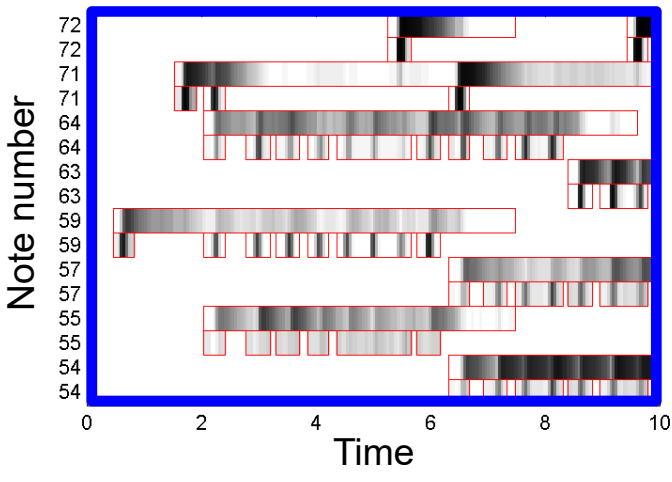
Activation initialization



Learnt templates



Learnt activations



Original



Model Onset



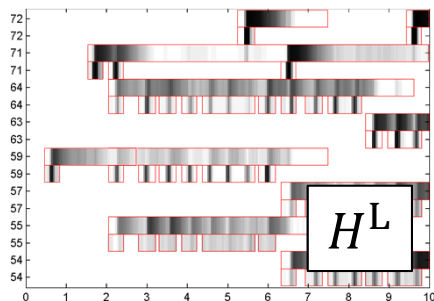
# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix

$$H^R$$

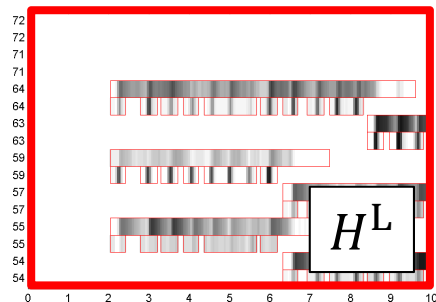
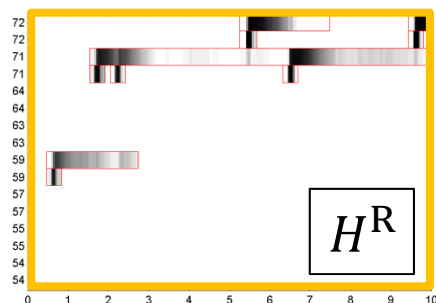


# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix

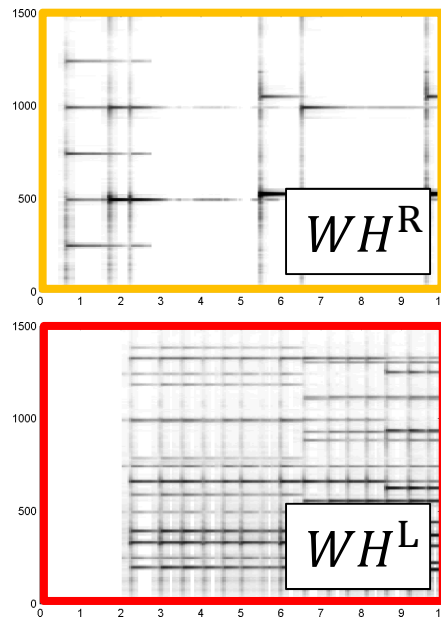
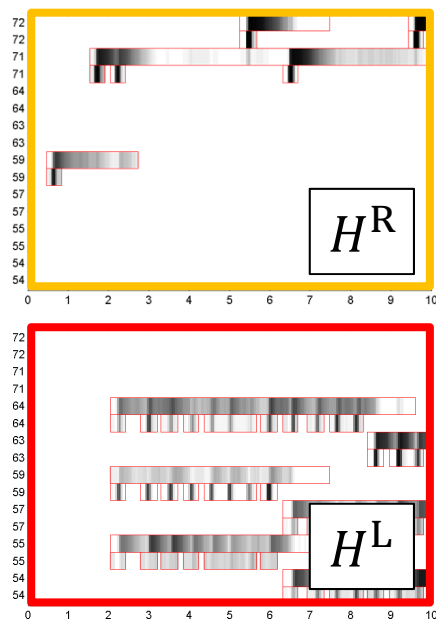


# Score-Informed Audio Decomposition

## Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right

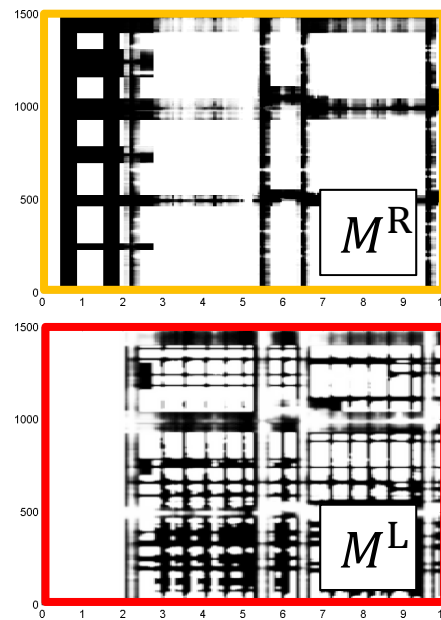
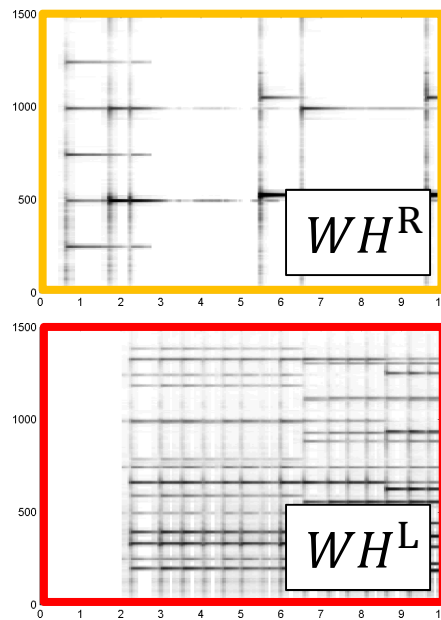
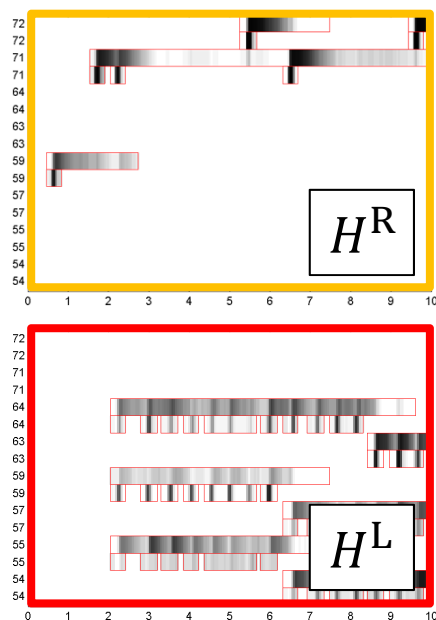


# Score-Informed Audio Decomposition

## Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right
3. Separation masks for left/right

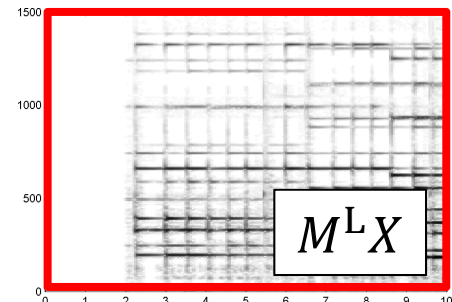
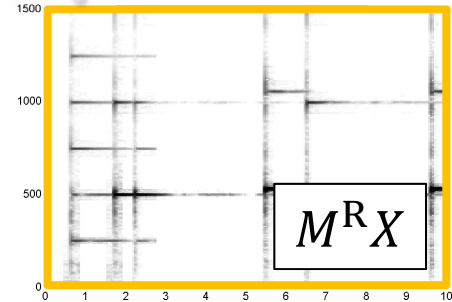
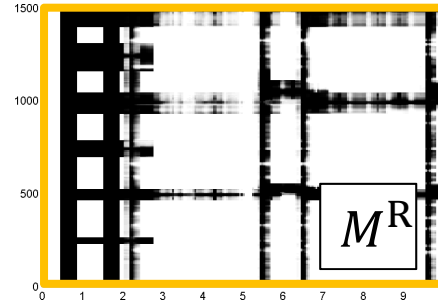
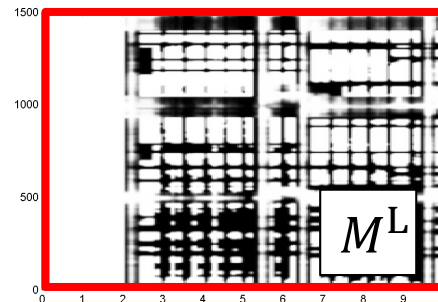
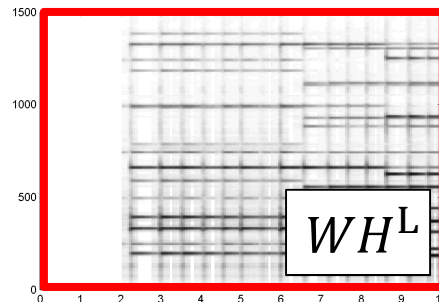
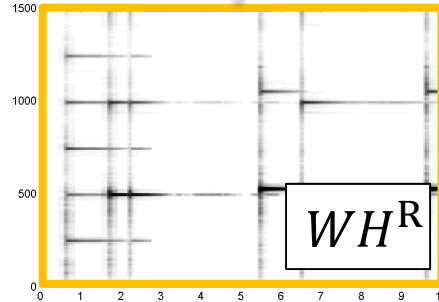
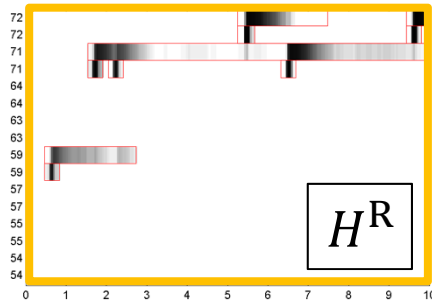


# Score-Informed Audio Decomposition

## Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right
3. Separation masks for left/right
4. Estimated spectrograms for left/right





# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

Chopin, Waltz Op. 64, No. 1

Molto Vivace

leggiero

Original



Ewert, Müller: Using Score-Informed Constraints for NMF-based Source Separation. Proc. ICASSP, 2012.

Further results available at

<http://www.mpi-inf.mpg.de/resources/MIR/ICASSP2012-ScoreInformedNMF/>

# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

Chopin, Waltz Op. 64, No. 1

Molto Vivace

Original

Left/right hand

Right hand

Left hand

Original



Left/right hand



Right hand



Left hand



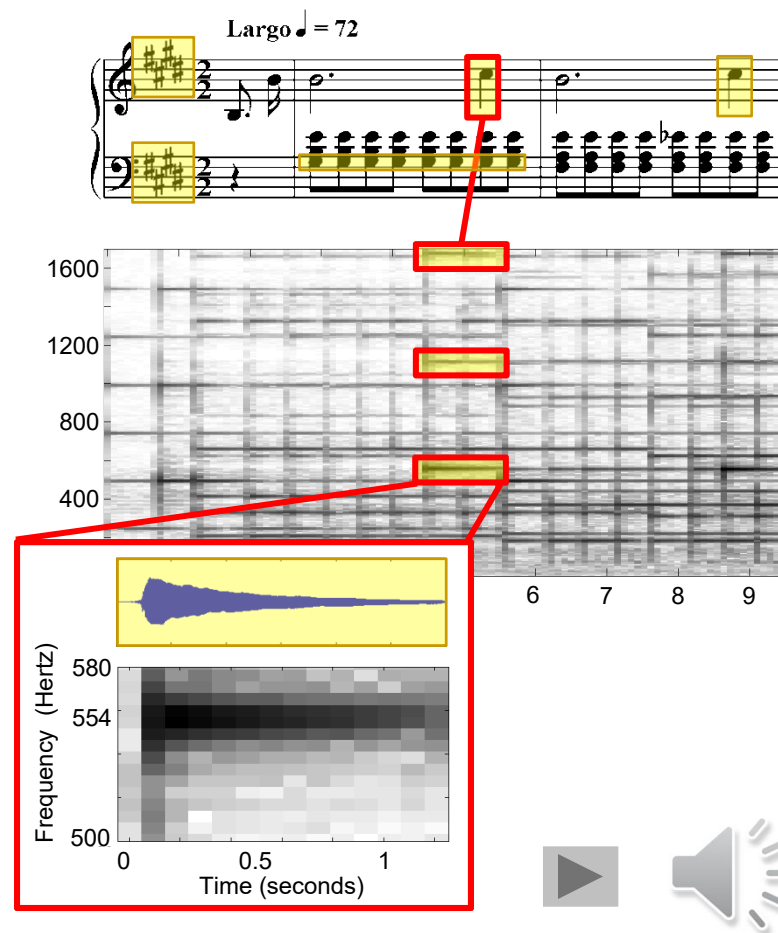
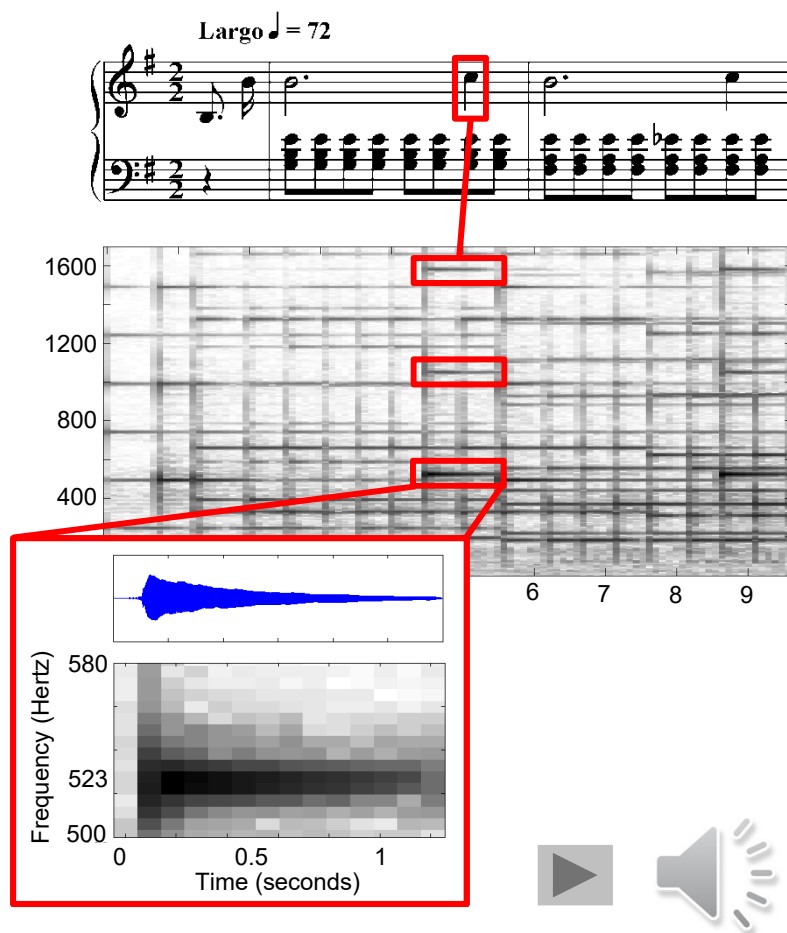
Ewert, Müller: Using Score-Informed Constraints for NMF-based Source Separation. Proc. ICASSP, 2012.

Further results available at

<http://www.mpi-inf.mpg.de/resources/MIR/ICASSP2012-ScoreInformedNMF/>

# Score-Informed Audio Decomposition

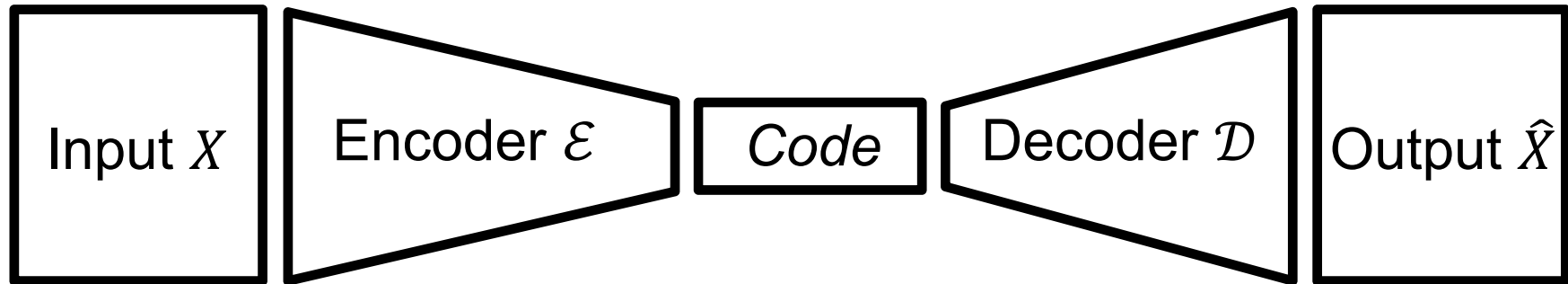
Application: Audio editing



## Conclusions (NMF)

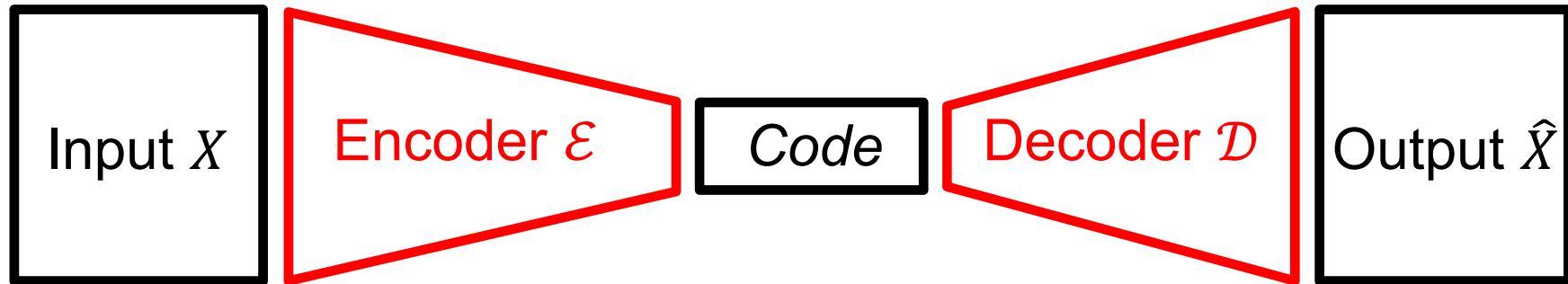
- NMF used for spectrogram decomposition
- Multiplicative update rules make it easy to constrain NMF model via zero initialization
- Exploiting score information to guide separation process (requires score–audio synchronization)
- Application: Separation of arbitrary note groups from given audio recording

# Autoencoder



- Specific type of neural network
- Encoder: Compress input  $X$  into a low-dimensional code
- Decoder: Reconstruct output  $\hat{X}$  from code

# Autoencoder



- Specific type of neural network
- Encoder: Compress input  $X$  into a low-dimensional code
- Decoder: Reconstruct output  $\hat{X}$  from code
- Goal: Learn **parameters** for **encoder** and **decoder** such that output is close to input with respect to some loss function:

$$\mathcal{L}(X, \hat{X}) \approx 0$$

# NMF and Autoencoder (AE)

Smaragdis, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.

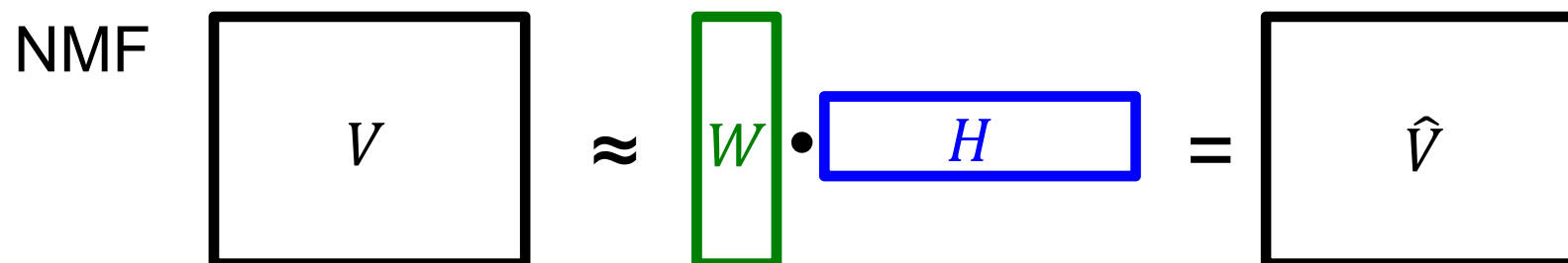
NMF

$$V \approx W \cdot H = \hat{V}$$

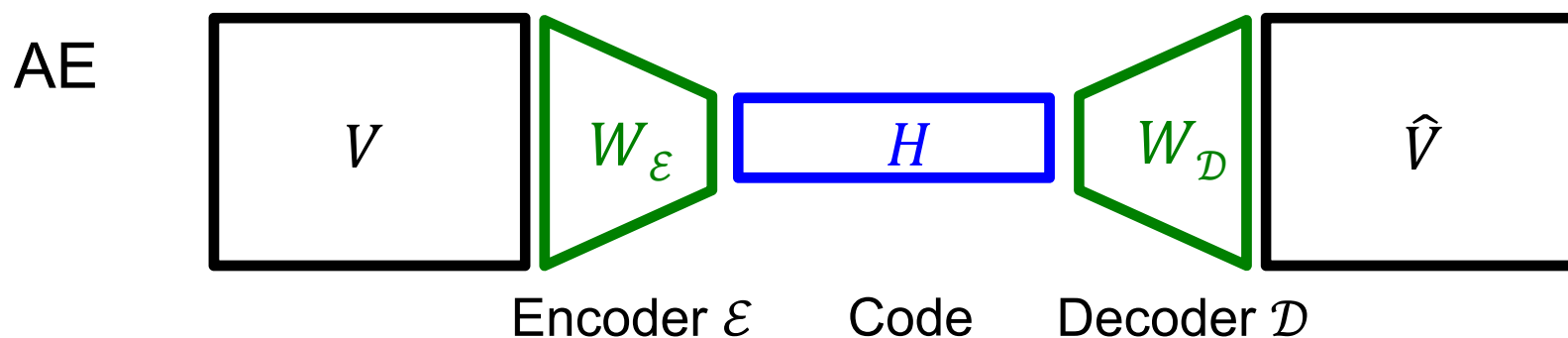
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$

# NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.



$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$

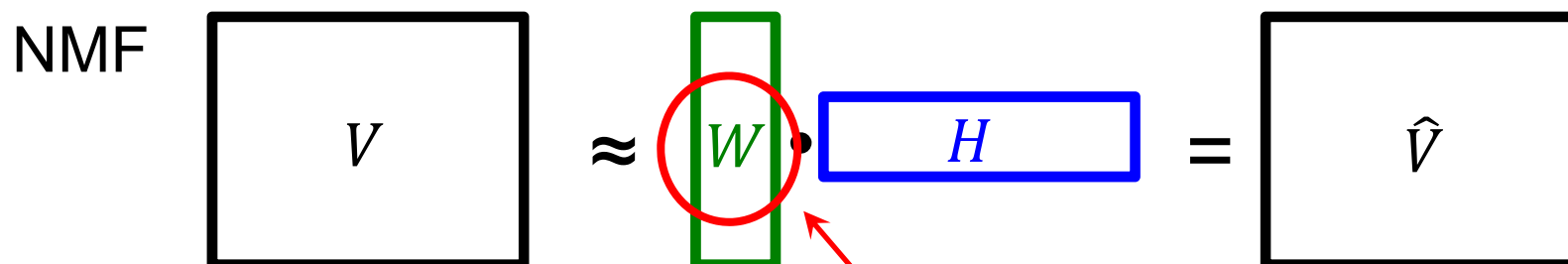


1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

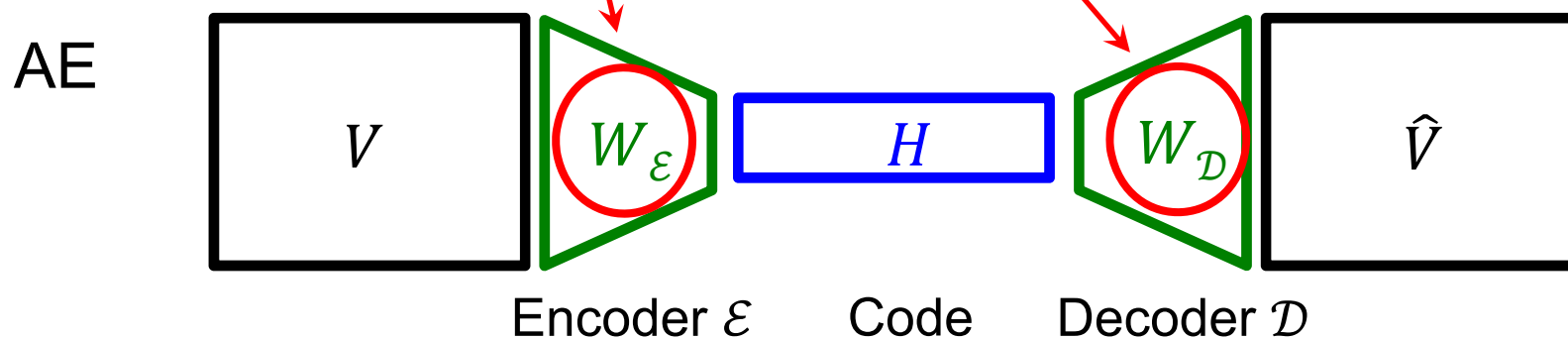


# NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.



$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$

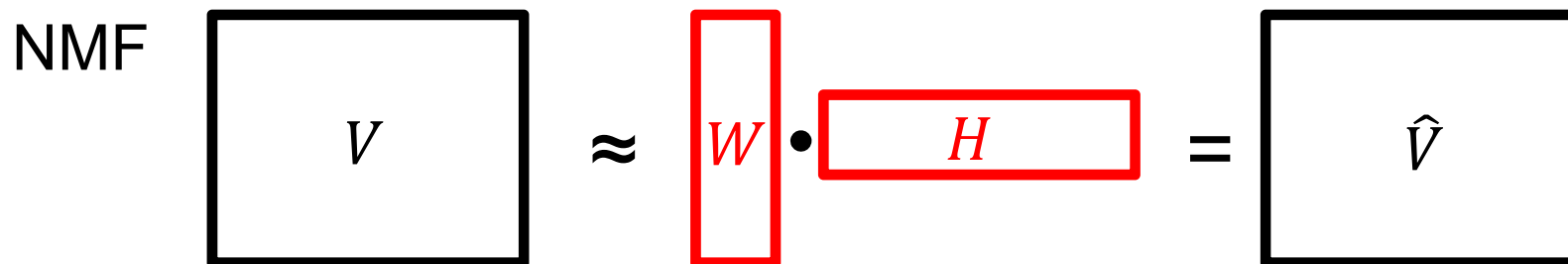


1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

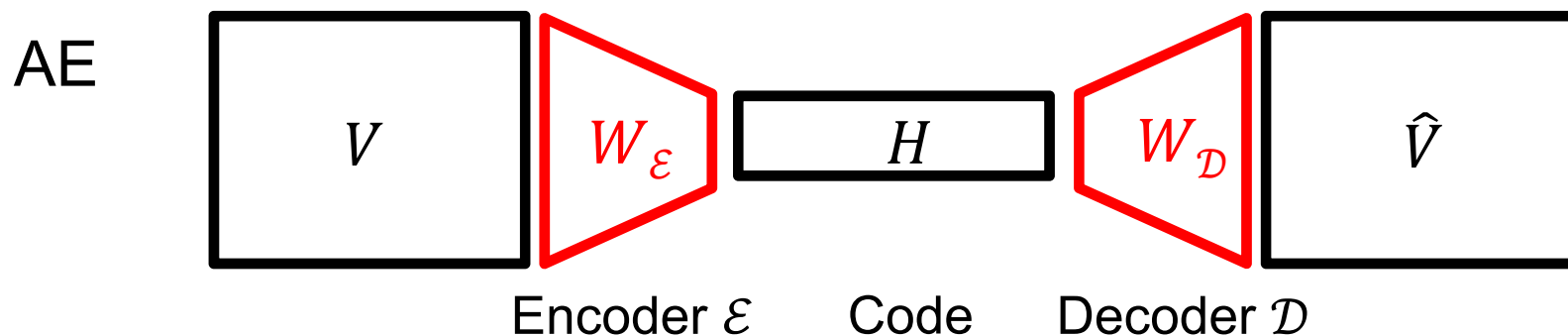
Fully connected network

# NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.



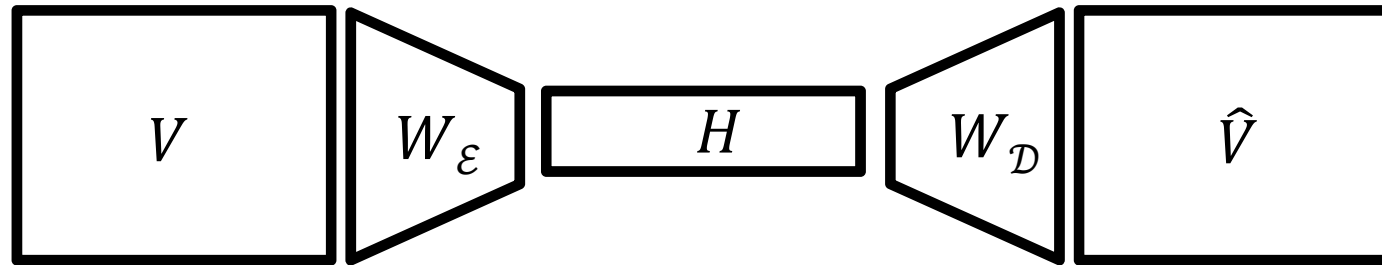
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

NMF: Learn  $H$  and  $W$   
AE: Learn  $W_\epsilon$  and  $W_D$

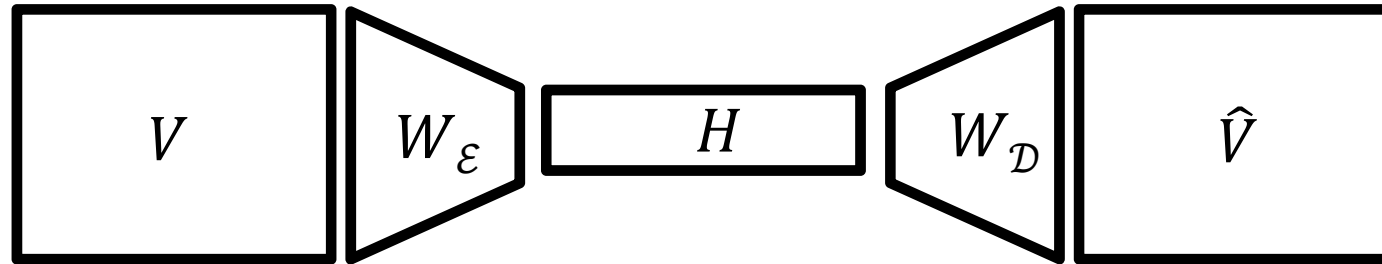
# Nonnegative Autoencoder (NAE)



1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

- How can one adjust the AE to simulate NMF?
- How can one achieve nonnegativity?
- How can one incorporate musical knowledge?
- ...

# Nonnegative Autoencoder (NAE)

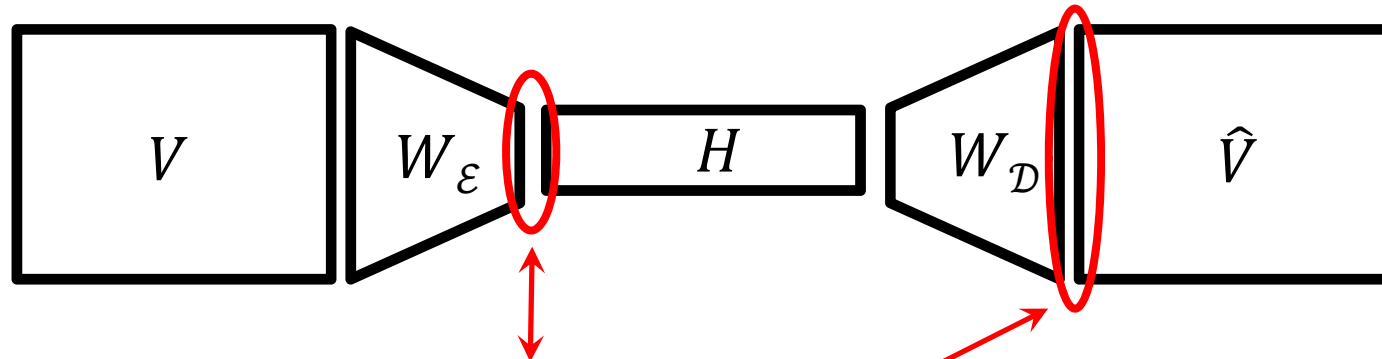


1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

- **Loss function:** same as in NMF

# Nonnegative Autoencoder (NAE)

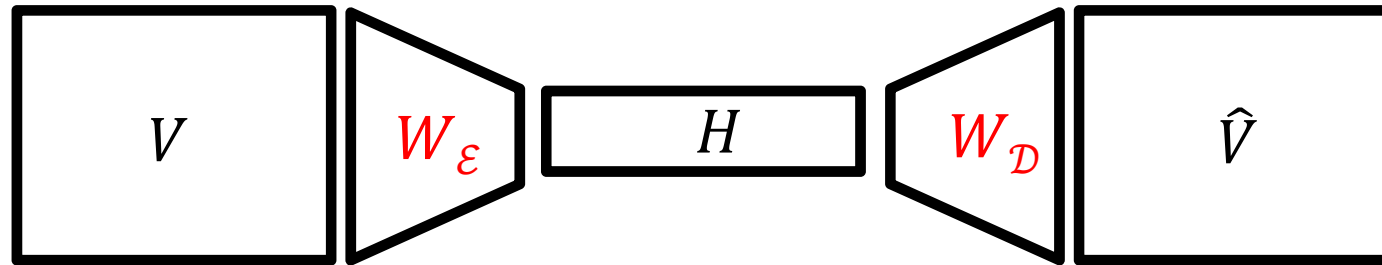


1. Layer:  $H = \max(W_\epsilon V, 0)$
2. Layer:  $\hat{V} = \max(W_D H, 0)$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

- Loss function: same as in NMF
- Activation function (**ReLU**) makes  $H$  and  $\hat{V}$  nonnegative

# Nonnegative Autoencoder (NAE)

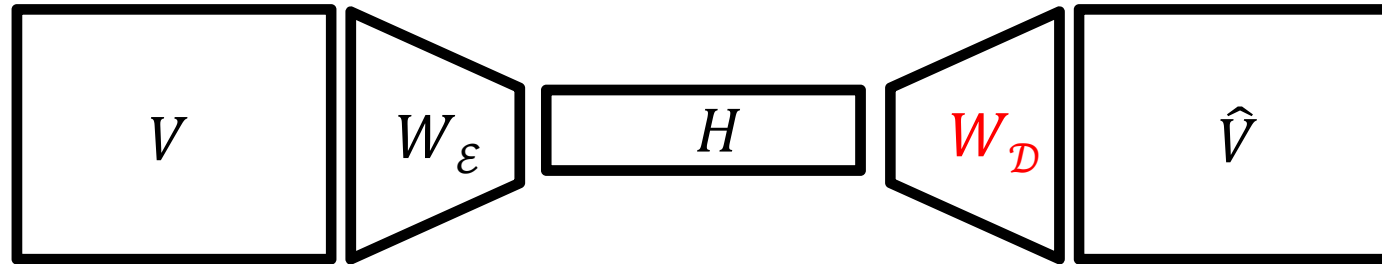


1. Layer:  $H = \max(W_\epsilon V, 0)$
  2. Layer:  $\hat{V} = \max(W_D H, 0)$
- $\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$

$$W_D \leftarrow \max\left(W_D - \gamma \frac{\partial \mathcal{L}}{\partial W_D}, 0\right)$$

- Loss function: same as in NMF
- Activation function (ReLU) makes  $H$  and  $\hat{V}$  nonnegative
- **Projected gradient descent** can be used to keep  $W_D$  (and  $W_\epsilon$ ) nonnegative

# Musical Constraints



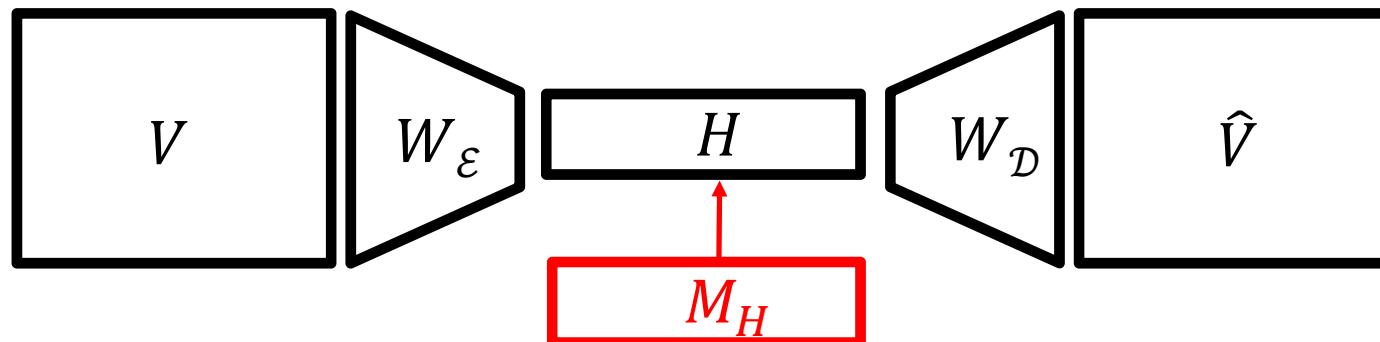
$$H = \max(W_\epsilon V, 0)$$

$$\hat{V} = \max(W_D H, 0)$$

- **Template constraints:** Project certain entries in  $W_D$  to zero values (using projected gradient decent)

# Musical Constraints

Ewert, Sandler: Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation. Proc. ICASSP, 2017.



$$H' = H \odot M_H$$
$$\hat{V} = \max(W_D H', 0)$$

- Template constraints: Project certain entries in  $W_D$  to zero values (using projected gradient decent)
- **Activation constraints: Use structured dropout by applying pointwise multiplication with binary mask  $M_H$**



# NAE with Multiplicative Update Rules

- Multiplicative update rules in NMF:
  - Preserve nonnegativity
  - Lead to fast convergence
- Question: Can one introduce multiplicative update rules to train network weights for NAE?
- Use in additive gradient descent

$$W^{(\ell+1)} = W^{(\ell)} - \gamma \cdot \frac{\partial \mathcal{L}}{\partial W}$$

a suitable (adaptive) learning rate  $\gamma$ .

# NAE with Multiplicative Update Rules

- Encoder:

$$H = W_{\mathcal{E}}V$$

- Structured Dropout:

$$H' = H \odot M_H$$

- Decoder:

$$\hat{V} = W_{\mathcal{D}}H'$$

# NAE with Multiplicative Update Rules

- Encoder:

$$H = W_{\mathcal{E}}V$$

$$W_{\mathcal{E},rk}^{(\ell+1)} = W_{\mathcal{E},rk}^{(\ell)} \cdot \frac{\left( \left( (W_{\mathcal{D}}^{\top} V) \odot M_H \right) V^{\top} \right)_{rk}}{\left( \left( (W_{\mathcal{D}}^{\top} W_{\mathcal{D}} H'^{(\ell)}) \odot M_H \right) V^{\top} \right)_{rk}}$$

- Structured Dropout:

$$H' = H \odot M_H$$

- Decoder:

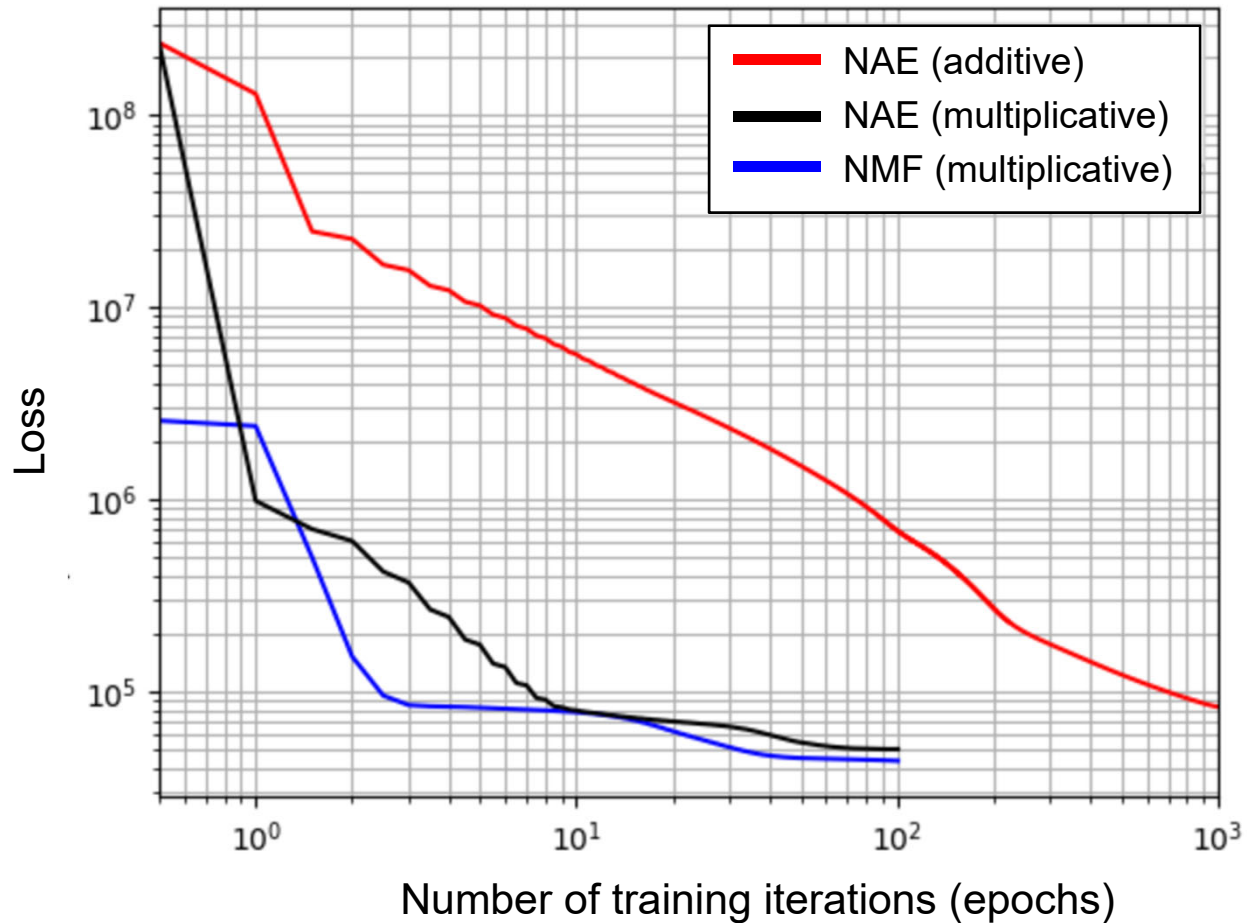
$$\hat{V} = W_{\mathcal{D}}H'$$

$$W_{\mathcal{D},kr}^{(\ell+1)} = W_{\mathcal{D},kr}^{(\ell)} \cdot \frac{(V H'^{\top})_{kr}}{(W_{\mathcal{D}}^{(\ell)} H' H'^{\top})_{kr}}$$

Similar idea and computation as for NMF.

Zunner: Neural Networks with Nonnegativity Constraints for Decomposing Music Recordings. Master Thesis, FAU, 2021.

# Approximation Loss



Zunner: Neural Networks with Nonnegativity Constraints for Decomposing Music Recordings. Master Thesis, FAU, 2021.

# Conclusions (NAE)

- Simulation of NMF:
  - Decoder corresponds to NMF templates
  - Encoder learns a kind of pseudo-inverse
  - Code corresponds to NMF activations
- Nonnegativity can be achieved via
  - activation function (ReLU)
  - projected gradient descent
  - multiplicative update rules
- Musical knowledge can be integrated via
  - removing network weights (template constraints)
  - structured dropout (activation constraints)

# Outlook

- More complex networks
  - Deeper networks (more layers)
  - Different layer types (CNN, RNN, ...) and activation functions
  - Modification of loss function and regularization terms
- Understanding encoder – decoder relationship
  - Nonnegativity
  - Pseudo-inverse
- Update rules
  - Constraints and conversion issues
  - Adaptive learning rates and projected gradient descent

# Audio Mosaicing (Style Transfer)

Target signal: Beatles–Let it be



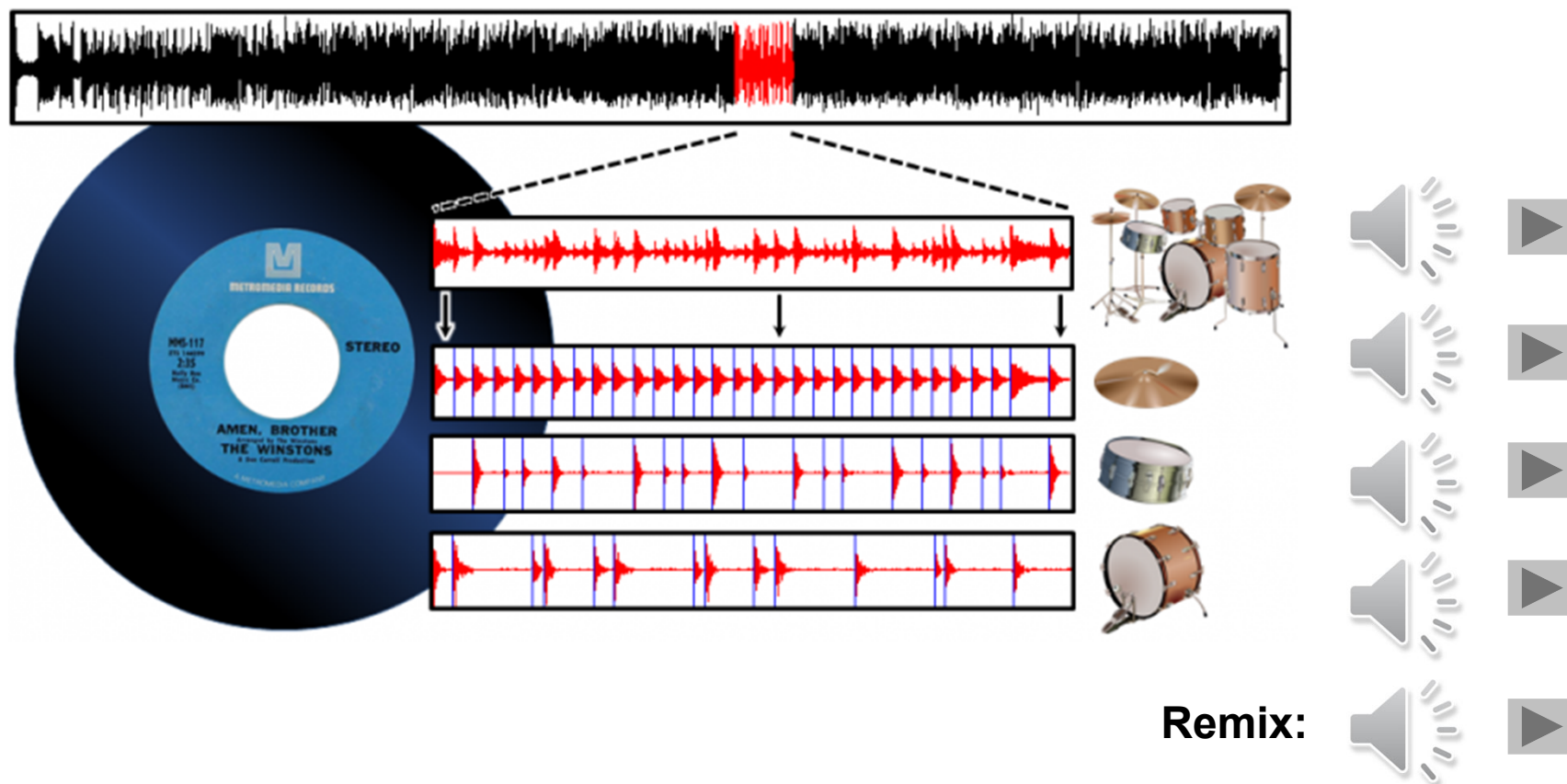
Source signal: Bees



Mosaic signal: **Let it Bee**

Driedger, Prätzlich, Müller: Let It Bee – Towards NMF-Inspired Audio Mosaicing, ISMIR 2015..

# Informed Drum-Sound Decomposition



Dittmar, Müller: Reverse Engineering the Amen Break – Score-Informed Separation and Restoration Applied to Drum Recordings, IEEE/ACM TASLP, 2016.

Suárez: DNN-Based Matrix Factorization with Applications to Drum Sound Decomposition. Master Thesis, FAU, 2020.



# Reconstruction of Sound Events

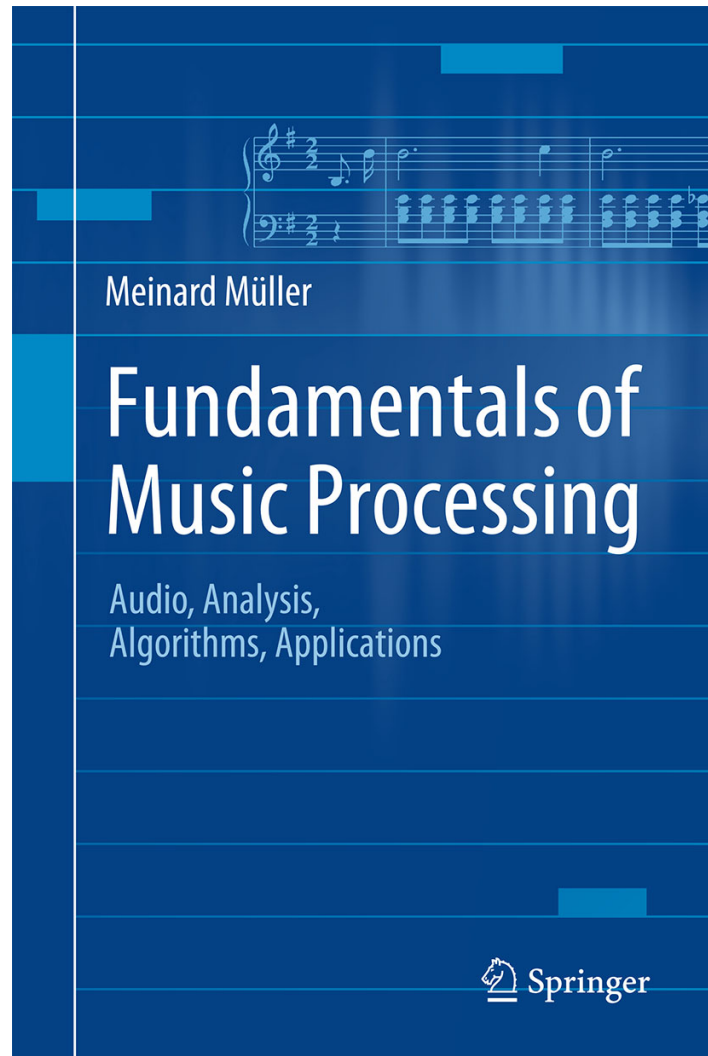
- Reconstruction via spectral masking (Wiener filtering)
- Alternative: Resynthesis approach
- Differentiable Digital Signal Processing (DDSP) combines classical DSP and deep learning
- Generative adversarial networks may help to reduce the artifacts

**Lecture 8: Recurrent and Generative Adversarial Network Architectures for Text-to-Speech**

# Selected Topics in Deep Learning for Audio, Speech, and Music Processing

1. Introduction to Audio and Speech Processing
2. Introduction to Music Processing
3. Permutation Invariant Training Techniques for Speech Separation
4. Deep Clustering for Single-Channel Ego-Noise Suppression
5. Music Source Separation
6. Nonnegative Autoencoders with Applications to Music Audio Decomposing
7. Attention in Sound Source Localization and Speaker Extraction
8. Recurrent and Generative Adversarial Network Architectures for Text-to-Speech
9. Connectionist Temporal Classification (CTC) Loss with Applications to Theme-Based Music Retrieval
10. From Theory to Practise

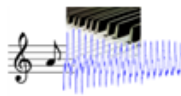

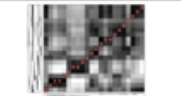
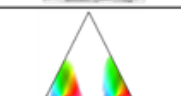

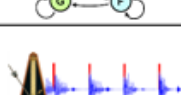


# Book: Fundamentals of Music Processing



Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
483 p., 249 illus., hardcover  
ISBN: 978-3-319-21944-8  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

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Chapter		Music Processing Scenario
1		Music Representations
2		Fourier Analysis of Signals
3		Music Synchronization
4		Music Structure Analysis
5		Chord Recognition
6		Tempo and Beat Tracking
7		Content-Based Audio Retrieval
8		Musically Informed Audio Decomposition

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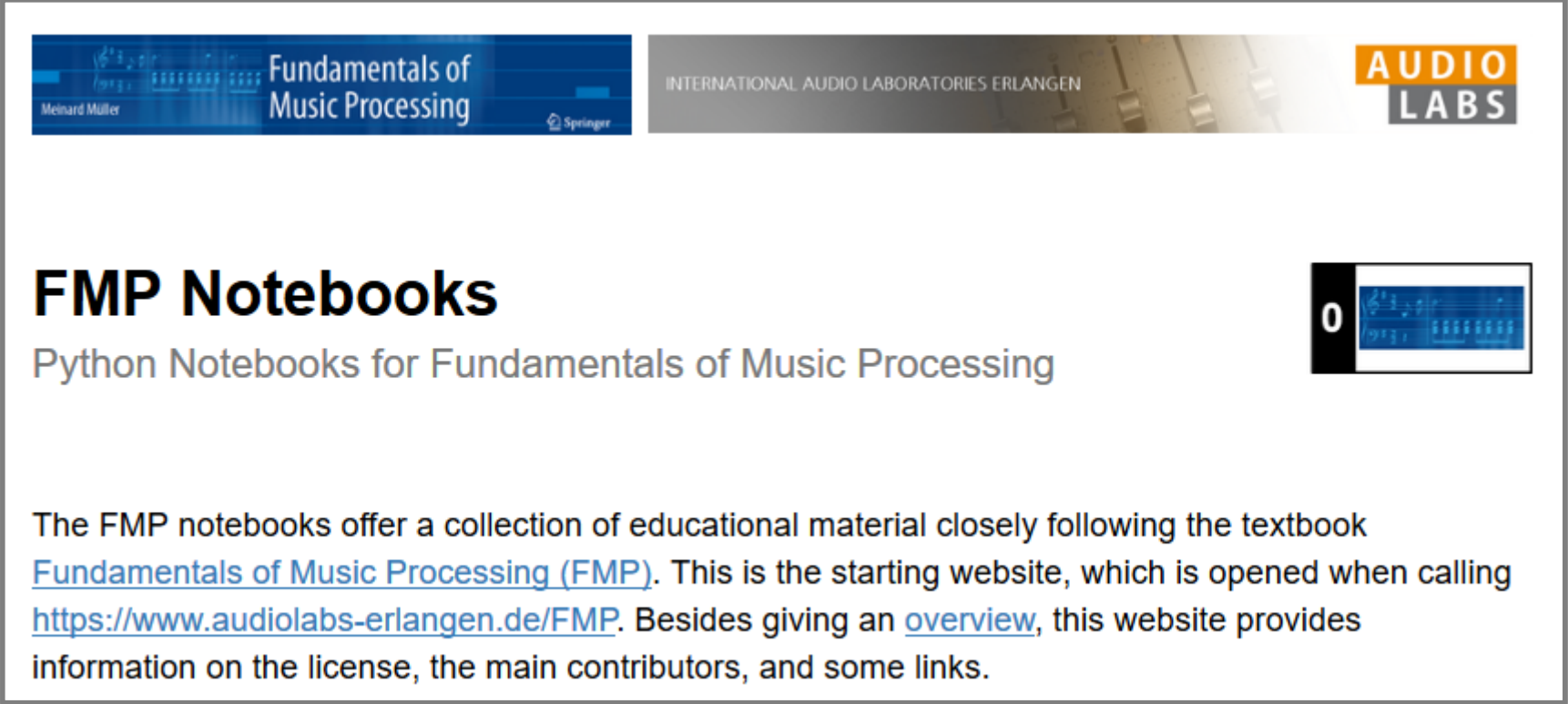
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# Software & Audio: FMP Notebooks



The screenshot shows the header of the FMP Notebooks website. On the left, there is a blue banner for the book "Fundamentals of Music Processing" by Meinard Müller, published by Springer. To the right of this banner is the text "INTERNATIONAL AUDIO LABORATORIES ERLANGEN" and the "AUDIO LABS" logo. Below the banner, the main heading "FMP Notebooks" is displayed in a large, bold, black font. Underneath it, the subtitle "Python Notebooks for Fundamentals of Music Processing" is shown in a smaller, grey font. To the right of the subtitle is a small icon of a notebook with a blue cover and a white page, with a black circle containing the number "0" to its left. Below the subtitle, a paragraph of text describes the FMP notebooks as educational material following the textbook "Fundamentals of Music Processing (FMP)". It provides the starting website URL <https://www.audiolabs-erlangen.de/FMP> and mentions that the website also offers an overview, license information, and contributor details.

<https://www.audiolabs-erlangen.de/FMP>