

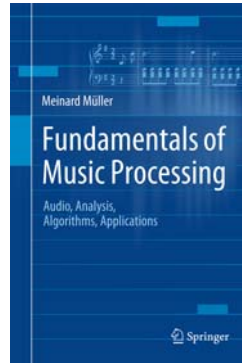
Lecture  
**Music Processing**

## Tempo and Beat Tracking

**Meinard Müller**  
 International Audio Laboratories Erlangen  
 meinard.mueller@audiolabs-erlangen.de



## Book: Fundamentals of Music Processing



Meinard Müller  
 Fundamentals of Music Processing  
 Audio, Analysis, Algorithms, Applications  
 483 p., 249 illus., hardcover  
 ISBN: 978-3-319-21944-8  
 Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

## Book: Fundamentals of Music Processing

Chapter	Music Processing Scenario
1	Music Representations
2	Fourier Analysis of Signals
3	Music Synchronization
4	Music Structure Analysis
5	Chord Recognition
6	Tempo and Beat Tracking
7	Content-Based Audio Retrieval
8	Musically Informed Audio Decomposition

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## Chapter 6: Tempo and Beat Tracking

- 6.1 Onset Detection
- 6.2 Tempo Analysis
- 6.3 Beat and Pulse Tracking
- 6.4 Further Notes



Tempo and beat are further fundamental properties of music. In Chapter 6, we introduce the basic ideas on how to extract tempo-related information from audio recordings. In this scenario, a first challenge is to locate note onset information—a task that requires methods for detecting changes in energy and spectral content. To derive tempo and beat information, note onset candidates are then analyzed with regard to quasiperiodic patterns. This leads us to the study of general methods for local periodicity analysis of time series.

## Introduction

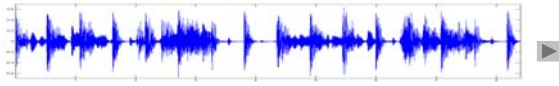
### Basic beat tracking task:

Given an audio recording of a piece of music, determine the periodic sequence of beat positions.

“Tapping the foot when listening to music”

## Introduction

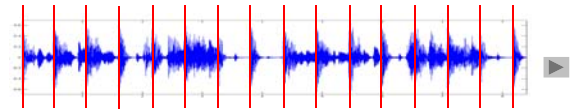
Example: Queen – Another One Bites The Dust



Time (seconds)

## Introduction

Example: Queen – Another One Bites The Dust



Time (seconds)



## Introduction

Example: Happy Birthday to you

Pulse level: **Measure**



Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py  
Birth - day dear \_\_\_\_\_, Hap - py Birth - day to you!

## Introduction

Example: Happy Birthday to you

Pulse level: **Tactus (beat)**



Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py  
Birth - day dear \_\_\_\_\_, Hap - py Birth - day to you!

## Introduction

Example: Happy Birthday to you

Pulse level: **Tatum (temporal atom)**




Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py  
Birth - day dear \_\_\_\_\_, Hap - py Birth - day to you!

## Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: ??? 

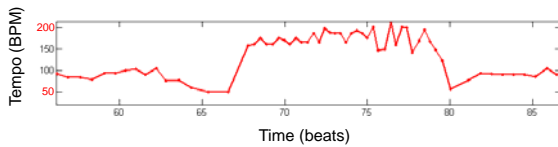
## Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: 50-200 BPM ▶

Tempo curve



## Introduction

Example: Borodin – String Quartet No. 2

Pulse level: Quarter note

Tempo: 120-140 BPM (roughly)

Beat tracker without any prior knowledge ▶

Beat tracker with prior knowledge on rough tempo range ▶

## Introduction

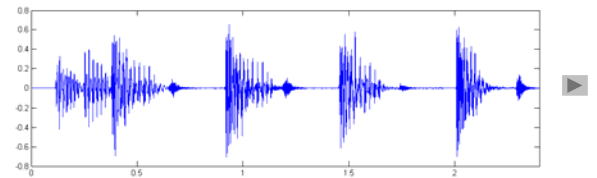
### Challenges in beat tracking

- Pulse level often unclear
- Local/sudden tempo changes (e.g. rubato)
- Vague information (e.g., soft onsets, extracted onsets corrupt)
- Sparse information (often only note onsets are used)

## Introduction

Tasks

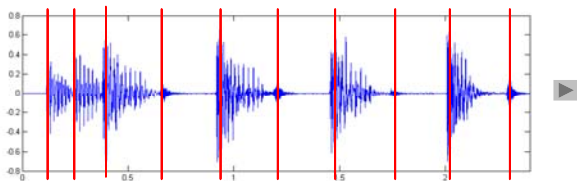
- Onset detection
- Beat tracking
- Tempo estimation



## Introduction

Tasks

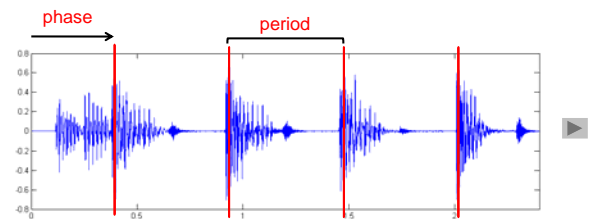
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## Introduction

Tasks

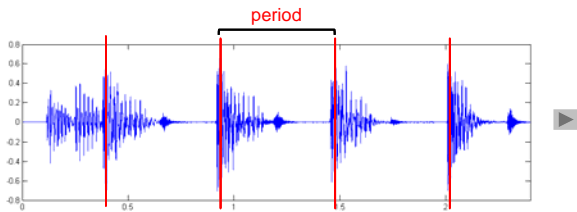
- Onset detection
- Beat tracking
- Tempo estimation



## Introduction

### Tasks

- Onset detection
  - Beat tracking
  - Tempo estimation
- $\text{Tempo} := 60 / \text{period}$   
 $\text{Beats per minute (BPM)}$

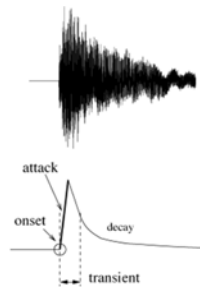


## Onset Detection

- Finding start times of perceptually relevant acoustic events in music signal
- Onset is the time position where a note is played
- Onset typically goes along with a change of the signal's properties:
  - energy or loudness
  - pitch or harmony
  - timbre

## Onset Detection

- Finding start times of perceptually relevant acoustic events in music signal
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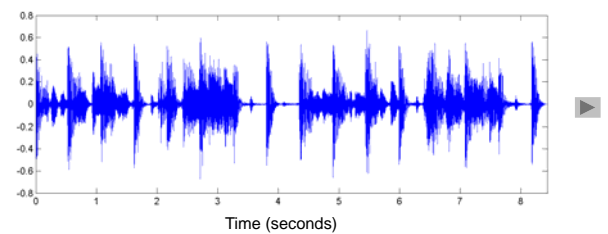


[Bello et al., IEEE-TASLP 2005]

## Onset Detection (Energy-Based)

### Steps

#### Waveform

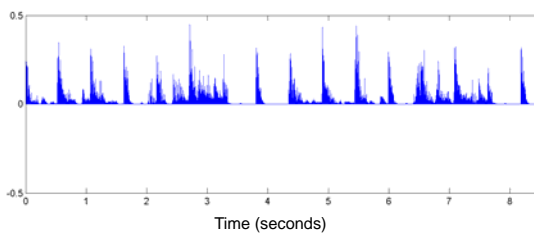


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring

#### Squared waveform

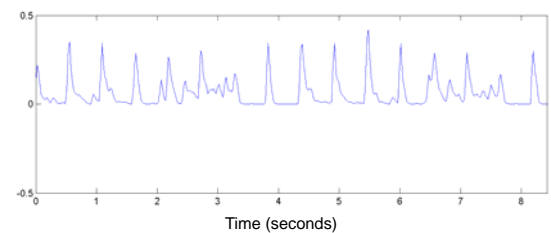


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring
2. Windowing

#### Energy envelope

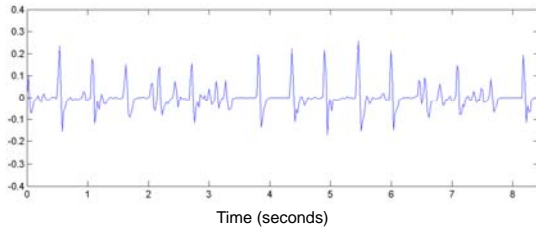


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring
2. Windowing
3. Differentiation Capturing energy changes

Differentiated energy envelope

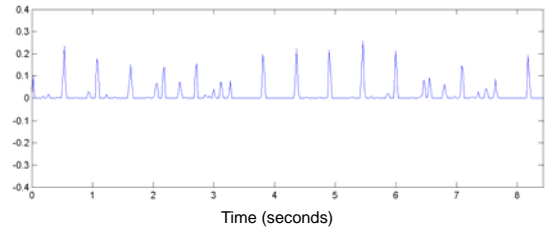


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring
2. Windowing
3. Differentiation
4. Half wave rectification Only energy increases are relevant for note onsets

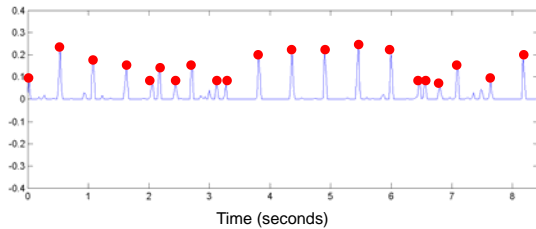
Novelty curve



## Onset Detection (Energy-Based)

### Steps

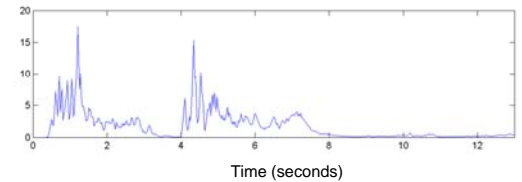
1. Amplitude squaring
2. Windowing
3. Differentiation Peak positions indicate note onset candidates
4. Half wave rectification
5. Peak picking



## Onset Detection (Energy-Based)



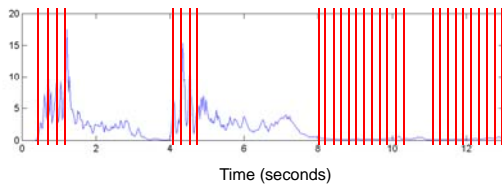
Energy envelope



## Onset Detection (Energy-Based)



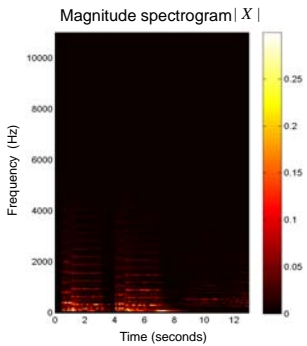
Energy envelope / note onsets positions



## Onset Detection

- Energy curves often only work for percussive music
- Many instruments such as strings have weak note onsets
- No energy increase may be observable in complex sound mixtures
- More refined methods needed that capture
  - changes of spectral content
  - changes of pitch
  - changes of harmony

## Onset Detection (Spectral-Based)

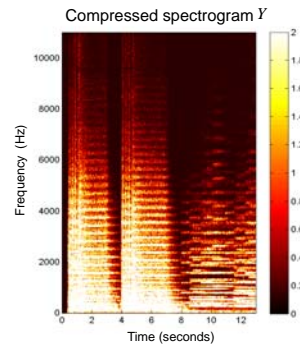


### Steps:

1. Spectrogram

- Aspects concerning pitch, harmony, or timbre are captured by spectrogram
- Allows for detecting local energy changes in certain frequency ranges

## Onset Detection (Spectral-Based)



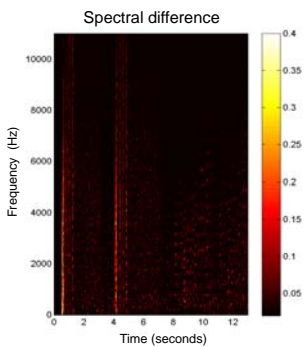
### Steps:

1. Spectrogram
2. Logarithmic compression

$$Y = \log(1 + C \cdot |X|)$$

- Accounts for the logarithmic sensation of sound intensity
- Dynamic range compression
- Enhancement of low-intensity values
- Often leading to enhancement of high-frequency spectrum

## Onset Detection (Spectral-Based)

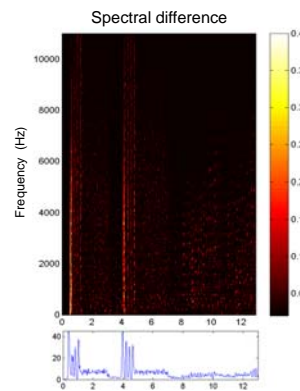


### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation

- First-order temporal difference
- Captures changes of the spectral content
- Only positive intensity changes considered

## Onset Detection (Spectral-Based)



### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation

- Frame-wise accumulation of all positive intensity changes
- Encodes changes of the spectral content

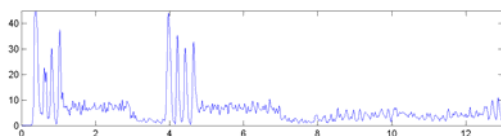
Novelty curve

## Onset Detection (Spectral-Based)

### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation

Novelty curve



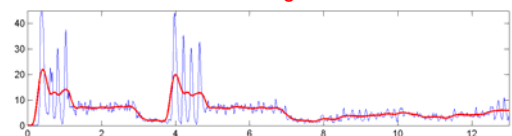
## Onset Detection (Spectral-Based)

### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization

Novelty curve

Subtraction of local average

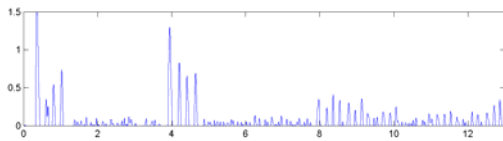


## Onset Detection (Spectral-Based)

### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization

Normalized novelty curve

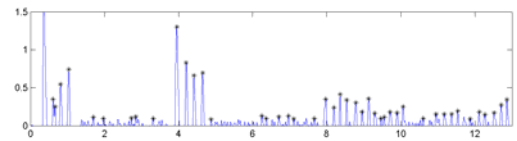


## Onset Detection (Spectral-Based)

### Steps:

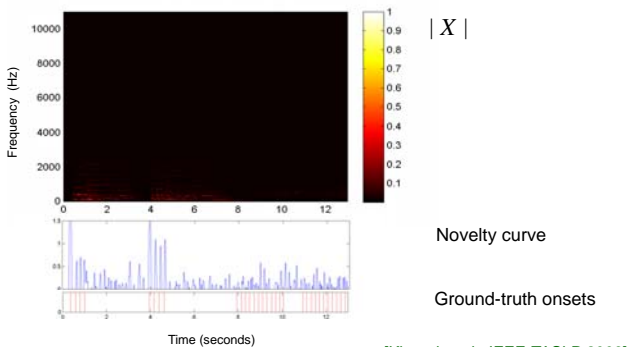
1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization
6. Peak picking

Normalized novelty curve



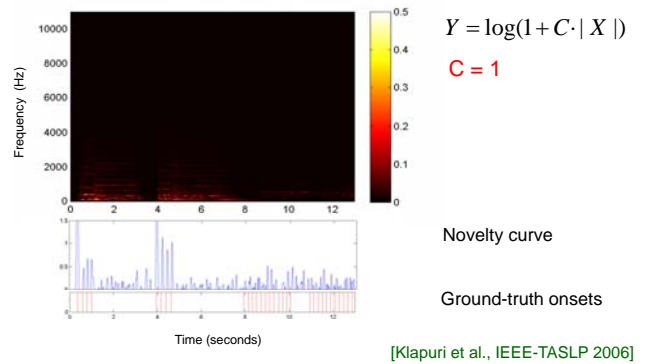
## Onset Detection (Spectral-Based)

Logarithmic compression is essential



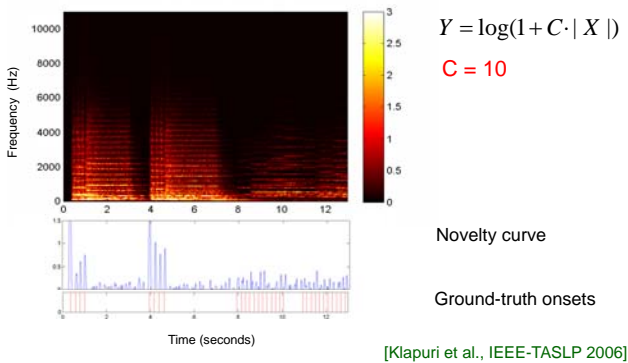
## Onset Detection (Spectral-Based)

Logarithmic compression is essential



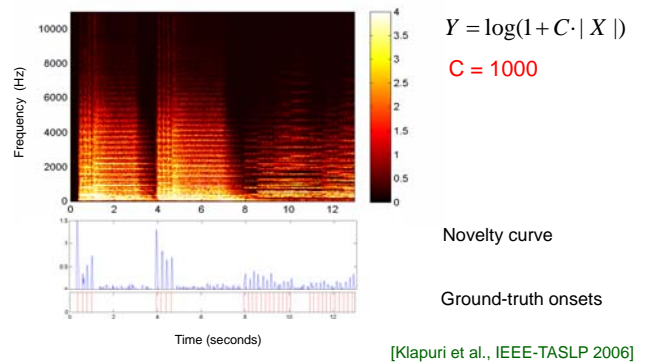
## Onset Detection (Spectral-Based)

Logarithmic compression is essential



## Onset Detection (Spectral-Based)

Logarithmic compression is essential



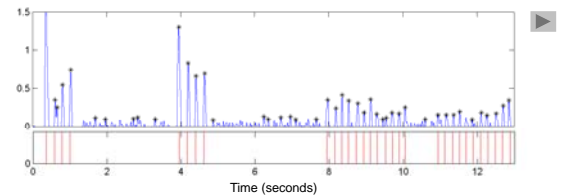
## Onset Detection (Spectral-Based)

- Spectrogram  $X = (X(t, k))_{t, k}$   $t \in [1 : T]$   
 $k \in [1 : K]$
- Compressed Spectrogram  $Y := \log(1 + C \cdot |X|)$   $C > 1$ ,
- Novelty curve  $\Delta : [1 : T - 1] \rightarrow \mathbb{R}$

$$\Delta(t) := \sum_{k=1}^K |Y(t+1, k) - Y(t, k)|_{\geq 0}$$

## Onset Detection

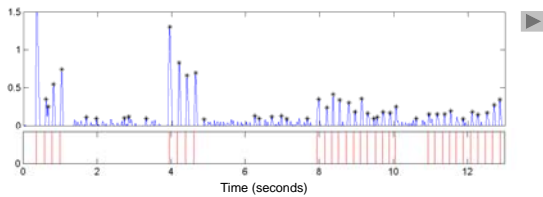
### Peak picking



- Peaks of the novelty curve indicate note onset candidates

## Onset Detection

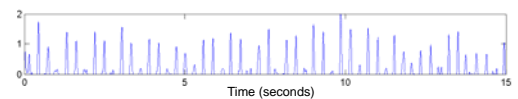
### Peak picking



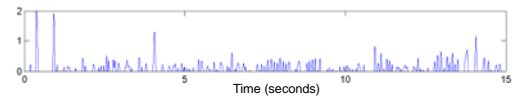
- Peaks of the novelty curve indicate note onset candidates
- In general many spurious peaks
- Usage of local thresholding techniques
- Peak-picking very fragile step in particular for soft onsets

## Onset Detection

### Shostakovich – 2<sup>nd</sup> Waltz



### Borodin – String Quartet No. 2



## Onset Detection

- Drumbeat
- Going Home
- Lyphard melodie
- Por una cabeza
- Donau

## Beat and Tempo

### What is a beat?

- Steady pulse that drives music forward and provides the temporal framework of a piece of music [Parcutt 1994]  
[Sethares 2007]  
[Large/Palmer 2002]
- Sequence of perceived pulses that are equally spaced in time [Lerdah/ Jackendoff 1983]
- The pulse a human taps along when listening to the music [Fitch/ Rosenfeld 2007]

The term **tempo** then refers to the speed of the pulse.



## Beat and Tempo

### Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

## Beat and Tempo

### Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

[Scheirer, JASA 1998]

[Ellis, JNMR 2007]

### Methods

- Comb-filter methods
- Autocorrelation
- Fourier transform

[Davies/Plumbley, IEEE-TASLP 2007]

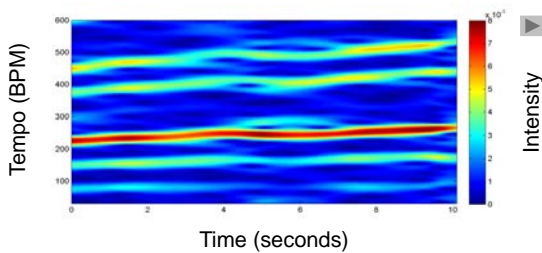
[Peeters, JASP 2007]

[Grosche/Müller, ISMIR 2009]

[Grosche/Müller, IEEE-TASLP 2011]

## Tempogram

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.



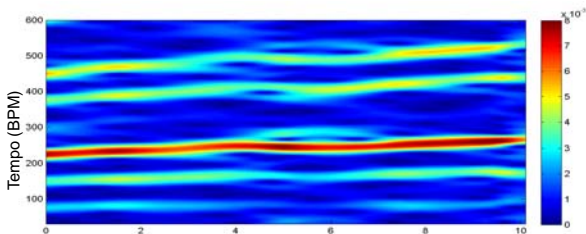
## Tempogram (Fourier)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

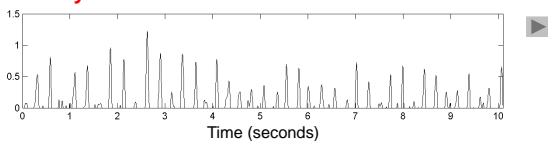
### Fourier-based method

- Compute a spectrogram (STFT) of the novelty curve
- Convert frequency axis (given in Hertz) into tempo axis (given in BPM)
- Magnitude spectrogram indicates local tempo

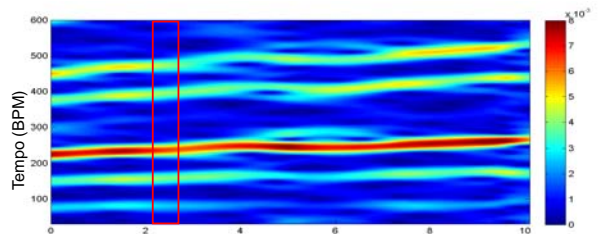
## Tempogram (Fourier)



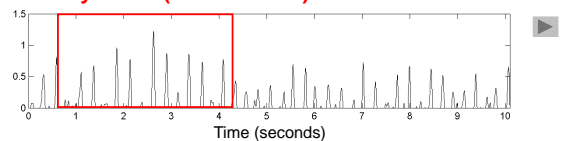
### Novelty curve



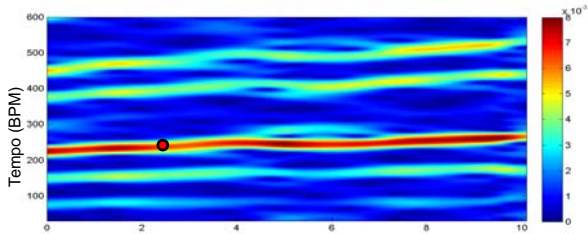
## Tempogram (Fourier)



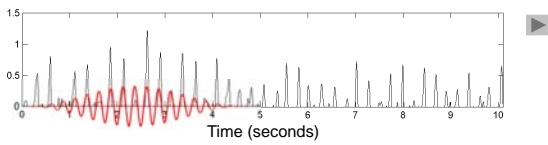
### Novelty curve (local section)



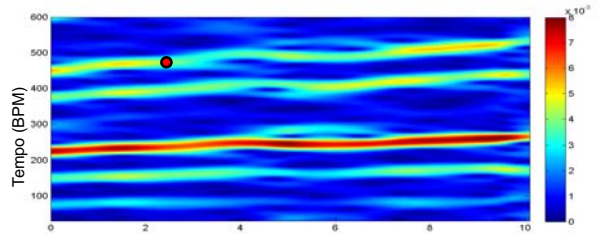
### Tempogram (Fourier)



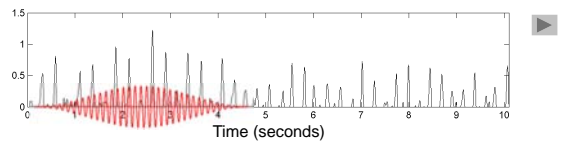
#### Windowed sinusoidal



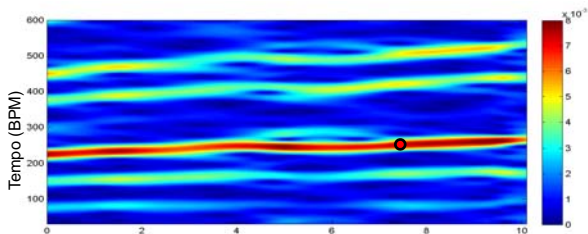
### Tempogram (Fourier)



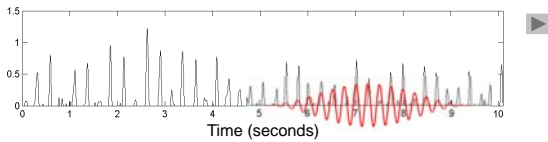
#### Windowed sinusoidal



### Tempogram (Fourier)



#### Windowed sinusoidal



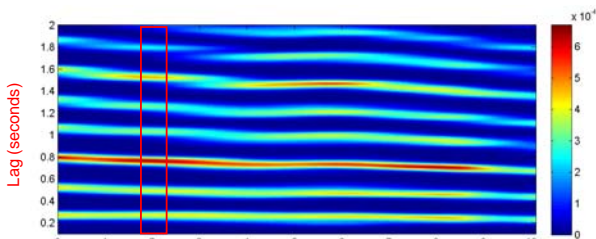
### Tempogram (Autocorrelation)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

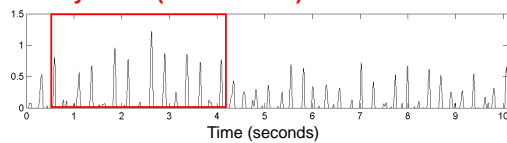
#### Autocorrelation-based method

- Compare novelty curve with time-lagged local sections of itself
- Convert lag-axis (given in seconds) into tempo axis (given in BPM)
- Autocorrelogram indicates local tempo

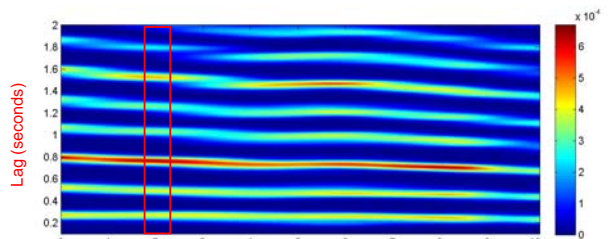
### Tempogram (Autocorrelation)



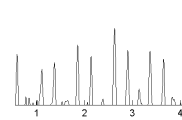
#### Novelty curve (local section)



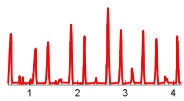
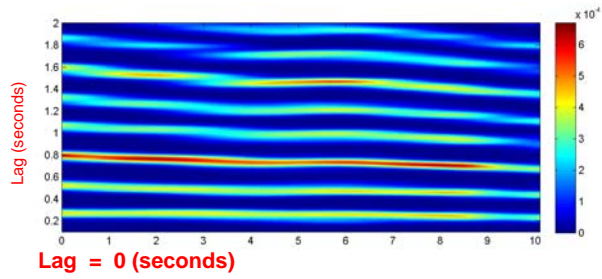
### Tempogram (Autocorrelation)



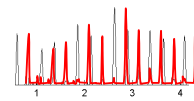
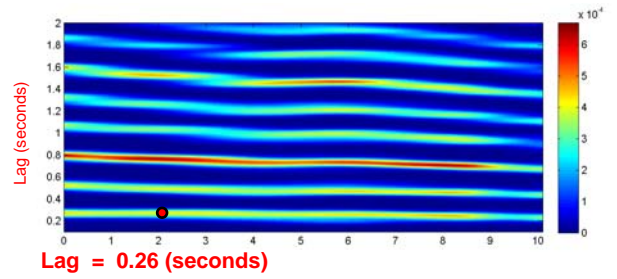
#### Windowed autocorrelation



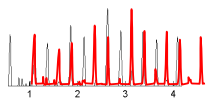
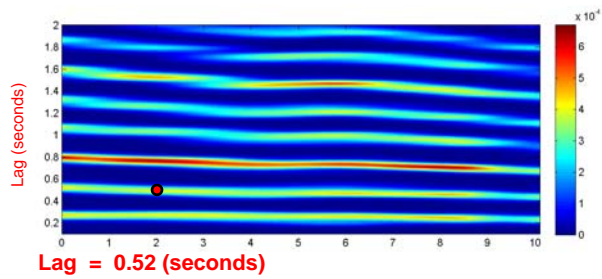
Tempogram (Autocorrelation)



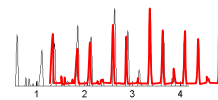
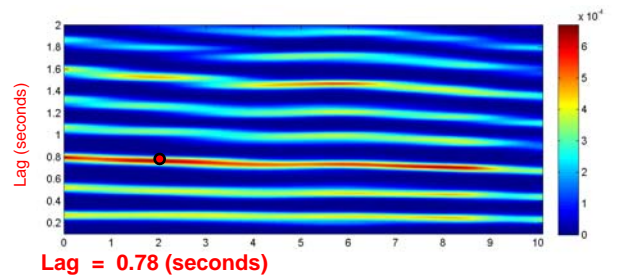
Tempogram (Autocorrelation)



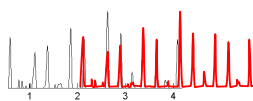
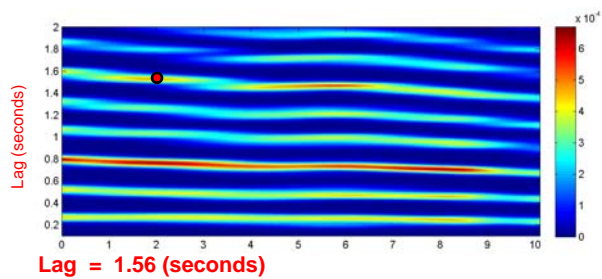
Tempogram (Autocorrelation)



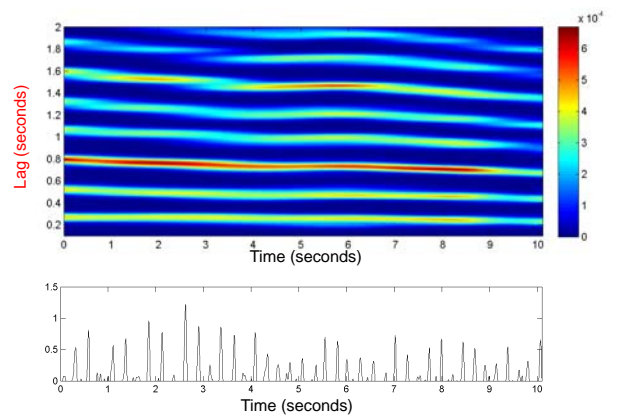
Tempogram (Autocorrelation)



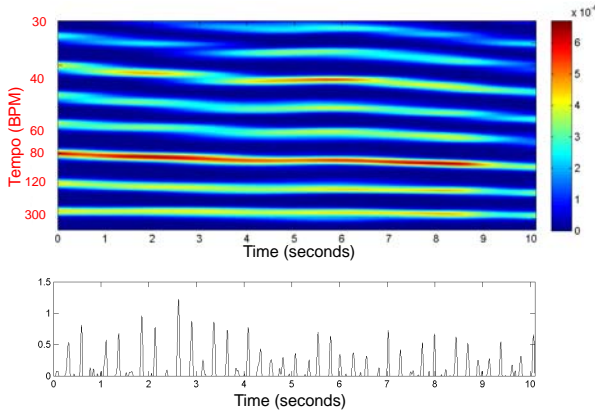
Tempogram (Autocorrelation)



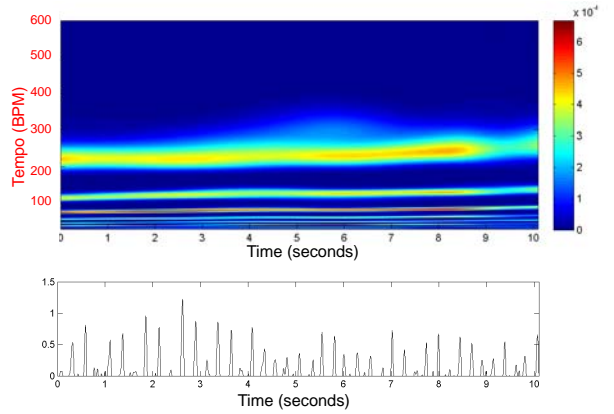
Tempogram (Autocorrelation)



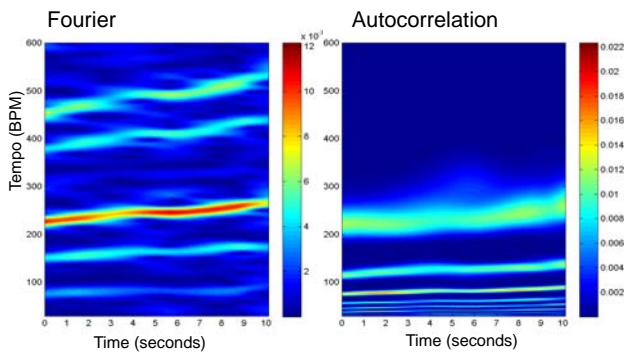
### Tempogram (Autocorrelation)



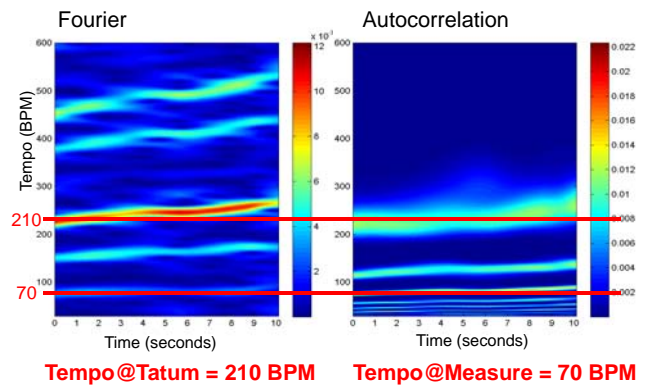
### Tempogram (Autocorrelation)



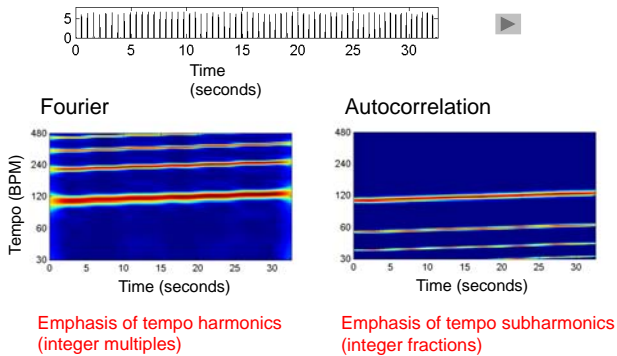
### Tempogram



### Tempogram



### Tempogram



[Peeters, JASP 2007][Grosche et al., ICASSP 2010]

### Tempogram (Summary)

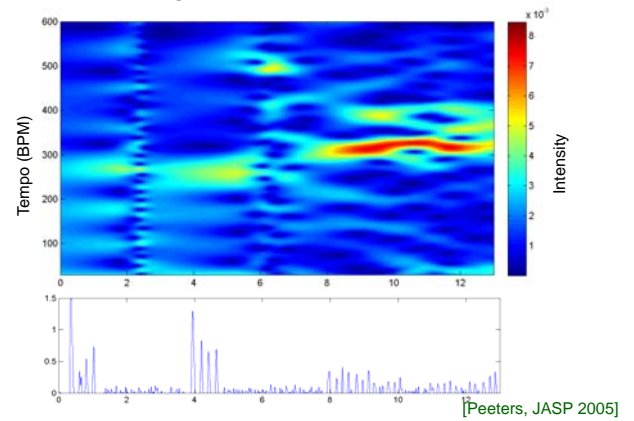
Fourier	Autocorrelation
Novelty curve is compared with sinusoidal kernels each representing a specific tempo	Novelty curve is compared with time-lagged local (windowed) sections of itself
Convert frequency (Hertz) into tempo (BPM)	Convert time-lag (seconds) into tempo (BPM)
Reveals novelty periodicities	Reveals novelty self-similarities
Emphasizes harmonics	Emphasizes subharmonics
Suitable to analyze tempo on tatum and tactus level	Suitable to analyze tempo on tactus and measure level

## Beat Tracking

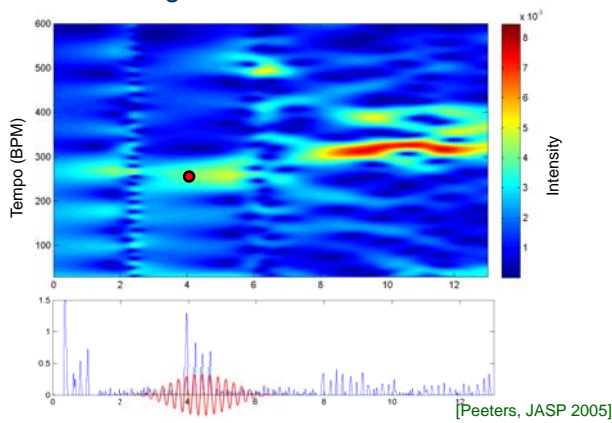
- Given the tempo, find the best sequence of beats
- Complex Fourier tempogram contains **magnitude** and **phase** information
- The **magnitude** encodes how well the novelty curve resonates with a sinusoidal kernel of a specific tempo
- The **phase** optimally aligns the sinusoidal kernel with the peaks of the novelty curve

[Peeters, JASP 2005]

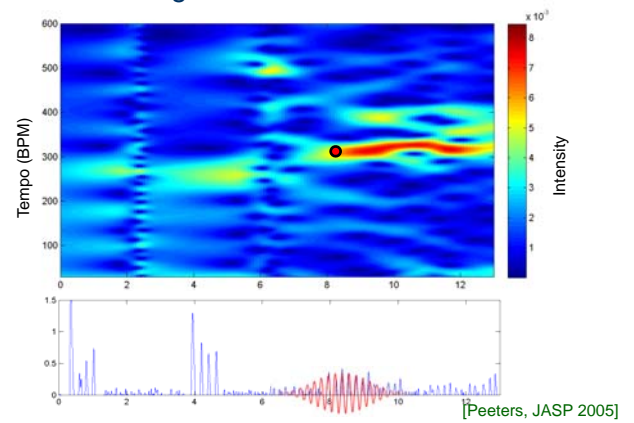
## Beat Tracking



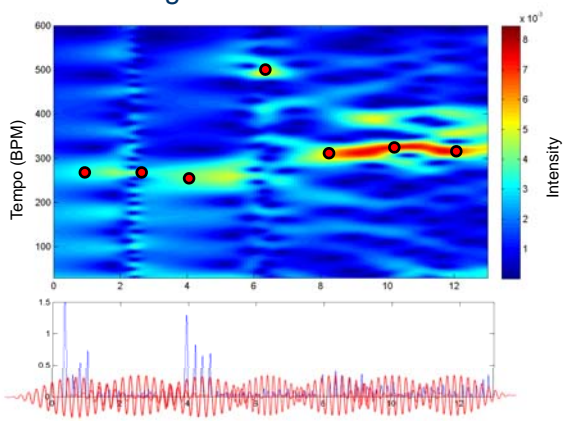
## Beat Tracking



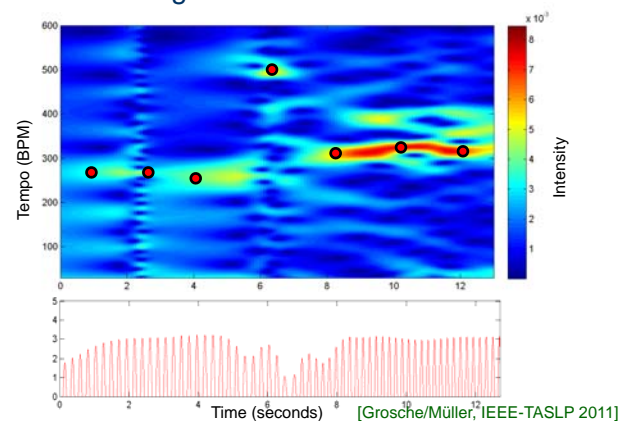
## Beat Tracking



## Beat Tracking



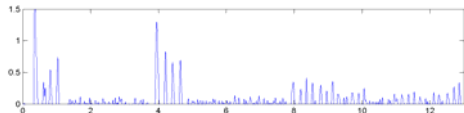
## Beat Tracking



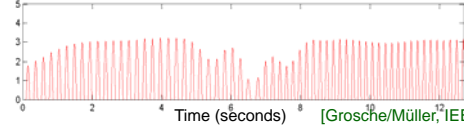
## Beat Tracking



### Novelty Curve



### Predominant Local Pulse (PLP)



[Grosche/Müller, IEEE-TASLP 2011]

## Beat Tracking

### Novelty Curve

- Indicates note onset candidates
- Extraction errors in particular for soft onsets
- Simple peak-picking problematic

### Predominant Local Pulse (PLP)

- Periodicity enhancement of novelty curve
- Accumulation introduces error robustness
- Locality of kernels handles tempo variations

[Grosche/Müller, IEEE-TASLP 2011]

## Beat Tracking

- Local tempo at time  $t$  :  $\tau_t \in \Theta$        $\Theta = [60:240]$  BPM

- Phase  $\varphi_t := \frac{1}{2\pi} \arccos\left(\frac{\text{Re}(\mathcal{T}(t, \tau_t))}{|\mathcal{T}(t, \tau_t)|}\right)$

- Sinusoidal kernel  $\kappa_t : \mathbb{Z} \rightarrow \mathbb{R}$

$$\kappa_t(n) := W(n-t) \cos(2\pi(\tau_t/60 \cdot n - \varphi_t)) \quad n \in \mathbb{Z}$$

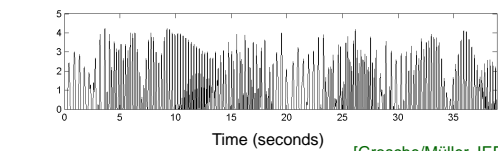
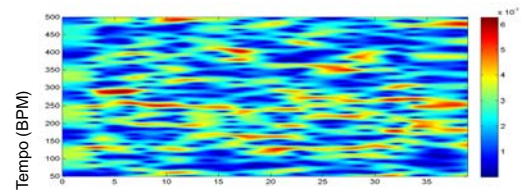
- Periodicity curve  $\Gamma : [1 : T] \rightarrow \mathbb{R}_{\geq 0}$

$$\Gamma(n) = \left| \sum_{t \in [1:T]} \kappa_t(n) \right|_{\geq 0} \quad n \in [1 : T]$$

[Grosche/Müller, IEEE-TASLP 2011]

## Beat Tracking

### Borodin – String Quartet No. 2

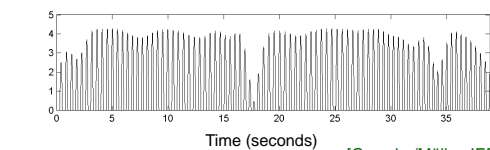
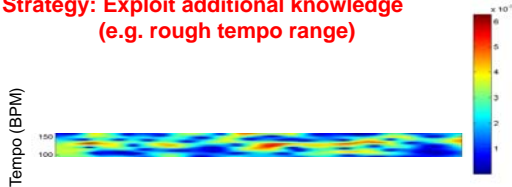


[Grosche/Müller, IEEE-TASLP 2011]

## Beat Tracking

### Borodin – String Quartet No. 2

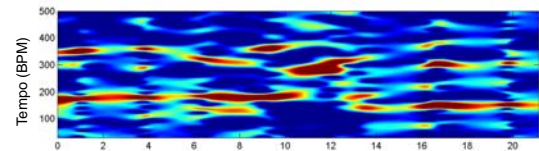
**Strategy: Exploit additional knowledge (e.g. rough tempo range)**



[Grosche/Müller, IEEE-TASLP 2011]

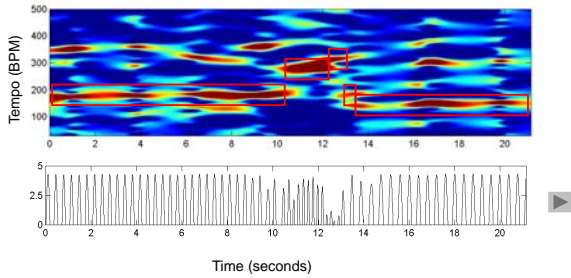
## Beat Tracking

### Brahms Hungarian Dance No. 5



## Beat Tracking

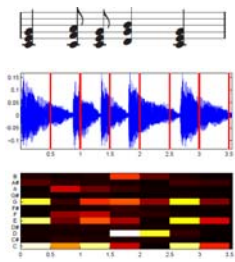
Brahms Hungarian Dance No. 5



## Applications

- Feature design (beat-synchronous features, adaptive windowing)
- Digital DJ / audio editing (mixing and blending of audio material)
- Music classification
- Music recommendation
- Performance analysis (extraction of tempo curves)

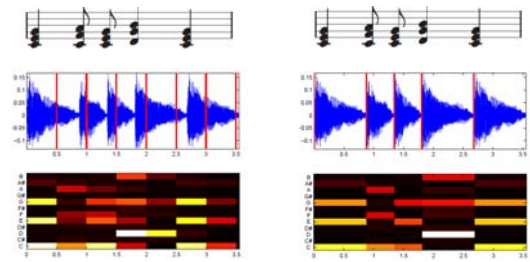
## Application: Feature Design



Fixed window size

[Ellis et al., ICASSP 2008] [Bello/Pickens, ISMIR 2005]

## Application: Feature Design

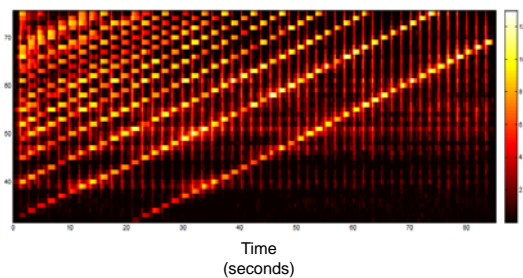


Fixed window size

Adaptive window size

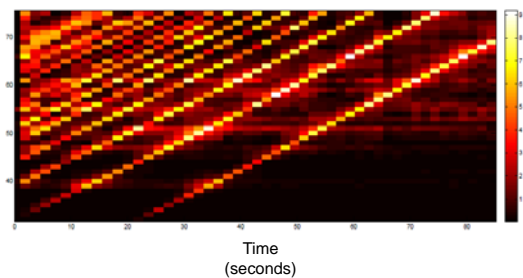
[Ellis et al., ICASSP 2008] [Bello/Pickens, ISMIR 2005]

## Application: Feature Design



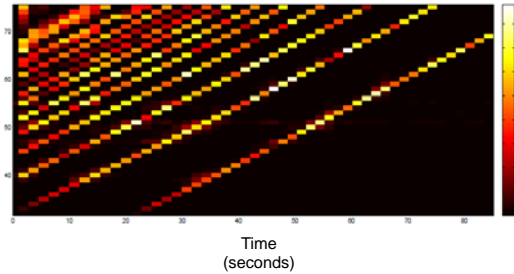
Fixed window size (100 ms)

## Application: Feature Design



Adaptive window size (roughly 1200 ms)  
Note onset positions define boundaries

## Application: Feature Design



Adaptive window size (roughly 1200 ms)  
Note onset positions define boundaries

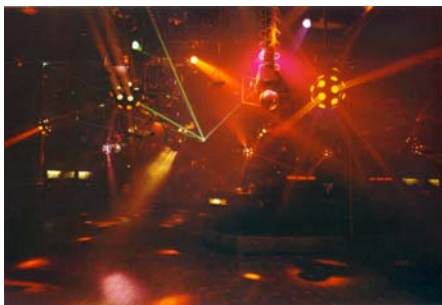
Denoising by excluding boundary neighborhoods

## Application: Audio Editing (Digital DJ)



<http://www.mixxx.org/>

## Application: Beat-Synchronous Light Effects



## Summary

1. Onset Detection
  - Novelty curve (*something is changing*)
  - Indicates note onset candidates
  - Hard task for non-percussive instruments (strings)
2. Tempo Estimation
  - Fourier tempogram
  - Autocorrelation tempogram
  - Musical knowledge (tempo range, continuity)
3. Beat tracking
  - Find most likely beat positions
  - Exploiting phase information from Fourier tempogram