

# PERFORMANCE ANALYSIS OF IPNLMS FOR IDENTIFICATION OF TIME-VARYING SYSTEMS

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## ABSTRACT

The tracking performance of adaptive filters is crucially important in practical applications involving time-varying systems. We present an analysis of the tracking performance for IPNLMS, one of the best known and best performing algorithms originally targeted at sparse system identification. We then validate our analytic results in practical simulations for echo cancellation for sparse and dispersive time-varying unknown echo path systems. These results show the analysis to be highly accurate in all the cases studied.

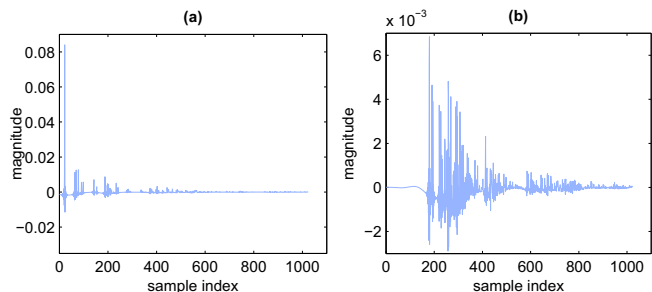
**Index Terms**— Sparse and dispersive impulse responses, Adaptive algorithms, Convergence analysis, Time-varying system identification, IPNLMS algorithm

## 1. INTRODUCTION

With the growing popularity of hands-free mobile telephony, effective acoustic echo cancellation (AEC) is vital to control the acoustic echo generated due to the coupling between the loudspeaker and microphone. The impulse response of the acoustic echo path is typically of length 100-400 ms [1]. Acoustic impulse responses (AIRs) are sensitive to movements of the acoustic source or changes in the acoustic environment and variations in temperature or pressure within an enclosed space. For this reason, adaptive filters have been used to achieve AEC in time-varying environments by tracking the acoustic echo path and thereby continuously predicting the acoustic echo that is received by the microphone.

The normalized least-mean-square (NLMS) algorithm is one of the most popular algorithm for AEC. Since its convergence performance reduces significantly in sparse system identification, sparse adaptive algorithms have been derived from NLMS, including proportionate NLMS (PNLMS) [2] and improved PNLMS (IPNLMS) [3]. By introducing a controlled mixture of proportionate (PNLMS) and non-proportionate (NLMS) adaptation, IPNLMS is a favorable choice as it performs well regardless of the level of sparseness in the AIR [3].

The sparseness of the AIR is dependent on (a) the distance from the loudspeaker to the microphone and (b) the nature and number of sound reflecting surfaces in the vicinity of the microphone. Both (a) and (b) may vary with time when a wireless microphone is used and when the terminal is mobile, respectively, and therefore the sparseness will also be time-varying. An everyday example of time-varying sparseness in the AIR is during a hands-free phone call when the caller starts in an elevator, then moves through the lobby of a building and finally moves outside onto the street. An illustration is shown in Fig. 1 using the image model [4], with different dis-



**Fig. 1.** Acoustic impulse responses obtained using the method of images [4] in a room with dimension of  $8 \times 10 \times 3$  m where the distances between the loudspeaker and microphone are (a) 0.9 m and (b) 7.7 m.

tances between loudspeaker and microphone in a room dimensions of  $8 \times 10 \times 3$  m.

In order to explore the algorithms' performances under such continuously time-varying condition, first-order Markov process [5, 6] is commonly used to model the unknown AIR. Tracking capability of time-variations by the LMS algorithm has already been the subject of several studies including [6, 7]. Recently, the transient behavior of a modified PNLMS algorithm was studied in [8].

The main contribution of this paper is the analysis of the tracking performance of IPNLMS for sparse and dispersive time-varying system. For the analysis we adopt the methodology proposed in [8]. The IPNLMS algorithm is first reviewed in Section 2. We then derive an expression to predict the mean-squared error performance of IPNLMS algorithm, in Section 3, for time-varying systems that can be described by a modified first-order Markov model. Simulation results shown in Section 4 demonstrate that the predicted performance and the actual performance (i.e., ensemble average of simulations) are very similar when the system changes in the context of AEC.

## 2. REVIEW OF IPNLMS

Defining the input signal  $\mathbf{x}(n) = [x(n) \ x(n-1) \ \dots \ x(n-L+1)]^T$  and  $\mathbf{h}(n) = [h_1(n) \ h_2(n) \ \dots \ h_L(n)]^T$  as the unknown impulse response, the desired output with additive noise  $w(n)$  is given by

$$y(n) = \mathbf{h}^T(n)\mathbf{x}(n) + w(n), \quad (1)$$

where  $L$  is the length of the room impulse response and  $[\cdot]^T$  is the transposition operator. The impulse response can be estimated by employing an adaptive filter  $\hat{\mathbf{h}}(n) = [\hat{h}_1(n) \ \hat{h}_2(n) \ \dots \ \hat{h}_L(n)]^T$ .

Many adaptive algorithms are described by the following set of equations:

$$e(n) = y(n) - \hat{\mathbf{h}}^T(n)\mathbf{x}(n), \quad (2)$$

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \frac{\mu \mathbf{Q}(n+1)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta}, \quad (3)$$

$$\mathbf{Q}(n+1) = \text{diag} \{q_1(n+1) \dots q_L(n+1)\}, \quad (4)$$

where  $\mu$  is a step-size and  $\delta$  is the regularization parameter. The diagonal step-size control matrix  $\mathbf{Q}(n)$ , which is algorithm dependent, enables the step-size control for each filter coefficient independently. Therefore,  $q_l(n)$  is commonly proportional to  $h_l(n)$ .

The IPNLMS [3] algorithm employs a combination of proportionate and non-proportionate adaptation, with the relative significance of each controlled by a factor  $\alpha$  such that the diagonal elements of  $\mathbf{Q}(n+1)$  are given as

$$q_l(n+1) = \frac{1-\alpha}{2L} + \frac{(1+\alpha)|\hat{h}_l(n)|}{2\|\hat{\mathbf{h}}(n)\|_1 + \delta_{\text{ip}}}, \quad 1 \leq l \leq L. \quad (5)$$

where  $\|\cdot\|_1$  is defined as the  $l_1$ -norm. Using a higher weighting towards the non-proportionate term, such as  $\alpha = 0, -0.5$  or  $-0.75$ , is a favorable choice for most applications [3].

### 3. CONVERGENCE ANALYSIS OF IPNLMS

In this Section we analyze the convergence performance of IPNLMS for time-varying systems. In Section 3.1, we described a time-varying system model that will be used in Section 3.2 for the theoretical analysis.

#### 3.1. Time-varying System Model

The modified first-order Markov model [5, 6] is widely employed to represent a time-varying system

$$\mathbf{h}(n+1) = \varepsilon \mathbf{h}(n) + \sqrt{1-\varepsilon^2} \mathbf{s}(n), \quad (6)$$

where  $\mathbf{s}(n)$  is a random sequence of length  $L$  with elements drawn from a normal (Gaussian) distribution with zero mean and variance  $\sigma_s^2$ . The parameter  $\varepsilon$  ( $0 \ll \varepsilon < 1$ ) controls the relative contributions to the instantaneous values of the ‘‘system memory’’ and ‘‘innovations’’ [5]. It can be noted that  $\varepsilon = 1$  represents a time-invariant environment. As time progresses, this dynamic model keeps  $E\{\mathbf{h}(n)\}$  constant and the covariance matrix of  $\mathbf{h}(n)$  tends to a finite steady-state value that is equal to the covariance matrix of the sequence  $\{\mathbf{s}(n)\}$ . Hence, the model always gives a dispersive system as  $n \rightarrow \infty$ . However, by initializing  $\mathbf{h}(0)$  to a sparse system and choosing a value close to 1 for  $\varepsilon$ , this model can be employed to simulate a slowly time-varying sparse system.

#### 3.2. Recursive Mean-square Error Analysis

With the weight deviation vector defined as

$$\mathbf{z}(n+1) = \mathbf{h}(n+1) - \hat{\mathbf{h}}(n+1), \quad (7)$$

using (1) and (2),  $e(n)$  can be rearranged as

$$e(n) = w(n) + \sum_{j=1}^L x_j(n)z_j(n), \quad (8)$$

where  $x_l(n) \triangleq x(n-l+1)$ . Hence, the mean-square output error (MSE) can be written as

$$\text{MSE}(n) = E\{e^2(n)\} = \sigma_w^2 + \sigma_x^2 \sum_{l=1}^L E\{z_l^2(n)\}, \quad (9)$$

where  $\sigma_w^2$  and  $\sigma_x^2$  are the variances of the additive noise and the input signal, respectively. Now, we proceed to find the expected values of the square weight deviations,  $E\{z_l^2(n)\}$ , in order to calculate (9).

By substituting (3) and (6) into (7), with  $e(n)$  defined as in (8), the component-wise weight deviation is given by

$$z_l(n+1) = z_l(n) + (\varepsilon - 1)h_l(n) + \sqrt{1-\varepsilon^2}s_l(n) - \left[ \frac{\mu q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta} \right] \left[ w(n) + \sum_{j=1}^L x_j(n)z_j(n) \right]. \quad (10)$$

It then follows from (10) that

$$\begin{aligned} z_l^2(n+1) &= z_l^2(n) + 2z_l(n)(\varepsilon - 1)h_l(n) + \\ &(\varepsilon - 1)^2 h_l^2(n) + 2z_l(n)\sqrt{1-\varepsilon^2}s_l(n) + \\ &2(\varepsilon - 1)h_l(n)\sqrt{1-\varepsilon^2}s_l(n) + (1-\varepsilon^2)s_l^2(n) - \\ &\left[ \frac{2\mu(\varepsilon - 1)h_l(n)q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta} \right] \left[ w(n) + \sum_{j=1}^L x_j(n)z_j(n) \right] - \\ &\left[ \frac{2\mu z_l(n)q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta} \right] \left[ w(n) + \sum_{j=1}^L x_j(n)z_j(n) \right] + \\ &\left[ \frac{\mu^2 q_l^2(n+1)x_l^2(n)}{(\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta)^2} \right] \left[ w(n) + \sum_{j=1}^L x_j(n)z_j(n) \right]^2 - \\ &\left[ \frac{2\mu\sqrt{1-\varepsilon^2}s_l(n)q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta} \right] \left[ w(n) + \sum_{j=1}^L x_j(n)z_j(n) \right]. \end{aligned} \quad (11)$$

For the subsequent theoretical analysis, we will rely on the following assumptions [8, 9, 10] that have been extensively used in the adaptive filtering literature to match reasonably well with their actual performance:

- I) The step-size  $\mu$  is chosen sufficiently small such that  $z_l(n)$  changes slowly relative to  $x_l(n)$ .
- II) The length of the adaptive filter  $L$  is equivalent to that of the unknown system.
- III) The expected value of the denominator term in (11) and the expected value of its squared can be assumed to be [8]

$$E\left\{ \mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta \right\} = \sigma_x^2 + \delta \quad (12)$$

$$E\left\{ \left( \mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n) + \delta \right)^2 \right\} = (\sigma_x^2 + \delta)^2. \quad (13)$$

- IV) Using the ‘separable approach’ theory developed in [8], for  $a, b \in \{1, 2\}$ ,

$$E\{q_l^a(n)\} = E\{q_l(n)\}^a \quad (14)$$

$$E\{q_l^a(n+1)z_l^b(n)\} = E\{q_l(n+1)\}^a E\{z_l^b(n)\} \quad (15)$$

- V) The  $l^{\text{th}}$  component of the weight deviation at each iteration,  $z_l(n)$ , follows a normal distribution with  $\bar{z}_l(n) \triangleq E\{z_l(n)\}$  and variance  $\sigma_l^2(n)$  [8]. This implies that the each adaptive filter coefficient  $\hat{h}_l(n)$  is also distributed as

$$\hat{h}_l(n) \sim \mathcal{N}(m_l(n), \sigma_l^2(n)), \quad (16)$$

with p.d.f

$$f(|\hat{h}_l(n)|) = \frac{1}{\sqrt{2\pi\sigma_l^2(n)}} \left[ e^{-\frac{(|\hat{h}_l(n)|-m_l(n))^2}{2\sigma_l^2(n)}} + e^{-\frac{(|\hat{h}_l(n)|+m_l(n))^2}{2\sigma_l^2(n)}} \right] U(\hat{h}_l(n)), \quad (17)$$

where  $m_l(n) = h_l(n) - \bar{z}_l(n)$ ,  $\sigma_l^2(n) \triangleq E\{z_l^2(n)\} - E^2\{z_l(n)\}$  and

$$U(\hat{h}_l(n)) = \begin{cases} 0, & \hat{h}_l(n) < 0 \\ 1, & \hat{h}_l(n) \geq 0; \end{cases} \quad (18)$$

It follows from (17) that the mean of this distribution is given by

$$\begin{aligned} E\{|\hat{h}_l(n)|\} &= \int_{-\infty}^{\infty} |\hat{h}_l(n)| f(|\hat{h}_l(n)|) d|\hat{h}_l(n)| \\ &= m_l(n) \operatorname{erf}\left(\frac{m_l(n)}{\sqrt{2\sigma_l^2(n)}}\right) + \sqrt{\frac{2}{\pi}}\sigma_l(n)e^{-\frac{m_l^2(n)}{2\sigma_l^2(n)}}, \end{aligned} \quad (19)$$

with  $\operatorname{erf}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ .

By employing these assumptions, the expectations  $E\{\cdot\}$  of the weight deviation in (10) and the square weight deviation in (11) are respectively given by the following recursive forms:

$$\begin{aligned} E\{z_l(n+1)\} &= E\{z_l(n)\} - \frac{\mu\sigma_x^2}{\sigma_x^2 + \delta} E\{q_l(n+1)\} E\{z_l(n)\}, \quad (20) \\ E\{z_l^2(n+1)\} &= E\{z_l^2(n)\} + 2(1-\varepsilon)\sigma_s^2 - \frac{2\mu\sigma_x^2}{\sigma_x^2 + \delta} E\{q_l(n+1)\} E\{z_l^2(n)\} + \\ &\quad \frac{\mu^2\sigma_x^2\sigma_w^2}{(\sigma_x^2 + \delta)^2} E\{q_l(n+1)\}^2 + \quad (21) \\ &\quad \frac{\mu^2\sigma_x^4}{(\sigma_x^2 + \delta)^2} E\{q_l(n+1)\}^2 \sum_{j=1}^L E\{z_j^2(n)\}, \end{aligned}$$

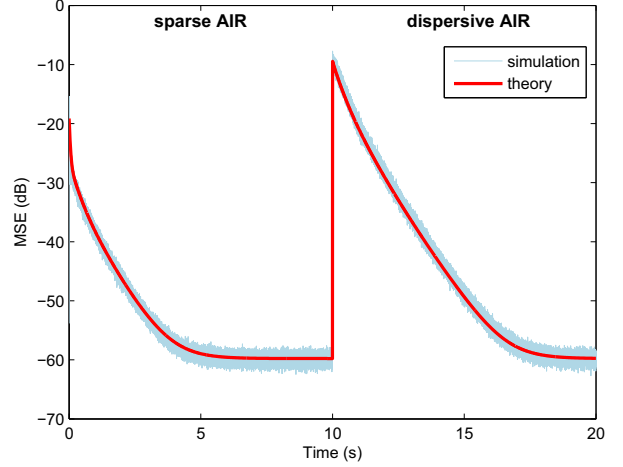
with their initial values assigned to  $E\{z_l(0)\} = h_l(0)$  and  $E\{z_l^2(0)\} = h_l^2(0)$ , and

$$E\{q_l(n+1)\} = \frac{1-\alpha}{2L} + \frac{(1+\alpha)E\{|\hat{h}_l(n)|\}}{2\sum_{j=1}^L E\{|\hat{h}_j(n)|\} + \delta_{ip}}. \quad (22)$$

Given (19)-(22), we can now recursively compute the MSE using (9).

#### 4. SIMULATIONS

The theoretical result derived in the previous Section is confirmed with the Monte Carlo simulations with 100 independent trials, for different time-varying systems scenarios in the context of AEC. In all simulations, the adaptive filter length was set to  $L = 1024$ , a zero mean white Gaussian noise (WGN) was used as the input



**Fig. 2.** MSE of IPNLMS with a sudden echo path change at 10 s. Impulse response is changed from that shown from Fig. 1 (a) to (b) and  $\mu = 0.1$ ,  $\alpha = -0.75$ ,  $\sigma_x^2 = 10^{-2}$ ,  $\sigma_w^2 = 10^{-6}$ ,  $\delta = \delta_{ip} = 10^{-4}$ .

signal  $x(n)$  while another WGN sequence  $w(n)$  was used with  $\sigma_w^2 = 10^{-6}$ . The proportionality control factor for IPNLMS was set to  $\alpha = -0.75$  and the regularization parameters were set to  $\delta = \delta_{ip} = 10^{-4}$ . Room impulse responses  $\mathbf{h}(n)$  have been used as described in Fig. 1. The sparseness measure of these impulse responses are computed using [11]

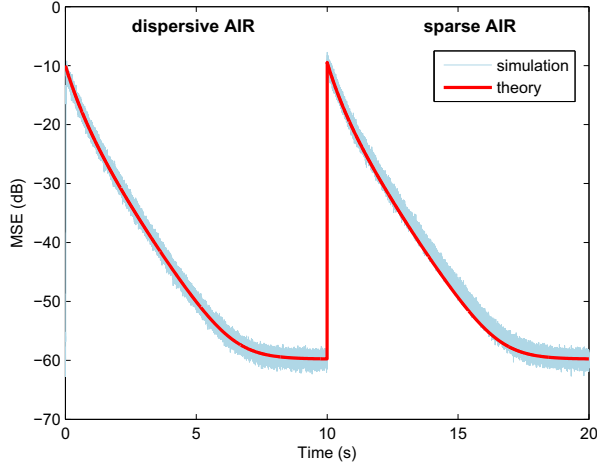
$$\xi(n) = \frac{L}{L - \sqrt{L}} \left\{ 1 - \frac{\|\mathbf{h}(n)\|_1}{\sqrt{L} \|\mathbf{h}(n)\|_2} \right\} \quad (23)$$

giving (a)  $\xi(n) = 0.83$  and (b)  $\xi(n) = 0.59$  respectively.

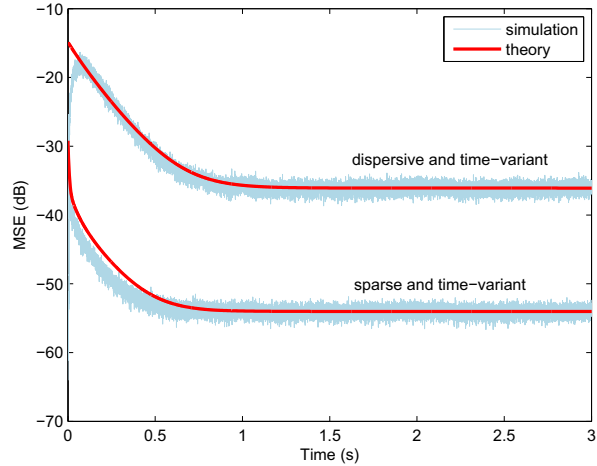
First, we compare the theoretical and simulated MSEs for two types of echo path changes. The step-size  $\mu = 0.1$ ,  $\varepsilon = 1$ , and  $\sigma_x^2 = 10^{-2}$  were used. In Fig. 2, we show the results obtained when the echo path changes from sparse to dispersive. It can be seen that the predicted MSE corresponds very well with the simulated MSE, even during the echo path change. In Fig. 3, the results are shown when the echo path changes from dispersive to sparse.

Now we assess the performance under few different first order Markov systems. The parameters were  $\mu = 0.7$ ,  $\sigma_s^2 = 1$  and  $\sigma_x^2 = 10^{-3}$ , while the other parameters were equal to those used in the previous experiment. The sparse and dispersive time-varying systems were modeled by initializing  $\mathbf{h}(0)$  to the sparse impulse response shown in Fig. 1(a) and the dispersive impulse response shown in Fig. 1(b). By setting the time-varying rate  $\varepsilon$  to  $1 - 10^{-9}$  for the sparse case and  $1 - 10^{-7}$  for the dispersive case, the systems change slowly over time such that the sparseness measure in the first 3 seconds of the sparse response ranges between 0.69 and 0.83 and of the dispersive response ranges between 0.34 and 0.59. As shown in Fig. 4, the MSE can be accurately predicted by (9). We also observed that the predicted MSE slightly deviates from the simulated MSE for the sparse time-varying system, during the initial stage. This is attributed in [8] to the assumption in (14).

Fig. 5 shows the MSEs for different  $\varepsilon$ , including  $\varepsilon = 1$ , which models a time-invariant system, and  $\varepsilon = 1 - 10^{-8}$  which models an equivalent scenario of a source moving approximately at  $0.35 \text{ ms}^{-1}$  in a room dimensions of  $8 \times 10 \times 3 \text{ m}$ . In all cases,  $\mathbf{h}(0)$  was initialized to a sparse impulse response. For these values of  $\varepsilon$  the predicted



**Fig. 3.** MSE of IPNLMS with a sudden echo path change at 10 s. Impulse response is changed from that shown from Fig. 1 (b) to (a) and  $\mu = 0.1$ ,  $\alpha = -0.75$ ,  $\sigma_x^2 = 10^{-2}$ ,  $\sigma_w^2 = 10^{-6}$ ,  $\delta = \delta_{ip} = 10^{-4}$ .

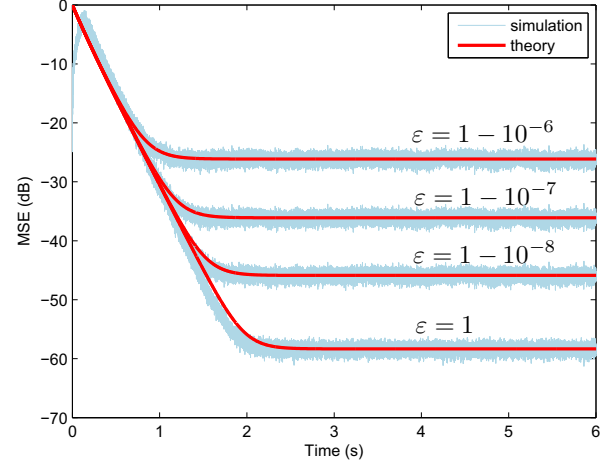


**Fig. 4.** MSE of IPNLMS for a sparse and a dispersive time-varying systems with  $\mu = 0.7$ ,  $\alpha = -0.75$ ,  $\varepsilon = 1 - 10^{-9}$ ,  $\sigma_x^2 = 10^{-3}$ ,  $\sigma_w^2 = 10^{-6}$ ,  $\sigma_s^2 = 1$ ,  $\delta = \delta_{ip} = 10^{-4}$ .

MSE provides results close to the MSE obtained by the simulations. In addition, we notice that the steady-state MSE increases when  $\varepsilon$  decreases (i.e. the system becomes more time-variant).

## 5. CONCLUSION

A performance analysis has been presented for IPNLMS, one of the best known sparse adaptive filtering algorithms. The analysis considers the tracking case in which the unknown system to be identified is not only sparse and dispersive but also time-varying. The analysis has been validated against simulation results in the context of AEC and shown to be accurate. The cases of step-changes in the echo path as well as slowly time-varying echo paths have been included in the study with varying levels of sparseness.



**Fig. 5.** MSE of IPNLMS for varying  $\varepsilon$  with  $\mu = 0.7$ ,  $\alpha = -0.75$ ,  $\sigma_x^2 = 10^{-3}$ ,  $\sigma_w^2 = 10^{-6}$ ,  $\sigma_s^2 = 1$ ,  $\delta = \delta_{ip} = 10^{-4}$ .

## 6. REFERENCES

- [1] A. Gilloire, "Experiments with sub-band acoustic echo cancellers for teleconferencing," in *Proc. IEEE Int. Conf. Acoustics Speech Signal Processing*, vol. 12, 1987, pp. 2141–2144.
- [2] D. L. Duttweiler, "Proportionate normalized least mean square adaptation in echo cancellers," *IEEE Trans. Speech Audio Processing*, vol. 8, no. 5, pp. 508–518, Sep. 2000.
- [3] J. Benesty and S. L. Gay, "An improved PNLMS algorithm," in *Proc. IEEE Int. Conf. Acoustics Speech Signal Processing*, vol. 2, 2002, pp. 1881–1884.
- [4] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Amer.*, vol. 65, no. 4, pp. 943–950, Apr. 1979.
- [5] N. J. Bershad, S. McLaughlin, and C. F. N. Cowan, "Performance comparison of RLS and LMS algorithms for tracking a first order Markov communications channel," in *Proc. IEEE Int. Symposium on Circuits and Syst.*, 1990, pp. 266–270.
- [6] A. W. H. Khong and P. A. Naylor, "Selective-tap adaptive filtering with performance analysis for identification of time-varying systems," *IEEE Trans. Audio, Speech, and Language Processing*, vol. 15, pp. 1681–1695, July 2007.
- [7] B. Widrow, J. M. McCool, M. G. Larimore, and C. R. Johnson, "Stationary and non-stationary learning characteristics of the LMS adaptive filter," in *Proc. IEEE*, vol. 64, no. 8, 1976, pp. 1151–1162.
- [8] K. Wagner and M. Doroslovacki, "Towards analytical convergence analysis of proportion-type NLMS algorithms," in *Proc. IEEE Int. Conf. Acoustics Speech Signal Processing*, 2008, pp. 3825–3828.
- [9] S. Haykin, *Adaptive Filter Theory*, 4th ed., ser. Information and System Science. Prentice Hall, 2002.
- [10] A. H. Sayed, *Fundamentals of adaptive filtering*. Wiley, 2003.
- [11] J. Benesty, Y. A. Huang, J. Chen, and P. A. Naylor, "Adaptive algorithms for the identification of sparse impulse responses," in *Selected methods for acoustic echo and noise control*, E. Hänsler and G. Schmidt, Eds. Springer, 2006, ch. 5, pp. 125–153.