DIRECTION-OF-ARRIVAL ESTIMATION USING ACOUSTIC VECTOR SENSORS IN THE PRESENCE OF NOISE

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ABSTRACT

A vector-sensor consisting of a monopole sensor collocated with orthogonally oriented dipole sensors can be used for direction-of-arrival (DOA) estimation. A method is proposed to estimate the DOA based on the direction of maximum power. Algorithms mentioned in earlier works are shown to be special cases of the proposed method. An iterative algorithm based on the principal of gradient ascent is presented for the solution of the maximum power problem. The proposed maximum-power method is shown to approach the Cramér-Rao lower bound (CRLB) with a suitable choice of parameter.

***Index Terms*—** DOA estimation, acoustic vector-sensors, angular error, Cramér-Rao lower bound

1. INTRODUCTION

In many scenarios, an estimation of the DOA of a source is necessary. Notable applications requiring DOA include navigation, surveillance, and beamforming; as well as target acquisition and tracking geared towards steering of automated cameras. The presence of noise (originating from the sensor or the environment), signal reflection, and reverberation all induce complications in the problem of DOA estimation. The accuracy of a given estimate can be evaluated by the angular error (AE), which is defined as the angle by which the estimate deviates from the true DOA. In certain scenarios, it is possible to obtain an unbiased DOA estimate which consistently converges to the true DOA value as the number of available measurements in time increases [1]. The rate of convergence may be described by the mean square angular error (MSAE). Estimation of DOA in reverberant environments and the resulting bias, have been investigated in [2] in the context of an intensity based algorithm.

In this paper we propose a new method for DOA estimation based on a sensor consisting of a monopole collocated with orthogonally oriented dipole elements, also known as a vector-sensor. Such a device may be implemented in a number of ways. In acoustical settings, the pressure of a plane wave is independent of its DOA, and an omnidirectional microphone consequently serves as a monopole. The wave's particle velocity components maintain a dipole directivity pattern, hence particle velocity sensors [3] may function as dipole elements (after appropriate scaling). Alternatively, differential microphones may be used to construct dipole directivity patterns. The latter are known to posses a high white noise gain (WNG), accentuating the importance of efficient performance in the presence of noise.

In [1], the CRLB for the MSAE of a vector-sensor operating in a scenario with uncorrelated Gaussian source and noise signals was calculated. Two different algorithms for DOA estimation algorithms were proposed, and their MSAE computed. In either case, the MSAE does not achieve the CRLB. In the current study, a DOA estimation method based on the direction of maximum power is proposed. Algorithms mentioned in earlier works [1, 4] are shown to be special instances of the proposed method. An iterative algorithm based on the principal of gradient ascent is presented for the solution of the maximum power problem. The proposed method is also shown to approach the CRLB with a suitable choice of parameter.

2. PROBLEM FORMULATION AND PRELIMINARIES

A signal propagates through space from a source towards a receiver. We wish to determine its DOA, which is represented by the unit vector $\mathbf{u}$ that points from the receiver to the source. The signal originates in the far-field and may be considered to be a plane wave at the location of the receiver. Based on the received measurements a DOA estimate $\hat{\mathbf{u}}$ (also being a unit vector) is produced.

2.1. Receiver Specification

The receiver consists of a monopole sensor element which is collocated with three dipole elements. The measurement of each sensor element can be expressed as $Ds[n] + e[n]$, where D is a directivity pattern, $s[n]$ is the signal, $e[n]$ signifies additive noise which may possibly induce error, and n represents discrete time. The monopole directivity pattern is uniform $D_{\text{mon}} = 1$, and the dipole directivity pattern is:

$$D_{\text{dip}} = \mathbf{q}^T \mathbf{u}\,, \tag{1}$$

where $\mathbf{q}$ is a unit vector representing the orientation of the dipole. The dipoles are oriented along the Cartesian axes: $\mathbf{q}_x = [1\ 0\ 0]^T$, $\mathbf{q}_y = [0\ 1\ 0]^T$ and $\mathbf{q}_z = [0\ 0\ 1]^T$. In the sequel, the monopole measurements will be represented by $p[n]$ and the dipoles measurements by $\mathbf{v}[n] = [v_x[n]\ v_y[n]\ v_z[n]]^T$ (corresponding to pressure and scaled particle velocity from which these quantities may be obtained). From the above, the measured signals are:

$$p[n] = s[n] + e_p[n] \tag{2a}$$

$$\mathbf{v}[n] = s[n]\,\mathbf{u} + \mathbf{e}_v[n]\,, \tag{2b}$$

where $e_p[n]$ is the additive monopole noise, and

$$\mathbf{e}_v[n] = \left[e_{v_x}[n]\ e_{v_y}[n]\ e_{v_z}[n]\right]^T$$

is the additive dipole noise.

2.2. Characterization of Signal and Noise

The signal $s[n]$ and noise components $e_p[n]$, $\mathbf{e}_v[n]$ are assumed to be stochastic processes which are wide sense stationary (WSS) and uncorrelated in time. They have zero mean and respective variances of σ_s^2, $\sigma_{e_p}^2$, and $\sigma_{e_v}^2$. Furthermore, the processes $s[n]$, $e_p[n]$, $e_{v_x}[n]$, $e_{v_y}[n]$, and $e_{v_z}[n]$ are all mutually uncorrelated. The above noise properties can be described succinctly as:

$$E\left\{\begin{bmatrix} e_p[n] \\ \mathbf{e}_v[n] \end{bmatrix}\right\} = \mathbf{0}_{4\times 1} \tag{3a}$$

$$E\left\{\begin{bmatrix} e_p[n] \\ \mathbf{e}_v[n] \end{bmatrix}\begin{bmatrix} e_p[n] \\ \mathbf{e}_v[n] \end{bmatrix}^T\right\} = \begin{bmatrix} \sigma_{e_p}^2 & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{3\times 1} & \sigma_{e_v}^2 \cdot \mathbf{I}_{3\times 3} \end{bmatrix} \delta[n-m]\,, \tag{3b}$$

where $\mathbf{0}_{j\times k}$ and $\mathbf{I}_{j\times k}$ respectively represent zero vectors and identity matrices of the designated dimensions, and $\delta[\circ]$ represents the Kronecker delta function.

Scenarios in which such noise characteristics can arise, comprise of settings of internal sensor noise and isotropic ambient noise. Reverberative environments, do not subscribe to these characteristics and are not discussed in the present study.

2.3. Quantities for Assessing Estimation Performance

The accuracy of DOA estimates can be evaluated by the AE which is the angle by which $\hat{\mathbf{u}}$ deviates from $\mathbf{u}$, defined formally as,

$$\text{AE} \equiv 2\sin^{-1}\left(\frac{\|\hat{\mathbf{u}} - \mathbf{u}\|}{2}\right)\,, \tag{4}$$

where $\| \circ \|$ is the Euclidian norm. The rate of convergence to the true value is described by the MSAE which is defined as,

$$\text{MSAE} \equiv \lim_{N\to\infty} (N E\{\text{AE}^2\})\,, \tag{5}$$

where N is the number of discrete time instants measured.

3. ESTIMATION BY DIRECTION OF MAXIMUM POWER

This section describes how the beampattern of a vector-sensor may be steered and shaped. This makes a search for direction of maximum power possible, introducing a method for DOA estimation.

3.1. Dipole steering and beam shaping

Although $v_x[n]$, $v_y[n]$, and $v_z[n]$ have fixed orientations along the Cartesian axes, they may be used to obtain the equivalent output of a dipole element $v_{\mathbf{q}}[n]$ steered towards an arbitrary direction $\mathbf{q}$ [5]. This is obtained by performing the following linear combination:

$$v_{\mathbf{q}}[n] = \mathbf{q}^T\mathbf{v}[n] = \mathbf{q}^T\mathbf{u}\,s[n] + \mathbf{q}^T\mathbf{e}_v[n]\,. \tag{6}$$

The right side, which can be verified from (2b), consists of a signal term $\mathbf{q}^T\mathbf{u}\,s[n]$ and a noise term $\mathbf{q}^T\mathbf{e}_v[n]$. The signal term complies with the form of a dipole directivity pattern (1) and the noise term has zero mean and a variance of $\sigma_{e_v}^2$ as can be ascertained from (3). Thus, applying linear weights to multiple fixed dipoles obtains the same effect as mechanical steering of a single dipole.

It is possible to produce various first-order directivity patterns [6] by combining dipole and monopole outputs as follows:

$$y_{\mathbf{q}}[n] = \alpha p[n] + (1-\alpha) v_{\mathbf{q}}[n]\,. \tag{7}$$

This weighting scheme ensures a unity boresight response. Setting the parameter α to 1 produces a pure monopole response, while setting it to 0, produces a pure dipole response. Other values which combine both monopole and dipole responses produce a shape known as a limaçon. The value 0.5 corresponds to cardioid directivity. Subcardioid and supercardioid patterns are produced by setting α within the respective ranges $(0.5,\,1)$ and $(0,\,0.5)$. The parameter α must satisfy $0 \le \alpha \le 1$ so that the direction of maximum response will be the boresight.

3.2. Proposed Maximum Power Method

It is reasonable to expect that steering a beam-pattern towards the direction-of-arrival should generally increase the power of the receiver output. In the suggested method for DOA estimation, a particular beam-pattern (i.e. α parameter) is chosen. Then $\hat{\mathbf{u}}$ is determined by finding the direction (unit vector $\mathbf{q}$) which attains the highest power. Formally, that is $\hat{\mathbf{u}} = \operatorname{argmax}_{\mathbf{q}}\{\frac{1}{N}\sum_0^{N-1} y_q^2[n]\}$ or more explicitly:

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\operatorname{argmax}}\left\{\frac{1}{N}\sum_{n=0}^{N-1}\Big(\alpha p[n] + (1-\alpha)\mathbf{q}^T\mathbf{v}[n]\Big)^2\right\} \tag{8a}$$

$$\text{subject to}\quad \mathbf{q}^T\mathbf{q} = 1\,. \tag{8b}$$

Ideally, it is desirable that (8) be solved without resorting to a grid search. The question of choosing an optimal value for α is postponed to Sec. 5.

Expanding the target function (braced term) of (8a) yields:

$$\frac{1}{N}\sum_{n=0}^{N-1}\Big(\alpha^2 p[n] + (1-\alpha)^2\mathbf{q}^T\mathbf{v}[n]\mathbf{v}^T[n]\mathbf{q} + 2\alpha(1-\alpha)\mathbf{q}^T\mathbf{v}[n]p[n]\Big)\,. \tag{9}$$

The term $\alpha^2 p^2[n]$ may be dropped as it is a constant with respect to $\mathbf{q}$ and has no effect upon maximization. For similar reasons it is permissible to multiply by any positive constant. A convenient choice is $(1-\alpha)^{-1}/2$, yielding the equivalent target function:

$$T(\mathbf{q}) = \alpha\,\mathbf{q}^T\hat{\mathbf{r}}_{pv} + \frac{1-\alpha}{2}\,\mathbf{q}^T\hat{\mathbf{R}}_{vv}\mathbf{q}\,, \tag{10}$$

where $\hat{\mathbf{R}}_{vv}$ and $\hat{\mathbf{r}}_{pv}$ represent the estimates of the covariance matrix and vector:

$$\hat{\mathbf{R}}_{vv} = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{v}[n]\mathbf{v}^T[n]$$

$$\hat{\mathbf{r}}_{pv} = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{v}[n]p[n]\,. \tag{11}$$

3.3. Relation to existing methods

We turn to examine two particular cases: i) $\alpha \to 1^-$, and ii) $\alpha = 0$.

i) For a near-monopole pattern, α approaches 1 (from below) and $(1-\alpha)$ approaches 0 (from above). Hence, the quadratic term $\mathbf{q}^T\hat{\mathbf{R}}_{vv}\mathbf{q}$ becomes in respect inconsequential. The optimization problem becomes:

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\operatorname{argmax}}\ \mathbf{q}^T\hat{\mathbf{r}}_{pv}$$

$$\text{s.t.}\quad \mathbf{q}^T\mathbf{q} = 1\,. \tag{12}$$

The solution to this problem is $\hat{\mathbf{u}} = \hat{\mathbf{r}}_{pv}/\|\hat{\mathbf{r}}_{pv}\|$ which will be denoted $\hat{\mathbf{u}}_{\text{mon}}$. It must be stressed that α is *not* exactly 1, i.e. the beam-pattern is not a *true* monopole. Were that the case, the power output would be identical in all directions rendering DOA estimation impossible. Rather, the beam-pattern is *nearly* monopole with α approaching 1 (from below), but never quite obtaining that value. The overwhelming majority of power received is contained in the term $\alpha^2 p^2[n]$ of (9) which has no impact on maximization, lending to a false allusion of uniform directivity.

An estimation algorithm corresponding to the above was suggested by Nehorai and Paldi [1] under the name "Intensity-Based Algorithm". Additionally, Davies [4] discusses properties of such an algorithm in the two-dimensional case. This is in essence a special case of the general maximum power method.

ii) For a dipole beam-pattern $\alpha = 0$ and so the term containing $\hat{\mathbf{r}}_{pv}$ is canceled. With the remaining expression, the optimization problem becomes:

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\text{argmax}}\ \mathbf{q}^T \hat{\mathbf{R}}_{vv} \mathbf{q}$$
$$\text{s.t.} \quad \mathbf{q}^T \mathbf{q} = 1\,. \tag{13}$$

The well-known solution to this problem (denoted $\hat{\mathbf{u}}_{\text{dip}}$) selects $\hat{\mathbf{u}}$ to be the eigenvector of $\hat{\mathbf{R}}_{vv}$ corresponding to the maximum eigenvalue. To comply with the constraint, this eigenvalue must have a unity Euclidian norm. An ambiguity of 180° remains since multiplying the solution by -1 also yields a valid solution. This is not at all surprising since the dipole is bidirectional possessing a "figure eight" pattern. The ambiguity can be resolved by constraining the solution to lie on the same hemisphere as $\hat{\mathbf{u}}_{\text{mon}}$, i.e. $\mathbf{q}^T \hat{\mathbf{u}}_{\text{mon}} \geq 0$. The algorithm proposed in [1] under the name "Velocity-Covariance-Based Algorithm" corresponds to the above and may be seen as a particular instance of the general approach taken herein.

4. ITERATIVE GRADIENT BASED SOLUTION

When $0 < \alpha < 1$, the beam-pattern is neither monopole nor dipole but rather a limaçon. We wish to solve:

$$\hat{\mathbf{u}}_{\text{gen}} = \underset{\mathbf{q}}{\text{argmax}}\ T(\mathbf{q}) \tag{14a}$$
$$\text{s.t.} \quad \mathbf{q}^T \mathbf{q} = 1\,. \tag{14b}$$

A direct solution for the general limaçon pattern which involves both a linear and a quadratic term is not manifest. One method is to formulate an approximation for the maximization problem. Alternatively, one may use numerical techniques which iteratively approach the desired solution.

We propose a solution based on the principal of gradient ascent. The target function $T(\mathbf{q})$ of (10) must be maximized in order to obtain maximum power . Its gradient,

$$\nabla_{\mathbf{q}} T(\mathbf{q}) = \alpha\, \hat{\mathbf{r}}_{pv} + (1-\alpha)\hat{\mathbf{R}}_{vv}\mathbf{q}\,, \tag{15}$$

corresponds to the direction of steepest ascent. It follows that starting with some initial vector $\mathbf{q}_0$, a perturbation in the direction of the gradient should bring about an increase in the value of the target function. Since the perturbed vector is likely to violate the constraint (14b), normalization is performed in a fashion similar to the celebrated "Scaled Projection Algorithm" of Cox [7]. Iterative repetition of this perturbation-normalization process brings about progressively better candidates for optimization of $T(\mathbf{q})$.

To complete the algorithm, two more issues need to be addressed. An initial value $\mathbf{q}_0$ needs to be specified, and a termination criteria determined. The initial value $\mathbf{q}_0 = \hat{\mathbf{u}}_{\text{mon}}$, proves to be a judicious choice. Since both $\hat{\mathbf{u}}_{\text{gen}}$ and $\hat{\mathbf{u}}_{\text{mon}}$ are estimates of $\mathbf{u}$, they are likely to be already close. Hence fewer iterations of the algorithm would be necessary to satisfactorily approach the desired value, or alternatively for the same number of iterations a closer approximation will be found.

The algorithm can be set to terminate after a fixed number of steps. Alternatively, the algorithm may be set to terminate when the change induced by the current step is less than a given threshold: $\|\mathbf{q}_{\text{current}} - \mathbf{q}_{\text{previous}}\|^2 < \epsilon^2$.

The algorithm is summarized in the schematic entitled Algorithm 1.

Algorithm 1: Iterative algorithm for finding direction of maximum power.

Input: $\hat{\mathbf{R}}_{vv}, \hat{\mathbf{r}}_{pv}, \alpha$
$\mathbf{q}_0 := \hat{\mathbf{u}} = \hat{\mathbf{r}}_{pv}/\|\hat{\mathbf{r}}_{pv}\|$
$k := 0$
$K :=$ maximum number of iterations
$\epsilon :=$ tolerance parameter
$\mu :=$ step size parameter
repeat
  $\mathbf{q}_{k+1} := \mathbf{q}_k + \mu\big(\alpha\, \hat{\mathbf{r}}_{pv} + (1-\alpha)\hat{\mathbf{R}}_{vv}\mathbf{q}_k\big)$
  $\mathbf{q}_{k+1} := \mathbf{q}_{k+1}/\|\mathbf{q}_{k+1}\|$
  $k := k+1$
until *($k = K$)* or alternatively *($\|\mathbf{q}_k - \mathbf{q}_{k-1}\|^2 < \epsilon^2$)*
Output: $\hat{\mathbf{u}} := \mathbf{q}_k$

The algorithm has the advantage of being relatively simple computationally. Furthermore, the algorithm's iterative nature lends itself to extension towards a recursive implementation. Such an implementation is useful for real-time processing of a constantly updated data-stream of measurements pertaining to a (possibly moving) target. In the case of a moving target, (9) needs to be modified to a weighted average which deemphasizes measurements from the distant past, inducing similar changes in (11).

5. EVALUATION

The MSAE for algorithms corresponding monopole and dipole receivers was computed in [1]. Also, for the case of Gaussian processes, the CRLB was computed for a sensor consisting of a monopole collocated with orthogonal dipoles. The results which have been modified for the time domain (as opposed to the complex phasor signals) are reproduced here in a slightly different formulation:

$$\text{MSAE}_{\text{mon}} = \frac{\sigma_{e_v}^2}{\sigma_s^2}\left(1 + \frac{\sigma_{e_p}^2}{\sigma_s^2}\right) \tag{16a}$$

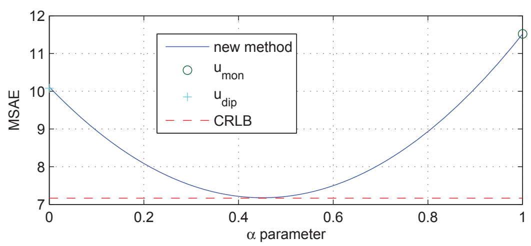
$$\text{MSAE}_{\text{dip}} = \frac{\sigma_{e_v}^2}{\sigma_s^2}\left(1 + \frac{\sigma_{e_v}^2}{\sigma_s^2}\right) \tag{16b}$$

$$\text{MSAE}_{\text{CRLB}} = \frac{\sigma_{e_v}^2}{\sigma_s^2}\left(1 + \frac{(\sigma_{e_p||e_v}^2)}{\sigma_s^2}\right)\,. \tag{16c}$$

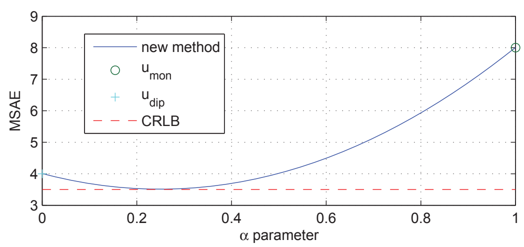
The symbol $\sigma_{e_p||e_v}^2$ denotes the quantity: $\sigma_{e_p||e_v}^2 = (\sigma_{e_p}^{-2} + \sigma_{e_v}^{-2})^{-1}$.

In order to evaluate the proposed method and its accompanying algorithm, a series of Monte-Carlo simulations were conducted. The DOA was randomly generated from a uniform distribution across the surface of the unit-sphere, and the signals $s[n]$, $e_p[n]$, and $\mathbf{e}_v[n]$ were randomly drawn from Gaussian distributions. The above quantities were used to produce the measurements $p[n]$ and $\mathbf{v}[n]$ using (2). Afterwards, the estimates $\hat{\mathbf{u}}_{\text{mon}}$ and $\hat{\mathbf{u}}_{\text{dip}}$ were calculated as described in Sec. 3.3. The new estimator $\hat{\mathbf{u}}_{\text{gen}}$ was computed with the algorithm described in Alg. 1 for a spread of parameter values $0 \le \alpha \le 1$.

This entire process was repeated MC = 100,000 times; the number of time samples per simulation set to N = 8,000; and the specified signal powers were alternately: (a) $\sigma_s^2 = 0.5$, $\sigma_{e_p}^2 = 1.1$, $\sigma_{e_v}^2 = 0.9$, and (b) $\sigma_s^2 = 0.5$, $\sigma_{e_p}^2 = 1.5$, $\sigma_{e_v}^2 = 0.5$. Based on the results of the procedure described above, graphs of the MSAE (sample-mean) of $\hat{\mathbf{u}}_{\text{gen}}$ are plotted as a function of α in Fig. 1. The theoretical values for MSAE_{mon}, MSAE_{dip}, and $\text{MSAE}_{\text{CRLB}}$ from (16) are displayed for comparison.



(a) $\sigma_s^2 = 0.5, \sigma_{e_p}^2 = 1.1, \sigma_{e_v}^2 = 0.9$.



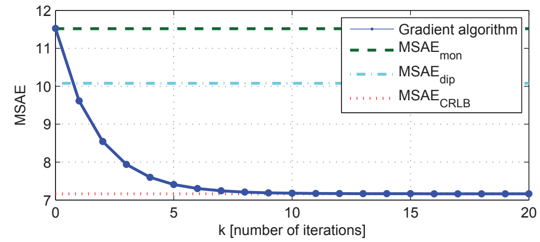
(b) $\sigma_s^2 = 0.5, \sigma_{e_p}^2 = 1.5, \sigma_{e_v}^2 = 0.5$.

Fig. 1: MSAE (sample-mean) of maximum power method compared with analytical values of MSAE_{mon}, MSAE_{dip}, and $\text{MSAE}_{\text{CRLB}}$.

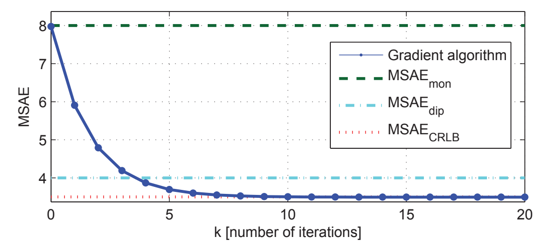
The figure demonstrates the potential improvement in MSAE brought about by using a limaçon pattern (MSAE_{gen}) as opposed to a near-monopole or pure dipole. A proper choice of α is critical in producing enhanced estimation. In these particular cases (a) $\alpha = 0.45$, and (b) $\alpha = 0.25$, appear to achieve the CRLB.

In Fig. 2, the sample-mean MSAE as a function of the iteration number is shown and compared with the theoretical values of MSAE_{mon}, MSAE_{dip}, and $\text{MSAE}_{\text{CRLB}}$. The simulations were conducted in a similar fashion to that described above. To obtain best results, the parameter α was set to $\alpha = 0.45$ and $\alpha = 0.25$ for the respective cases. The step-size parameter was set as $\mu = 1$.

Inspection of the equations (16) indicates that when $\sigma_{e_p}^2 \gg \sigma_{e_v}^2$ or conversely $\sigma_{e_v}^2 \gg \sigma_{e_p}^2$ then $\hat{\mathbf{u}}_{\text{mon}}$ and $\hat{\mathbf{u}}_{\text{dip}}$ will respectively approach the CRLB [1]. Also, for scenarios of high SNR ($\sigma_s^2 \gg \min\{\sigma_{e_p}^2, \sigma_{e_v}^2\}$), any improvement in MSAE could only be marginal. Hence, the maximum-power method is most useful when $\sigma_{e_p}^2$ and $\sigma_{e_v}^2$ are of similar magnitude and the SNR is relatively low.



(a) $\sigma_s^2 = 0.5, \sigma_{e_p}^2 = 1.1, \sigma_{e_v}^2 = 0.9, \alpha = 0.45$.



(b) $\sigma_s^2 = 0.5, \sigma_{e_p}^2 = 1.5, \sigma_{e_v}^2 = 0.5, \alpha = 0.25$.

Fig. 2: Sample-mean MSAE of gradient algorithm.

6. CONCLUSIONS

A method for DOA estimation based on the direction of maximum power has been proposed. This method, which can be seen as a generalization of previous methods, was shown to approach the CRLB with a proper choice of parameter. A computationally simple algorithm was presented which can iteratively approach the direction of maximum power. The algorithm is suitable for real time computation and may readily be extended to tracking problems.

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